On-line Lecture 1

¶See Section 11.1.1, Flow equations

▶Boussinesq approximation: density variation only in gravitation (buoyancy) term

$$\frac{\partial \rho_0 \bar{v}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\rho_0 \bar{v}_i \bar{v}_j\right) = -\frac{\partial \bar{p}}{\partial x_i} + \mu \frac{\partial^2 \bar{v}_i}{\partial x_j \partial x_j} - \rho_0 \frac{\partial \overline{v_i' v_j'}}{\partial x_j} - \rho_0 \beta(\bar{\theta} - \theta_0) g_i$$

On-line Lecture 1

¶See Section 11.1.1, Flow equations

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$$\frac{\partial \rho_0 \bar{v}_i}{\partial t} + \frac{\partial}{\partial x_j}(\rho_0 \bar{v}_i \bar{v}_j) = -\frac{\partial \bar{p}}{\partial x_i} + \mu \frac{\partial^2 \bar{v}_i}{\partial x_j \partial x_j} - \rho_0 \frac{\partial \overline{v_i' v_j'}}{\partial x_j} - \rho_0 \beta(\bar{\theta} - \theta_0) g_i$$

$\bar{p}$ is hydrodynamic pressure: $\rho f_i \to (\rho - \rho_0) g_i$

On-line Lecture 1

¶See Section 11.1.1, Flow equations

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$$d\rho = \left(\frac{\partial \rho}{\partial \theta}\right)_p d\theta + \left(\frac{\partial \rho}{\partial p}\right)_\theta dp$$

On-line Lecture 1

¶See Section 11.1.1, Flow equations

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On-line Lecture 1

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On-line Lecture 1

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$$\beta = -\frac{1}{\rho_0}\left(\frac{\partial \rho}{\partial \theta}\right)_p \Rightarrow$$

$$d\rho = -\rho_0 \beta d\theta \Rightarrow \rho - \rho_0 = -\beta \rho_0 (\theta - \theta_0)$$

On-line Lecture 1

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$$\rho_0 f_i = (\rho - \rho_0) g_i = -\rho_0 \beta (\bar{\theta} - \theta_0) g_i$$

¶See Section 11.1.2, Temperature equation
▶Temperature equation

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▶Temperature equation

$$\frac{\partial \theta}{\partial t} + \frac{\partial v_i \theta}{\partial x_i} = \alpha \frac{\partial^2 \theta}{\partial x_i \partial x_i}$$

¶See Section 11.1.2, Temperature equation

▶Temperature equation

$$\frac{\partial \theta}{\partial t} + \frac{\partial v_i \theta}{\partial x_i} = \alpha \frac{\partial^2 \theta}{\partial x_i \partial x_i}$$

where $\alpha = k/(\rho c_p)$.

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▶Temperature equation

$$\frac{\partial \theta}{\partial t} + \frac{\partial v_i \theta}{\partial x_i} = \alpha \frac{\partial^2 \theta}{\partial x_i \partial x_i}$$

where $\alpha = k/(\rho c_p)$.

Introducing $\theta = \bar{\theta} + \theta'$ gives the mean temperature equation

$$\frac{\partial \bar{v}_i \bar{\theta}}{\partial x_i} = \alpha \frac{\partial^2 \bar{\theta}}{\partial x_i \partial x_i} - \frac{\partial \overline{v_i' \theta'}}{\partial x_i} \tag{30.1}$$

►Total (viscous plus turbulent) flux: momentum and temperature equation

$$-\frac{q_{2,tot}}{\rho c_p} = -\frac{q_{2,visc}}{\rho c_p} - \frac{q_{2,turb}}{\rho c_p} = \alpha \frac{\partial \bar{\theta}}{\partial x_2} - \overline{v_2' \theta'}, \quad \alpha = \frac{k}{\rho c_p}$$

$$\tau_{tot} = \tau_{visc} + \tau_{turb} = \mu \frac{\partial \bar{v}_1}{\partial x_2} - \rho \overline{v_1' v_2'}$$

▶Total (viscous plus turbulent) flux: momentum and temperature equation

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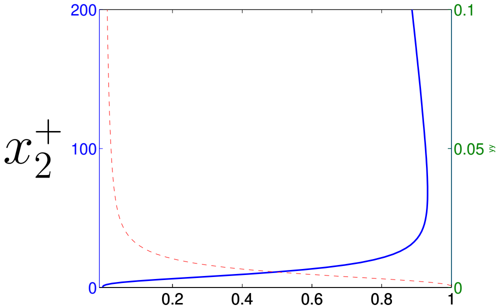

Reynolds shear stress.

—: $-\rho\overline{v_1'v_2'}/\tau_w$

- - -: $\mu(\partial\bar{v}_1/\partial x_2)/\tau_w$.

▶Total (viscous plus turbulent) flux: momentum and temperature equation

$$-\frac{q_{2,tot}}{\rho c_p} = -\frac{q_{2,visc}}{\rho c_p} - \frac{q_{2,turb}}{\rho c_p} = \alpha \frac{\partial \bar{\theta}}{\partial x_2} - \overline{v_2' \theta'}, \quad \alpha = \frac{k}{\rho c_p}$$

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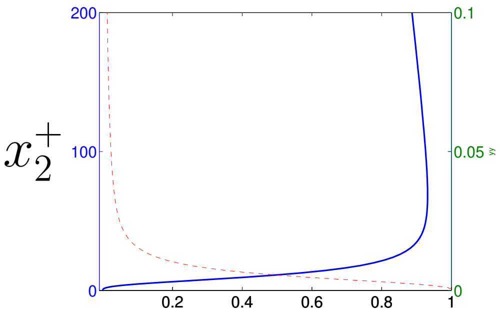

Reynolds shear stress.

—: $-\rho \overline{v_1' v_2'} / \tau_w$

- - -: $\mu (\partial \bar{v}_1 / \partial x_2) / \tau_w$.

