Since $f_k = 0.4$, it means that the turbulent diffusion in the k and ε equations are $1/0.4^2 \simeq 6$ times larger in [168] than in [203]. The consequence is that peaks in k and ε (and also ν_t) are reduced in the former case compared to the latter (this is the physical role played by diffusion: it transports k from regions of high k to regions of low k). This explains why the peaks of k are much larger in [203] compared to in [168].

Hence, in the original PANS model (Eq U.3), the RANS turbulent viscosity appears in the turbulent diffusion of k (and ε), because the turbulent diffusion term reads (recall that $f_k = k/k_{total} = k/k_{RANS}$ where k_{RANS} denotes the turbulent kinetic energy in a RANS simulation)

$$\begin{aligned} &\frac{\partial}{\partial x_j}\left(\frac{\nu_t f_k^2}{1.4}\frac{\partial k}{\partial x_j}\right) = \frac{\partial}{\partial x_j}\left(\frac{c_\mu k^2}{\varepsilon f_k^2 1.4}\frac{\partial k}{\partial x_j}\right) \\ &= \frac{\partial}{\partial x_j}\left(\frac{c_\mu k_{RANS}^2}{\varepsilon 1.4}\frac{\partial k}{\partial x_j}\right) = \frac{\partial}{\partial x_j}\left(\frac{\nu_{t,RANS}}{1.4}\frac{\partial k}{\partial x_j}\right) \end{aligned} \tag{U.5}$$

cf. Eqs. 18 and 19 in [144]. Thus the *total* (i.e. RANS) viscosity is responsible for the transport of the *modeled* turbulent kinetic energy.

U.4 Location of interface in DES and DDES

The results analyzed above were from LES simulations [168,203] (i.e. the PANS model was used in LES mode). Now we will analyze results from Zonal PANS [164] where the interface is prescribed along a fixed grid line (No $j = 32$). Let's compare that with DES and DDES. Load the file `vectz_zonal_pans.dat` in `pl_vect_hump.m`. Recall that $\Delta z = 0.2/32$ (note that in [168, 203] $\Delta z = 0.2/64$)

In SA-DES, the interface is defined as the location where the wall distance is equal to $C_{DES}\Delta$ where $\Delta = \max\{\Delta_x, \Delta_y, \Delta_z\}$, see Eq. 20.3. How does this compare with gridline number $j = 32$? Compare the DES lengthscale, $C_{DES}\Delta$, with the lengthscale of PANS, i.e.

$$\ell_{PANS} = C_\mu^{3/4}\frac{k_{total}^{3/2}}{\varepsilon} \tag{U.6}$$

see Eq. 12 in [168]

In SST-DES, the location of the interface is computed using k and ω. Compute ω from $\varepsilon/(\beta^* k)$ and compute the location using Eq. 20.8. How does the location compare with gridline $j = 32$ and SA-DES?

In DDES, the boundary layer is shielded with a damping function. In SST-DES, the shielding function (see Eq. 20.9) may be one of the blending functions, F_1 or F_2 (see Eq. 20.5). Let's use F_2 as the shielding function as in [78]. Does DDES work, i.e. does it make the model to be in RANS mode in the entire boundary layer? What about the separation region?

U.5 Compute f_k

The function f_k is used in the PANS model. see Section 23. In [168, 203], $f_k = 0.4$ in the entire domain. In the Zonal PANS simulations [164], $f_k = 0.4$ in the LES region or it is computed from Eq. U.7; in the RANS region near the lower wall $f_k = 1$ (interface at grid line No $j = 32$). The expression for computing f_k reads [209]

$$f_k = c_\mu^{-2/3}\frac{\Delta}{L_t}, \quad L_t = \frac{k_{total}^{3/2}}{\langle\varepsilon\rangle} \tag{U.7}$$