Identify the first term in Eqs. K.2 and K.3 in the momentum equation for v_1, Eq. K.1. Write out the momentum equation also for $\bar{v}_2$ and identify the other two terms in Eqs. K.2 and K.3. Note that $\mathbf{F}_N$ and $\mathbf{F}_S$ are forces per unit volume ($[N/m^3]$).

Assignment 1.3. Plot the vector field $\mathbf{F}_N$ to find out some features. When $\overline{v_2'^2}$ reaches a maximum or a minimum along a grid line normal to the wall, what happens with the vector field? Zoom-in on interesting regions.

Assignment 1.4. Plot also vector fields of the shear stress, $\mathbf{F}_S$ (see Eq. K.3), the pressure gradient and the viscous terms. Zoom up in interesting regions. Anything interesting?

K.2.2 The turbulent kinetic energy equation

The exact transport equation for the turbulent kinetic energy, k, reads

$$\begin{aligned}\frac{\partial}{\partial x_j}\left(\bar{v}_j k\right) &= \nu \frac{\partial^2 k}{\partial x_j \partial x_j} + P_k + D_k - \varepsilon \\ P_k &= -\overline{v_i' v_j'}\frac{\partial \bar{v}_i}{\partial x_j}\end{aligned} \tag{K.4}$$

Assignment 1.5. Plot the production term along the two grid lines. Explain why it is large at some locations and small at others. The production term consists of the sum of four terms, two of which involve the shear stress while the other include the normal stresses. Compare the contributions due the shear stress and the normal stresses.

Assignment 1.6. Plot the dissipation and compare it with the production. Do you have local equilibrium (i.e. $P^k \simeq \varepsilon$) anywhere?

K.2.3 The Reynolds stress equations

The modelled transport equation for the Reynolds stresses can be written as

$$\begin{aligned}\frac{\partial}{\partial x_k}\left(\bar{v}_k \overline{v_i' v_j'}\right) &= \nu \frac{\partial^2 \overline{v_i' v_j'}}{\partial x_k \partial x_k} + P_{ij} + \Phi_{ij} + D_{ij} - \varepsilon_{ij} \\ P_{ij} &= -\overline{v_i' v_k'}\frac{\partial \bar{v}_j}{\partial x_k} - \overline{v_j' v_k'}\frac{\partial \bar{v}_i}{\partial x_k}\end{aligned} \tag{K.5}$$

The pressure-strain term, Φ_{ij}, and the diffusion term, D_{ij}, need to be modelled. Here we use the models in Eqs. 11.87, 11.53, 11.86, 11.91 and 11.92.

1. In the damping function, f (see Eq. 11.88), $|x_i - x_{i,n}|$ denotes the distance to the nearest wall. If, for example, the lower wall is the closest wall to node (I, J), then

$$|x_i - x_{i,n}| = \left\{(x(I,J) - x(I,1))^2 + (y(I,J) - y(I,1))^2\right\}^{1/2} \tag{K.6}$$

2. If we assume, again, that the lower wall is the closest wall to cell (I, J) and that the lower wall is horizontal, then $n_{i,w} = (0, 1)$. To compute $n_{i,w}$ for the general case (see Eqs. 11.91 and 11.92), compute first the vector which is parallel to the wall, $s_{i,w}$, and compute then $n_{i,w}$ from $s_{i,w}$ (see Eq. K.11)