¶See Section 11.2, The exact $\overline{v_i'v_j'}$ equation

¶See Section 11.2, The exact $\overline{v'_i v'_j}$ equation

- Set up the momentum equation for the instantaneous velocity $v_i = \bar{v}_i + v'_i \rightarrow$ Eq. (A)

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- Set up the momentum equation for the instantaneous velocity $v_i = \bar{v}_i + v'_i \rightarrow$ Eq. (A)
- Time average $\rightarrow$ equation for $\bar{v}_i$, Eq. (B)

¶See Section 11.2, The exact $\overline{v_i' v_j'}$ equation

- Set up the momentum equation for the instantaneous velocity $v_i = \bar{v}_i + v_i' \rightarrow$ Eq. (A)
- Time average $\rightarrow$ equation for $\bar{v}_i$, Eq. (B)
- Subtract Eq. (B) from Eq. (A) $\rightarrow$ equation for v_i', Eq. (C)

¶See Section 11.2, The exact $\overline{v'_i v'_j}$ equation

- Set up the momentum equation for the instantaneous velocity $v_i = \bar{v}_i + v'_i \to$ Eq. (A)
- Time average $\to$ equation for $\bar{v}_i$, Eq. (B)
- Subtract Eq. (B) from Eq. (A) $\to$ equation for v'_i, Eq. (C)
- Do the same procedure for $v_j \to$ equation for v'_j, Eq. (D)

¶See Section 11.2, The exact $\overline{v_i'v_j'}$ equation

- Set up the momentum equation for the instantaneous velocity $v_i = \bar{v}_i + v_i' \rightarrow$ Eq. (A)
- Time average $\rightarrow$ equation for $\bar{v}_i$, Eq. (B)
- Subtract Eq. (B) from Eq. (A) $\rightarrow$ equation for v_i', Eq. (C)
- Do the same procedure for $v_j \rightarrow$ equation for v_j', Eq. (D)
- Multiply Eq. (C) with v_j' and Eq. (D) with v_i', time average and add them together $\rightarrow$ equation for $\overline{v_i'v_j'}$

¶See Section 11.2, The exact $\overline{v_i'v_j'}$ equation

- Set up the momentum equation for the instantaneous velocity $v_i = \bar{v}_i + v_i' \rightarrow$ Eq. (A)
- Time average $\rightarrow$ equation for $\bar{v}_i$, Eq. (B)
- Subtract Eq. (B) from Eq. (A) $\rightarrow$ equation for v_i', Eq. (C)
- Do the same procedure for $v_j \rightarrow$ equation for v_j', Eq. (D)
- Multiply Eq. (C) with v_j' and Eq. (D) with v_i', time average and add them together $\rightarrow$ equation for $\overline{v_i'v_j'}$

In Section 9 these steps are given in some detail.

¶See Section 11.2, The exact $\overline{v_i' v_j'}$ equation

- Set up the momentum equation for the instantaneous velocity $v_i = \bar{v}_i + v_i' \rightarrow$ Eq. (A)
- Time average $\rightarrow$ equation for $\bar{v}_i$, Eq. (B)
- Subtract Eq. (B) from Eq. (A) $\rightarrow$ equation for v_i', Eq. (C)
- Do the same procedure for $v_j \rightarrow$ equation for v_j', Eq. (D)
- Multiply Eq. (C) with v_j' and Eq. (D) with v_i', time average and add them together $\rightarrow$ equation for $\overline{v_i' v_j'}$

In Section 9 these steps are given in some detail.
The final $\overline{v_i' v_j'}$-equation (Reynolds Stress equation) reads (see Eq. 9.12)

▶ $\overline{v_i'v_j'}$-equation

$$\underbrace{\bar{v}_k \frac{\partial \overline{v_i'v_j'}}{\partial x_k}}_{C_{ij}} = \underbrace{-\overline{v_i'v_k'}\frac{\partial \bar{v}_j}{\partial x_k} - \overline{v_j'v_k'}\frac{\partial \bar{v}_i}{\partial x_k}}_{P_{ij}} + \underbrace{\overline{\frac{p'}{\rho}\left(\frac{\partial v_i'}{\partial x_j} + \frac{\partial v_j'}{\partial x_i}\right)}}_{\Pi_{ij}}$$
$$\underbrace{-\frac{\partial}{\partial x_k}\left[\overline{v_i'v_j'v_k'} + \frac{\overline{p'v_j'}}{\rho}\delta_{ik} + \frac{\overline{p'v_i'}}{\rho}\delta_{jk}\right]}_{D_{ij,t}} + \underbrace{\nu\frac{\partial^2 \overline{v_i'v_j'}}{\partial x_k \partial x_k}}_{D_{ij,\nu}} \tag{30.2}$$
$$\underbrace{-g_i\beta\overline{v_j'\theta'} - g_j\beta\overline{v_i'\theta'}}_{G_{ij}} - \underbrace{2\nu\overline{\frac{\partial v_i'}{\partial x_k}\frac{\partial v_j'}{\partial x_k}}}_{\varepsilon_{ij}}$$

