AD MTF271 Group discussions at Zoom lectures

MTF271 Lecture 1

1. How is the buoyancy term, ρg_i, re-written in incompressible flow?
2. Show the principles how to derive the transport equation for $\overline{v'_i v'_j}$, Eq. 11.11 (see Section 11.2 on p. 122)
3. What is the expression for the total heat flux that appears in the $\bar{\theta}$ equation? Compare with the total shear stress in the momentum equation. Where is the viscous term large? At what x_2 (and x_2^+) does the turbulent part become larger than the viscous part?
4. Which terms in the $\overline{v'_i v'_j}$ equation need to be modeled? Explain the physical meaning of the different terms in the $\overline{v'_i v'_j}$ equation.
5. Show how the turbulent diffusion (i.e. the term which includes the triple correlation) in the k equation is modeled.

MTF271 Lecture 2

1. How is the production term modeled in the $k-\varepsilon$ model? Show how it can be expressed in $\bar{s}_{ij}$
2. Given the modeled k equation, derive the modeled ε equation.
3. Derive the Boussinesq assumption (see Section 11.6). How is the turbulent heat flux, $\overline{v_i'\theta'}$, modeled in the Boussinesq approach?
4. Discuss and show how the dissipation term, ε_{ij}, is modeled.
5. Use physical reasoning to derive a model for the diagonal components of the pressure-strain term (slow part).
6. Using physical reasoning, the model for the pressure-strain term above is formulated only for the normal stresses. Show that if the model is expressed in the principal directions, then a model for the shear stress is also obtained.

MTF271 Lecture 3

1. Describe the derivation of the algebraic stress model (ASM). What main assumption is made?

2. Show the physical reasoning leading to the modeled slow pressure strain term, $\Phi_{22,1w}$, for wall effects. What sign does it have? Give also the expressions for $\Phi_{11,1w}$ and $\Phi_{33,1w}$

3. The modeled slow and rapid pressure strain term read $\Phi_{ij,1} = -c_1\rho\frac{\varepsilon}{k}\left(\overline{v_i'v_j'} - \frac{2}{3}\delta_{ij}k\right)$ and $\Phi_{ij,2} = -c_2\left(P_{ij} - \frac{2}{3}\delta_{ij}P^k\right)$, respectively. Give the expression for the production terms, modeled pressure-strain terms and modeled dissipation terms for a simple boundary layer. In some stress equations there is no production terms nor any dissipation term. How come? Which is the main source term (or sink term) in these equations?

4. Describe the physical effect of stable stratification and unstable stratification on turbulence.

5. Consider buoyancy-dominated flow with x_3 vertically upwards. The production term for the $\overline{v_i'v_j'}$ and the $\overline{v_i'\theta'}$ equations read

$$G_{ij} = -g_i\beta\overline{v_j'\theta'} - g_j\beta\overline{v_i'\theta'}, \quad P_{i\theta} = -\overline{v_i'v_k'}\frac{\partial\bar{\theta}}{\partial x_k}$$

respectively (we assume that the velocity gradient is negligible). Discuss how the Reynolds stress model dampens and increases the vertical fluctuation in stable and unstable stratification, respectively, as it should.

MTF271 Lecture 4

1. Consider streamline curvature for a streamline formed as a circular arc (convex curvature). Show that the turbulence is dampened if $\partial v_\theta/\partial r > 0$ and that it is enhanced if the sign of $\partial v_\theta/\partial r$ is negative.

2. Streamline curvature: now consider a boundary layer where the streamlines are curved away from the wall (concave curvature). Show that the Reynolds stress model gives an enhanced turbulence production (as it should) because of positive feedback between the production terms. Why is the effect of streamline curvature in the $k-\varepsilon$ model much smaller?

3. Consider stagnation flow. Show that in the Reynolds stress model, there is only a small production of turbulence whereas eddy-viscosity models (such as the $k-\varepsilon$ model) give a large production of turbulence.

4. What is a realizability constraint? There are two main realizability constraints, one on the normal and one on the shear stresses: give the form of these constraints.

5. Show that the Boussinesq assumption may give negative normal stresses. In which coordinate system is the risk largest for negative normal stresses?

6. Consider the symmetric strain-rate tensor $\bar{s}_{ij}$. What is the physical meaning of ite eigenvalues?

MTF271 Lecture 5

1. Which equations are solved in the V2F model?
2. What does the acronym SST mean? The SST model is a combination of the $k-\varepsilon$ and the $k-\omega$ model. In which region is each model used and why? How is ω expressed in k and ε?
3. How is the transport equation for ω derived?
4. In the SST model, two blending function, F_1 and F_2, are used; explain what is the object of F_1 or F_2.
5. What is the purpose of the shear stress limiter in the SST model?

MTF271 Lecture 6

1. What characterizes turbulence? Explain the characteristics. What is a turbulent eddy?

2. Explain the cascade process. How large are the largest scales? What is dissipation? What dimensions does it have? Which eddies extract energy from the mean flow? Why are these these eddies "best" at extracting energy from the mean flow?

3. Make a figure of the energy spectrum. The energy spectrum consists of three subregions: which? describe their characteristics. Show the flow of turbulent kinetic energy in the energy spectrum. Given the energy spectrum, $E(\kappa)$, how is the turbulent kinetic energy, k, computed? Use dimensional analysis to derive the $-5/3$ Kolmogorov law.

