T.7 SAS turbulent length scales

Compute the 1D von Kármán length scale defined as

$$L_{vK,1D} = \kappa \left| \frac{\partial \langle \bar{v}_1 \rangle / \partial x_2}{\partial^2 \langle \bar{v}_1 \rangle / \partial x_2^2} \right| \tag{T.1}$$

Note that you should take the derivatives of the *averaged* $\bar{v}_1$ velocity, i.e. of $\langle \bar{v}_1 \rangle$. Zoom up near the wall. How does it behave (i.e. what is n in $\mathcal{O}(x_2^n)$? What should n be?

Compare that with the von Kármán length scale defined from instantaneous $\bar{v}_1$, i.e.

$$L_{vK,1D,inst} = \kappa \left| \left\langle \frac{\partial \bar{v}_1 / \partial x_2}{\partial^2 \bar{v}_1 / \partial x_2^2} \right\rangle \right| \tag{T.2}$$

How does it compare with $L_{vK,1D}$?

When we're doing real 3D simulations, the first and second derivative must be defined in 3D. One way of defining the von Kármán length scale in 3D is [158, 159]

$$\begin{aligned} L_{vK,3D,inst} &= \kappa \left| \frac{S}{U''} \right| \\ S &= (2\bar{s}_{ij}\bar{s}_{ij})^{0.5} \\ U'' &= \left(\frac{\partial^2 \bar{v}_i}{\partial x_j \partial x_j} \frac{\partial^2 \bar{v}_i}{\partial x_j \partial x_j} \right)^{0.5} \end{aligned} \tag{T.3}$$

The second derivative is then computed as

$$\begin{aligned} U''^2 &= \left(\frac{\partial^2 \bar{v}_1}{\partial x_1^2} + \frac{\partial^2 \bar{v}_1}{\partial x_2^2} + \frac{\partial^2 \bar{v}_1}{\partial x_3^2} \right)^2 \\ &+ \left(\frac{\partial^2 \bar{v}_2}{\partial x_1^2} + \frac{\partial^2 \bar{v}_2}{\partial x_2^2} + \frac{\partial^2 \bar{v}_2}{\partial x_3^2} \right)^2 \\ &+ \left(\frac{\partial^2 \bar{v}_3}{\partial x_1^2} + \frac{\partial^2 \bar{v}_3}{\partial x_2^2} + \frac{\partial^2 \bar{v}_3}{\partial x_3^2} \right)^2 \end{aligned} \tag{T.4}$$

Plot the von Kármán length scale using Eqs. T.3 and T.4. Compare them with Eq. T.1. What's the difference? What effect do the different length scales give for P_{SAS} (i.e. T_1 in Eq. 22.5) and what effect does it give to ω?

Another way to compute the second derivative is [204]

$$U'' = \left(\frac{\partial^2 \bar{v}_i}{\partial x_j \partial x_k} \frac{\partial^2 \bar{v}_i}{\partial x_j \partial x_k} \right)^{0.5} \tag{T.5}$$