▶ $\overline{v_i'v_j'}$-equation

$$\underbrace{\bar{v}_k \frac{\partial \overline{v_i'v_j'}}{\partial x_k}}_{C_{ij}} = \underbrace{-\overline{v_i'v_k'}\frac{\partial \bar{v}_j}{\partial x_k} - \overline{v_j'v_k'}\frac{\partial \bar{v}_i}{\partial x_k}}_{P_{ij}} + \underbrace{\overline{\frac{p'}{\rho}\left(\frac{\partial v_i'}{\partial x_j} + \frac{\partial v_j'}{\partial x_i}\right)}}_{\Pi_{ij}}$$
$$\underbrace{-\frac{\partial}{\partial x_k}\left[\overline{v_i'v_j'v_k'} + \frac{\overline{p'v_j'}}{\rho}\delta_{ik} + \frac{\overline{p'v_i'}}{\rho}\delta_{jk}\right]}_{D_{ij,t}} + \underbrace{\nu\frac{\partial^2 \overline{v_i'v_j'}}{\partial x_k \partial x_k}}_{D_{ij,\nu}}$$
$$\underbrace{-g_i\beta\overline{v_j'\theta'} - g_j\beta\overline{v_i'\theta'}}_{G_{ij}} - \underbrace{2\nu\overline{\frac{\partial v_i'}{\partial x_k}\frac{\partial v_j'}{\partial x_k}}}_{\varepsilon_{ij}} \qquad (30.2)$$

- Unknown terms

▶ $\overline{v_i'v_j'}$-equation

$$\underbrace{\bar{v}_k\frac{\partial\overline{v_i'v_j'}}{\partial x_k}}_{C_{ij}} = \underbrace{-\overline{v_i'v_k'}\frac{\partial\bar{v}_j}{\partial x_k} - \overline{v_j'v_k'}\frac{\partial\bar{v}_i}{\partial x_k}}_{P_{ij}} + \underbrace{\overline{\frac{p'}{\rho}\left(\frac{\partial v_i'}{\partial x_j} + \frac{\partial v_j'}{\partial x_i}\right)}}_{\Pi_{ij}}$$
$$\underbrace{-\frac{\partial}{\partial x_k}\left[\overline{v_i'v_j'v_k'} + \frac{\overline{p'v_j'}}{\rho}\delta_{ik} + \frac{\overline{p'v_i'}}{\rho}\delta_{jk}\right]}_{D_{ij,t}} + \underbrace{\nu\frac{\partial^2\overline{v_i'v_j'}}{\partial x_k\partial x_k}}_{D_{ij,\nu}} \tag{30.2}$$
$$\underbrace{-g_i\beta\overline{v_j'\theta'} - g_j\beta\overline{v_i'\theta'}}_{G_{ij}} - \underbrace{2\nu\overline{\frac{\partial v_i'}{\partial x_k}\frac{\partial v_j'}{\partial x_k}}}_{\varepsilon_{ij}}$$

- Unknown terms

Π_{ij} Pressure-strain

► $\overline{v_i'v_j'}$-equation

$$\underbrace{\bar{v}_k\frac{\partial\overline{v_i'v_j'}}{\partial x_k}}_{C_{ij}} = \underbrace{-\overline{v_i'v_k'}\frac{\partial\bar{v}_j}{\partial x_k} - \overline{v_j'v_k'}\frac{\partial\bar{v}_i}{\partial x_k}}_{P_{ij}} + \underbrace{\overline{\frac{p'}{\rho}\left(\frac{\partial v_i'}{\partial x_j} + \frac{\partial v_j'}{\partial x_i}\right)}}_{\Pi_{ij}}$$
$$\underbrace{-\frac{\partial}{\partial x_k}\left[\overline{v_i'v_j'v_k'} + \frac{\overline{p'v_j'}}{\rho}\delta_{ik} + \frac{\overline{p'v_i'}}{\rho}\delta_{jk}\right]}_{D_{ij,t}} + \underbrace{\nu\frac{\partial^2\overline{v_i'v_j'}}{\partial x_k\partial x_k}}_{D_{ij,\nu}} \tag{30.2}$$
$$\underbrace{-g_i\beta\overline{v_j'\theta'} - g_j\beta\overline{v_i'\theta'}}_{G_{ij}} - \underbrace{2\nu\overline{\frac{\partial v_i'}{\partial x_k}\frac{\partial v_j'}{\partial x_k}}}_{\varepsilon_{ij}}$$

- Unknown terms

Π_{ij} Pressure-strain

$D_{ij,t}$ Turbulent diffusion

▶ $\overline{v_i'v_j'}$-equation

$$\underbrace{\bar{v}_k \frac{\partial \overline{v_i' v_j'}}{\partial x_k}}_{C_{ij}} = \underbrace{-\overline{v_i' v_k'} \frac{\partial \bar{v}_j}{\partial x_k} - \overline{v_j' v_k'} \frac{\partial \bar{v}_i}{\partial x_k}}_{P_{ij}} + \underbrace{\overline{\frac{p'}{\rho}\left(\frac{\partial v_i'}{\partial x_j} + \frac{\partial v_j'}{\partial x_i}\right)}}_{\Pi_{ij}}$$
$$\underbrace{-\frac{\partial}{\partial x_k}\left[\overline{v_i' v_j' v_k'} + \frac{\overline{p' v_j'}}{\rho}\delta_{ik} + \frac{\overline{p' v_i'}}{\rho}\delta_{jk}\right]}_{D_{ij,t}} + \underbrace{\nu \frac{\partial^2 \overline{v_i' v_j'}}{\partial x_k \partial x_k}}_{D_{ij,\nu}}$$
$$\underbrace{-g_i \beta \overline{v_j' \theta'} - g_j \beta \overline{v_i' \theta'}}_{G_{ij}} - \underbrace{2\nu \overline{\frac{\partial v_i'}{\partial x_k} \frac{\partial v_j'}{\partial x_k}}}_{\varepsilon_{ij}} \tag{30.2}$$

- Unknown terms

Π_{ij} Pressure-strain

$D_{ij,t}$ Turbulent diffusion

ε_{ij} Dissipation

¶See Section 11.3, The exact $\overline{v_i'\theta'}$ equation

► $\overline{v_i'\theta'}$ equation

¶See Section 11.3, The exact $\overline{v_i'\theta'}$ equation

▶$\overline{v_i'\theta'}$ equation

$$\frac{\partial \theta'}{\partial t} + \frac{\partial}{\partial x_k}\left(v_k'\bar{\theta} + \bar{v}_k\theta' + v_k'\theta'\right) = \alpha\frac{\partial^2\theta'}{\partial x_k \partial x_k} - \frac{\partial \overline{v_k'\theta'}}{\partial x_k} \tag{30.3}$$