4. What does isotropic turbulence mean? What about the shear stresses?

5. How is the energy transfer from eddy-to-eddy, ε_κ, estimated? Show how the ratio of the large eddies to the dissipative eddies depends on the Reynolds number (see Eq. 5.16).

MTF271 Lecture 7

1. Show the difference between volume averaging (filtering) in LES and time-averaging in RANS. Show that avereging once or twice is different in RANS and LES.

2. Consider the spatial derivative of the pressure in the filtered Navier-Stokes: show that the derivative can be moved outside the filtering integral (it gives an additional second-order term).

3. The filtered non-linear term has the form

$$\overline{\frac{\partial v_i v_j}{\partial x_j}}$$

Show that it can be re-written as

$$\frac{\partial \bar{v}_i \bar{v}_j}{\partial x_j}$$

giving an additional term

$$-\frac{\partial}{\partial x_j}(\overline{v_i v_j}) + \frac{\partial}{\partial x_j}(\bar{v}_i \bar{v}_j) = -\frac{\partial \tau_{ij}}{\partial x_j}$$

on the right side.

4. Consider a 1D finite volume grid. Carry out a second filtering of $\bar{v}$ at node I and show that $\bar{\bar{v}}_I \neq \bar{v}_I$.

5. Consider the energy spectrum. Show the three different regions (the large energy-containing scales, the $-5/3$ range and the dissipating scales). Where should the cut-off be located? Show where the SGS scales, grid (i.e resolved) scales and the cut-off, κ_c are located in the spectrum.

6. Show how a sinus wave $\sin(\kappa_c x)$ corresponding to cut-off is represented on a grid with two and four cells, respectively. How is κ_c related to the grid size Δx for these cases?

7. Taking guidance from the RANS k equation, formulate the one-equation k_{sgs} equation

8. Consider the energy spectrum and discuss the physical meaning of $P_{k_{sgs}}$ and ε_{sgs}.

9. Discuss the energy path in connection to the source and sink terms in the $\bar{k}$, $\bar{K}$ and the k_{sgs} equations, see Figs. S.4 and S.5.

10. Show how the velocity is decomposed in RANS and LES, both in physical space and wavennumber space (i.e. how are k, $\bar{k}$ and k_{sgs} computed from the energy spectrum?).

MTF271 Lecture 8

1. Consider the dynamic model. What is a test filter?
2. Draw an energy spectrum and show which wavenumber range $\bar{k}$, k_{sgs}, $k_{sgs,test}$ cover.
3. Formulate the Smagorinsky model for the grid filter SGS stress, τ_{ij}, and the test filter SGS stress, T_{ij}.
4. Show that when a first-order upwind schemes is used for the convection term, an additional diffusion term and dissipation terms appear because of a numerical SGS viscosity

MTF271 Lecture 9

1. Explain and discuss the two-point correlation and auto-correlation. How are the integral length and timescale defined?
2. When doing LES, how fine does the mesh need to be in the near-wall region? Why does it need to be that fine?
3. Mention four different ways to estimate the resolution of an LES that you have made; see Section 18.26 Which method is good/bad? Which is best?
4. What is DES? The destruction term in the RANS S-A model reads $\left(\frac{\tilde{\nu}_t}{d}\right)^2$; how is it computed in the S-A DES model?
5. How is the length scale computed in a $k-\varepsilon$ two-equation DES model? Where in a boundary layer does the DES model switch from RANS to LES?
6. The modified (reduced) length scale in two-equation DES models can be introduced in different ways. It it usually introduced in one transport equation. Which one and which term? Apart from this transport equation, it is sometimes used in a another equation. Which one? What is the effect on the modeled, turbulent quantities?

MTF271 Lecture 10

1. Give a brief description of the $k - \omega$ SST DES model.
2. What is DDES? Why was it invented?
3. Describe hybrid LES-RANS based on a one-equation model.
4. Describe the SAS model. How is the von Kármán length scale defined? An additional source term is introduced in the ω equation: what is the form of this term? Describe how this source reduces the turbulent viscosity. When is the term large and small, respectively?

MTF271 Lecture 11

1. What is the physical meaning of f_k in PANS?

2. The PANS equations are given in Eqs. 23.19. Assume that $f_\varepsilon = 1$. Consider the destruction term in the ε equation and the coefficient $C^*_{\varepsilon 2}$. Explain what happens if f_k is reduced from 1 (RANS mode) to $f_k = 0.4$ (LES mode).

3. Consider the PANS model: in the derivation, we assumed that f_k is constant. Relaxing that condition is useful when prescribing k and ε prescribed at inlet and at embedded RANS/LES interfaces. Derive the additional term in the k equation (Eq. 23.24). Show that the sign of the gradient at the inlet (and embedded interface) does indeed give a reduction of k as it should.

MTF271 Lecture 12

1. Give a short description of the method to generate synthetic turbulent inlet fluctuations. What form on the spectrum is assumed? How is the wavenumber, κ_e, for the energy-containing eddies, determined? How are the maximum and minimum wavelengths, κ_{max}, κ_{min}, determined? With this method, the generated shear stress is zero: why? How is the correlation in time achieved?

2. What is embedded LES? Give an example when we may use it.