¶See Section 11.3, The exact $\overline{v_i'\theta'}$ equation

▶ $\overline{v_i'\theta'}$ equation

$$\frac{\partial \theta'}{\partial t} + \frac{\partial}{\partial x_k}(v_k'\bar{\theta} + \bar{v}_k\theta' + v_k'\theta') = \alpha \frac{\partial^2 \theta'}{\partial x_k \partial x_k} - \frac{\partial \overline{v_k'\theta'}}{\partial x_k} \tag{30.3}$$

$$\frac{\partial v_i'}{\partial t} + \frac{\partial}{\partial x_k}(v_k'\bar{v}_i + \bar{v}_k v_i' + v_k' v_i') = -\frac{1}{\rho}\frac{\partial p'}{\partial x_i} + \nu \frac{\partial^2 v_i'}{\partial x_k \partial x_k} - \frac{\partial \overline{v_i' v_j'}}{\partial x_k} - g_i \beta \theta' \tag{30.4}$$

¶See Section 11.3, The exact $\overline{v_i'\theta'}$ equation

▶$\overline{v_i'\theta'}$ equation

$$\frac{\partial\theta'}{\partial t}+\frac{\partial}{\partial x_k}(v_k'\bar{\theta}+\bar{v}_k\theta'+v_k'\theta')=\alpha\frac{\partial^2\theta'}{\partial x_k\partial x_k}-\frac{\partial\overline{v_k'\theta'}}{\partial x_k} \tag{30.3}$$

$$\frac{\partial v_i'}{\partial t}+\frac{\partial}{\partial x_k}(v_k'\bar{v}_i+\bar{v}_k v_i'+v_k'v_i')=-\frac{1}{\rho}\frac{\partial p'}{\partial x_i}+\nu\frac{\partial^2 v_i'}{\partial x_k\partial x_k}-\frac{\partial\overline{v_i'v_j'}}{\partial x_k}-g_i\beta\theta' \tag{30.4}$$

Multiply Eq. 30.3 with v_i' and multiply Eq. 30.4 with θ', add them together and time average

¶See Section 11.3, The exact $\overline{v_i'\theta'}$ equation

▶$\overline{v_i'\theta'}$ equation

$$\frac{\partial \theta'}{\partial t} + \frac{\partial}{\partial x_k}(v_k'\bar{\theta} + \bar{v}_k\theta' + v_k'\theta') = \alpha\frac{\partial^2\theta'}{\partial x_k \partial x_k} - \frac{\partial \overline{v_k'\theta'}}{\partial x_k} \tag{30.3}$$

$$\frac{\partial v_i'}{\partial t} + \frac{\partial}{\partial x_k}(v_k'\bar{v}_i + \bar{v}_k v_i' + v_k' v_i') = -\frac{1}{\rho}\frac{\partial p'}{\partial x_i} + \nu\frac{\partial^2 v_i'}{\partial x_k \partial x_k} - \frac{\partial \overline{v_i' v_j'}}{\partial x_k} - g_i\beta\theta' \tag{30.4}$$

Multiply Eq. 30.3 with v_i' and multiply Eq. 30.4 with θ', add them together and time average

$$\begin{aligned}
&\overline{v_i'\frac{\partial}{\partial x_k}(v_k'\bar{\theta} + \bar{v}_k\theta' + v_k'\theta') + \theta'\frac{\partial}{\partial x_k}(\bar{v}_i v_k' + \bar{v}_k v_i' + v_i' v_k')} \\
&= -\overline{\frac{\theta'}{\rho}\frac{\partial p'}{\partial x_i}} + \alpha\overline{v_i'\frac{\partial^2\theta'}{\partial x_k \partial x_k}} + \nu\overline{\theta'\frac{\partial^2 v_i'}{\partial x_k \partial x_k}} - g_i\beta\overline{\theta'\theta'}
\end{aligned}$$

¶See Section 11.3, The exact $\overline{v_i'\theta'}$ equation

▶$\overline{v_i'\theta'}$ equation

$$\frac{\partial\theta'}{\partial t}+\frac{\partial}{\partial x_k}(v_k'\bar{\theta}+\bar{v}_k\theta'+v_k'\theta')=\alpha\frac{\partial^2\theta'}{\partial x_k\partial x_k}-\frac{\partial\overline{v_k'\theta'}}{\partial x_k} \tag{30.3}$$

$$\frac{\partial v_i'}{\partial t}+\frac{\partial}{\partial x_k}(v_k'\bar{v}_i+\bar{v}_k v_i'+v_k'v_i')=-\frac{1}{\rho}\frac{\partial p'}{\partial x_i}+\nu\frac{\partial^2 v_i'}{\partial x_k\partial x_k}-\frac{\partial\overline{v_i'v_j'}}{\partial x_k}-g_i\beta\theta' \tag{30.4}$$

Multiply Eq. 30.3 with v_i' and multiply Eq. 30.4 with θ', add them together and time average

$$\begin{aligned}&\overline{v_i'\frac{\partial}{\partial x_k}(v_k'\bar{\theta}+\bar{v}_k\theta'+v_k'\theta')+\theta'\frac{\partial}{\partial x_k}(\bar{v}_i v_k'+\bar{v}_k v_i'+v_i'v_k')}\\&=-\frac{\overline{\theta'}}{\rho}\overline{\frac{\partial p'}{\partial x_i}}+\alpha\overline{v_i'\frac{\partial^2\theta'}{\partial x_k\partial x_k}}+\nu\overline{\theta'\frac{\partial^2 v_i'}{\partial x_k\partial x_k}}-g_i\beta\overline{\theta'\theta'}\end{aligned}$$

$$\frac{\partial}{\partial x_k}\bar{v}_k\overline{v_i'\theta'} = \underbrace{-\overline{v_i'v_k'}\frac{\partial\bar{\theta}}{\partial x_k} - \overline{v_k'\theta'}\frac{\partial\bar{v}_i}{\partial x_k}}_{P_{i\theta}} \underbrace{-\overline{\frac{\theta'}{\rho}\frac{\partial p'}{\partial x_i}}}_{\Pi_{i\theta}} \underbrace{-\frac{\partial}{\partial x_k}\overline{v_k'v_i'\theta'}}_{D_{i\theta,t}}$$
$$+\underbrace{(\nu+\alpha)\frac{\partial^2\overline{v_i'\theta'}}{\partial x_k\partial x_k}}_{D_{i\theta,\nu}} - \underbrace{(\nu+\alpha)\overline{\frac{\partial v_i'}{\partial x_k}\frac{\partial\theta'}{\partial x_k}}}_{\varepsilon_{i\theta}} \underbrace{-g_i\beta\overline{\theta'^2}}_{G_{i\theta}}$$

- Unknown terms

- Unknown terms

$\Pi_{i\theta}$ Scramble

- Unknown terms

$\Pi_{i\theta}$ Scramble

$D_{i\theta,t}$ Turbulent diffusion

- Unknown terms

$\Pi_{i\theta}$ Scramble

$D_{i\theta,t}$ Turbulent diffusion

$\varepsilon_{i\theta}$ Dissipation

¶The original derivation of the k equation is shown in Section 8.2.

►In 11.4, The k equation, we derive the k equation as follows

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$$+ \nu \frac{\partial^2 \overline{v_i' v_j'}}{\partial x_k \partial x_k} - g_i \beta \overline{v_j' \theta'} - g_j \beta \overline{v_i' \theta'} - 2\nu \overline{\frac{\partial v_i'}{\partial x_k} \frac{\partial v_j'}{\partial x_k}}$$

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$$+\nu\frac{\partial^2\overline{v_i'v_j'}}{\partial x_k\partial x_k}-g_i\beta\overline{v_j'\theta'}-g_j\beta\overline{v_i'\theta'}-2\nu\overline{\frac{\partial v_i'}{\partial x_k}\frac{\partial v_j'}{\partial x_k}}$$

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$$\begin{aligned}\bar{v}_k \frac{\partial \overline{v'_i v'_j}}{\partial x_k} = & -\overline{v'_i v'_k}\frac{\partial \bar{v}_j}{\partial x_k} - \overline{v'_j v'_k}\frac{\partial \bar{v}_i}{\partial x_k} + \overline{\frac{p'}{\rho}\left(\frac{\partial v'_i}{\partial x_j} + \frac{\partial v'_j}{\partial x_i}\right)} - \frac{\partial}{\partial x_k}\left[\overline{v'_i v'_j v'_k} + \frac{\overline{p' v'_j}}{\rho}\delta_{ik} + \frac{\overline{p' v'_i}}{\rho}\delta_{jk}\right] \\ & + \nu \frac{\partial^2 \overline{v'_i v'_j}}{\partial x_k \partial x_k} - g_i \beta \overline{v'_j \theta'} - g_j \beta \overline{v'_i \theta'} - 2\nu \overline{\frac{\partial v'_i}{\partial x_k}\frac{\partial v'_j}{\partial x_k}}\end{aligned}$$

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$$\underbrace{\bar{v}_j \frac{\partial k}{\partial x_j}}_{C^k} = -\underbrace{\overline{v'_j v'_k}\frac{\partial \bar{v}_j}{\partial x_k}}_{P^k} \underbrace{- \frac{\partial}{\partial x_k}\left[\overline{v'_k\left(\frac{1}{2}v'_i v'_i + \frac{p'}{\rho}\right)}\right]}_{D_t^k} + \underbrace{\nu \frac{\partial^2 k}{\partial x_k \partial x_k}}_{D_\nu^k} \underbrace{- g_i \beta \overline{v'_i \theta'}}_{G^k} - \underbrace{\nu \overline{\frac{\partial v'_i}{\partial x_k}\frac{\partial v'_i}{\partial x_k}}}_{\varepsilon} \tag{30.5}$$

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►The Boussinesq assumption: a model for $\overline{v_i' v_j'}$

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►The Boussinesq assumption: a model for $\overline{v_i'v_j'}$

The diffusion term of time-averaged Navier-Stokes

$$\frac{\partial}{\partial x_j}\left\{\nu\left(\frac{\partial \bar{v}_i}{\partial x_j}+\frac{\partial \bar{v}_j}{\partial x_i}\right)-\overline{v_i'v_j'}\right\}\Rightarrow\frac{\partial}{\partial x_j}\left\{(\nu+\nu_t)\left(\frac{\partial \bar{v}_i}{\partial x_j}+\frac{\partial \bar{v}_j}{\partial x_i}\right)\right\}$$

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On-line Lecture 2

¶See Section 11.7.1, Production terms

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- First let's repeat the definition of the strain-rate and vorticity tensors, see Eq. 1.11

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- Recall that the product $\bar{s}_{ij}\bar{\Omega}_{ij} = 0$ (product of symmetric and anti-symmetric tensor, see Section 1.5)

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$$P^k = -\overline{v_i'v_j'}\frac{\partial \bar{v}_i}{\partial x_j} = \nu_t\left[\left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i}\right) - \frac{2}{3}\delta_{ij}k\right]\frac{\partial \bar{v}_i}{\partial x_j}$$

On-line Lecture 2

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$$\begin{aligned} P^k = -\overline{v_i' v_j'}\frac{\partial \bar{v}_i}{\partial x_j} &= \nu_t\left[\left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i}\right) - \frac{2}{3}\delta_{ij}k\right]\frac{\partial \bar{v}_i}{\partial x_j} \\ &= \nu_t\left[\left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i}\right)\right]\frac{\partial \bar{v}_i}{\partial x_j} - \frac{2}{3}\nu_t\delta_{ij}k\frac{\partial \bar{v}_i}{\partial x_j} \end{aligned}$$

On-line Lecture 2

¶See Section 11.7.1, Production terms

- First let's repeat the definition of the strain-rate and vorticity tensors, see Eq. 1.11

$$\frac{\partial \bar{v}_i}{\partial x_j} = \bar{s}_{ij} + \bar{\Omega}_{ij}, \quad \bar{s}_{ij} = \frac{1}{2}\left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i}\right), \quad \bar{\Omega}_{ij} = \frac{1}{2}\left(\frac{\partial \bar{v}_j}{\partial x_i} - \frac{\partial \bar{v}_j}{\partial x_i}\right)$$

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&= \nu_t \left[\left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i}\right)\right]\frac{\partial \bar{v}_i}{\partial x_j} - \frac{2}{3}\nu_t \delta_{ij} k \frac{\partial \bar{v}_i}{\partial x_j} \\
&= \nu_t \left[\left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i}\right)\right]\frac{\partial \bar{v}_i}{\partial x_j} - \cancelto{0}{\frac{2}{3}\nu_t \frac{\partial \bar{v}_i}{\partial x_i}}
\end{aligned}$$

On-line Lecture 2

¶See Section 11.7.1, Production terms

- First let's repeat the definition of the strain-rate and vorticity tensors, see Eq. 1.11

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&= 2\nu_t\bar{s}_{ij}(\bar{s}_{ij} + \bar{\Omega}_{ij})
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On-line Lecture 2

¶See Section 11.7.1, Production terms

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&= 2\nu_t\bar{s}_{ij}(\bar{s}_{ij} + \bar{\Omega}_{ij}) = 2\nu_t\bar{s}_{ij}\bar{s}_{ij}
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On-line Lecture 2

¶See Section 11.7.1, Production terms

- First let's repeat the definition of the strain-rate and vorticity tensors, see Eq. 1.11

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\end{aligned}$$

▶Diffusion term in k eq, Eq. 30.5, must be modelled.

▶The exact k equation:

$$\bar{v}_j \frac{\partial k}{\partial x_j} = -\overline{v'_j v'_k} \frac{\partial \bar{v}_j}{\partial x_k} - \frac{\partial}{\partial x_k} \left[\overline{v'_k \left(\frac{1}{2} v'_i v'_i + \frac{p'}{\rho} \right)} \right] + \nu \frac{\partial^2 k}{\partial x_k \partial x_k} - g_i \beta \overline{v'_i \theta'} - \nu \overline{\frac{\partial v'_i}{\partial x_k} \frac{\partial v'_i}{\partial x_k}}$$

▶The constitutive model for heat conduction, Fourier's law, (see Section 2.2)

$$q_i = -k \frac{\partial \bar{\theta}}{\partial x_i}.$$

Flux of k:

▶ Diffusion term in k eq, Eq. 30.5, must be modelled.

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Flux of k:

$$d^k_{j,t} = \frac{1}{2}\overline{v'_j v'_i v'_i} = -\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_j}$$

$$\Rightarrow -\frac{1}{2}\frac{\partial\overline{v'_j v'_i v'_i}}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_j}\right)$$

▶Diffusion term in k eq, Eq. 30.5, must be modelled.

▶The exact k equation:

$$\bar{v}_j \frac{\partial k}{\partial x_j} = -\overline{v'_j v'_k} \frac{\partial \bar{v}_j}{\partial x_k} - \frac{\partial}{\partial x_k} \left[\overline{v'_k \left(\frac{1}{2} v'_i v'_i + \frac{p'}{\rho} \right)} \right] + \nu \frac{\partial^2 k}{\partial x_k \partial x_k} - g_i \beta \overline{v'_i \theta'} - \nu \overline{\frac{\partial v'_i}{\partial x_k} \frac{\partial v'_i}{\partial x_k}}$$

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▶The heat flux is an unknown both in the mean temperature equation, Eq. 30.1, and in the exact k equation above. Taking guidance from Fourier's law . It is modeled as

►Diffusion term in k eq, Eq. 30.5, must be modelled.
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$$\bar{v}_j\frac{\partial k}{\partial x_j} = -\overline{v_j'v_k'}\frac{\partial \bar{v}_j}{\partial x_k} - \frac{\partial}{\partial x_k}\left[\overline{v_k'\left(\frac{1}{2}v_i'v_i' + \frac{p'}{\rho}\right)}\right] + \nu\frac{\partial^2 k}{\partial x_k \partial x_k} - g_i\beta\overline{v_i'\theta'} - \nu\overline{\frac{\partial v_i'}{\partial x_k}\frac{\partial v_i'}{\partial x_k}}$$

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$$q_i = -k\frac{\partial \bar{\theta}}{\partial x_i}.$$

Flux of k:

$$d_{j,t}^k = \frac{1}{2}\overline{v_j'v_i'v_i'} = -\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_j}$$

$$\Rightarrow -\frac{1}{2}\frac{\partial \overline{v_j'v_i'v_i'}}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\frac{\nu_t}{\sigma_k}\frac{\partial k}{\partial x_j}\right)$$

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$$\overline{v_i'\theta'} = -\alpha_t\frac{\partial \bar{\theta}}{\partial x_i}, \quad \alpha_t = \frac{\nu_t}{\sigma_t}$$

¶See Section 11.8, The $k - \varepsilon$ model

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▶Modeled k equation

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▶Modeled k equation

$$\bar{v}_j \frac{\partial k}{\partial x_j} = 2\nu_t \bar{s}_{ij} \bar{s}_{ij} + \frac{\partial}{\partial x_j} \left\{ \left(\nu + \frac{\nu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right\} + g_i \beta \frac{\nu_t}{\sigma_\theta} \frac{\partial \bar{\theta}}{\partial x_i} - \varepsilon$$

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▶Modeled k equation

$$\bar{v}_j \frac{\partial k}{\partial x_j} = 2\nu_t \bar{s}_{ij}\bar{s}_{ij} + \frac{\partial}{\partial x_j}\left\{\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right\} + g_i \beta \frac{\nu_t}{\sigma_\theta}\frac{\partial \bar{\theta}}{\partial x_i} - \varepsilon$$

▶Exact k equation

$$\underbrace{\bar{v}_j \frac{\partial k}{\partial x_j}}_{C^k} = -\underbrace{\overline{v'_j v'_k}\frac{\partial \bar{v}_j}{\partial x_k}}_{P^k} \underbrace{-\frac{\partial}{\partial x_k}\left[\overline{v'_k\left(\frac{1}{2}v'_i v'_i + \frac{p'}{\rho}\right)}\right]}_{D^k_t} + \underbrace{\nu \frac{\partial^2 k}{\partial x_k \partial x_k}}_{D^k_\nu} \underbrace{-g_i \beta \overline{v'_i t \theta'}}_{G^k} - \underbrace{\nu \overline{\frac{\partial v'_i}{\partial x_k}\frac{\partial v'_i}{\partial x_k}}}_{\varepsilon}$$

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¶See Section 11.5, The ε equation

▶ε equation

¶See Section 11.8, The $k-\varepsilon$ model

▶Modeled k equation

$$\bar{v}_j\frac{\partial k}{\partial x_j}=2\nu_t\bar{s}_{ij}\bar{s}_{ij}+\frac{\partial}{\partial x_j}\left\{\left(\nu+\frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right\}+g_i\beta\frac{\nu_t}{\sigma_\theta}\frac{\partial\bar{\theta}}{\partial x_i}-\varepsilon$$

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$$C^\varepsilon=P^\varepsilon+D^\varepsilon+G^\varepsilon-\Psi^\varepsilon$$

¶See Section 11.8, The $k-\varepsilon$ model

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¶See Section 11.5, The ε equation

▶ε equation

$$C^\varepsilon = P^\varepsilon + D^\varepsilon + G^\varepsilon - \Psi^\varepsilon$$

Use the same source terms as in k equation and add turbulent time-scale ε/k to get the right dimensions:

$$P^\varepsilon + G^\varepsilon - \Psi^\varepsilon = \frac{\varepsilon}{k}\left(c_{\varepsilon 1}P^k + c_{\varepsilon 3}G^k - c_{\varepsilon 2}\varepsilon\right)$$

¶See Section 11.8, The $k - \varepsilon$ model

▶Modeled k equation

$$\bar{v}_j \frac{\partial k}{\partial x_j} = 2\nu_t \bar{s}_{ij}\bar{s}_{ij} + \frac{\partial}{\partial x_j}\left\{\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right\} + g_i \beta \frac{\nu_t}{\sigma_\theta}\frac{\partial \bar{\theta}}{\partial x_i} - \varepsilon$$

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▶The final form modelled ε equation

¶See Section 11.8, The $k-\varepsilon$ model

▶Modeled k equation

$$\bar{v}_j \frac{\partial k}{\partial x_j} = 2\nu_t \bar{s}_{ij}\bar{s}_{ij} + \frac{\partial}{\partial x_j}\left\{\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right\} + g_i\beta\frac{\nu_t}{\sigma_\theta}\frac{\partial\bar{\theta}}{\partial x_i} - \varepsilon$$

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▶The final form modelled ε equation

$$\frac{\partial \varepsilon}{\partial t} + \bar{v}_j\frac{\partial \varepsilon}{\partial x_j} = c_{\varepsilon 1}\frac{\varepsilon}{k}P^k + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] + c_{\varepsilon 3}\frac{\varepsilon}{k}G^k - c_{\varepsilon 2}\frac{\varepsilon}{k}\varepsilon$$

¶See Section 11.8, The $k-\varepsilon$ model

▶Modeled k equation

$$\bar{v}_j \frac{\partial k}{\partial x_j} = 2\nu_t \bar{s}_{ij}\bar{s}_{ij} + \frac{\partial}{\partial x_j}\left\{\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right\} + g_i\beta\frac{\nu_t}{\sigma_\theta}\frac{\partial\bar{\theta}}{\partial x_i} - \varepsilon$$

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Use the same source terms as in k equation and add turbulent time-scale ε/k to get the right dimensions:

$$P^\varepsilon + G^\varepsilon - \Psi^\varepsilon = \frac{\varepsilon}{k}\left(c_{\varepsilon 1}P^k + c_{\varepsilon 3}G^k - c_{\varepsilon 2}\varepsilon\right)$$

▶The final form modelled ε equation

$$\frac{\partial\varepsilon}{\partial t} + \bar{v}_j\frac{\partial\varepsilon}{\partial x_j} = c_{\varepsilon 1}\frac{\varepsilon}{k}P^k + \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial\varepsilon}{\partial x_j}\right] + c_{\varepsilon 3}\frac{\varepsilon}{k}G^k - c_{\varepsilon 2}\frac{\varepsilon}{k}\varepsilon$$

¶See Section 11.7.3, Dissipation term, ε_{ij}

$$\underbrace{\bar{v}_k \frac{\partial \overline{v_i' v_j'}}{\partial x_k}}_{C_{ij}} = \underbrace{-\overline{v_i' v_k'} \frac{\partial \bar{v}_j}{\partial x_k} - \overline{v_j' v_k'} \frac{\partial \bar{v}_i}{\partial x_k}}_{P_{ij}} + \underbrace{\overline{\frac{p'}{\rho}\left(\frac{\partial v_i'}{\partial x_j} + \frac{\partial v_j'}{\partial x_i}\right)}}_{\Pi_{ij}}$$

$$\underbrace{-\frac{\partial}{\partial x_k}\left[\overline{v_i' v_j' v_k'} + \frac{\overline{p' v_j'}}{\rho}\delta_{ik} + \frac{\overline{p' v_i'}}{\rho}\delta_{jk}\right]}_{D_{ij,t}} + \underbrace{\nu \frac{\partial^2 \overline{v_i' v_j'}}{\partial x_k \partial x_k}}_{D_{ij,\nu}}$$

$$\underbrace{-g_i \beta \overline{v_j' \theta'} - g_j \beta \overline{v_i' \theta'}}_{G_{ij}} - \underbrace{2\nu \overline{\frac{\partial v_i'}{\partial x_k} \frac{\partial v_j'}{\partial x_k}}}_{\varepsilon_{ij}}$$

¶See Section 11.7.3, Dissipation term, ε_{ij}

$$\underbrace{\bar{v}_k \frac{\partial \overline{v'_i v'_j}}{\partial x_k}}_{C_{ij}} = \underbrace{-\overline{v'_i v'_k} \frac{\partial \bar{v}_j}{\partial x_k} - \overline{v'_j v'_k} \frac{\partial \bar{v}_i}{\partial x_k}}_{P_{ij}} + \underbrace{\overline{\frac{p'}{\rho} \left(\frac{\partial v'_i}{\partial x_j} + \frac{\partial v'_j}{\partial x_i} \right)}}_{\Pi_{ij}}$$

$$\underbrace{-\frac{\partial}{\partial x_k} \left[\overline{v'_i v'_j v'_k} + \frac{\overline{p' v'_j}}{\rho} \delta_{ik} + \frac{\overline{p' v'_i}}{\rho} \delta_{jk} \right]}_{D_{ij,t}} + \underbrace{\nu \frac{\partial^2 \overline{v'_i v'_j}}{\partial x_k \partial x_k}}_{D_{ij,\nu}}$$

$$\underbrace{-g_i \beta \overline{v'_j \theta'} - g_j \beta \overline{v'_i \theta'}}_{G_{ij}} - \underbrace{2\nu \overline{\frac{\partial v'_i}{\partial x_k} \frac{\partial v'_j}{\partial x_k}}}_{\varepsilon_{ij}}$$

►The dissipation term, ε_{ij}, in the $\overline{v'_i v'_j}$ equation (eq. 30.2), is modeled as follows:

¶See Section 11.7.3, Dissipation term, ε_{ij}

$$\underbrace{\bar{v}_k \frac{\partial \overline{v'_i v'_j}}{\partial x_k}}_{C_{ij}} = \underbrace{-\overline{v'_i v'_k}\frac{\partial \bar{v}_j}{\partial x_k} - \overline{v'_j v'_k}\frac{\partial \bar{v}_i}{\partial x_k}}_{P_{ij}} + \underbrace{\overline{\frac{p'}{\rho}\left(\frac{\partial v'_i}{\partial x_j} + \frac{\partial v'_j}{\partial x_i}\right)}}_{\Pi_{ij}}$$

$$\underbrace{-\frac{\partial}{\partial x_k}\left[\overline{v'_i v'_j v'_k} + \frac{\overline{p' v'_j}}{\rho}\delta_{ik} + \frac{\overline{p' v'_i}}{\rho}\delta_{jk}\right]}_{D_{ij,t}} + \underbrace{\nu \frac{\partial^2 \overline{v'_i v'_j}}{\partial x_k \partial x_k}}_{D_{ij,\nu}}$$

$$\underbrace{-g_i \beta \overline{v'_j \theta'} - g_j \beta \overline{v'_i \theta'}}_{G_{ij}} - \underbrace{2\nu \overline{\frac{\partial v'_i}{\partial x_k}\frac{\partial v'_j}{\partial x_k}}}_{\varepsilon_{ij}}$$

►The dissipation term, ε_{ij}, in the $\overline{v'_i v'_j}$ equation (eq. 30.2), is modeled as follows:
Small-scale turbulence is isotropic

1.

¶See Section 11.7.3, Dissipation term, ε_{ij}

$$\underbrace{\bar{v}_k \frac{\partial \overline{v_i' v_j'}}{\partial x_k}}_{C_{ij}} = \underbrace{-\overline{v_i' v_k'} \frac{\partial \bar{v}_j}{\partial x_k} - \overline{v_j' v_k'} \frac{\partial \bar{v}_i}{\partial x_k}}_{P_{ij}} + \underbrace{\overline{\frac{p'}{\rho}\left(\frac{\partial v_i'}{\partial x_j} + \frac{\partial v_j'}{\partial x_i}\right)}}_{\Pi_{ij}}$$

$$\underbrace{-\frac{\partial}{\partial x_k}\left[\overline{v_i' v_j' v_k'} + \frac{\overline{p' v_j'}}{\rho}\delta_{ik} + \frac{\overline{p' v_i'}}{\rho}\delta_{jk}\right]}_{D_{ij,t}} + \underbrace{\nu \frac{\partial^2 \overline{v_i' v_j'}}{\partial x_k \partial x_k}}_{D_{ij,\nu}}$$

$$\underbrace{-g_i \beta \overline{v_j' \theta'} - g_j \beta \overline{v_i' \theta'}}_{G_{ij}} - \underbrace{2\nu \overline{\frac{\partial v_i'}{\partial x_k} \frac{\partial v_j'}{\partial x_k}}}_{\varepsilon_{ij}}$$

►The dissipation term, ε_{ij}, in the $\overline{v_i' v_j'}$ equation (eq. 30.2), is modeled as follows: Small-scale turbulence is isotropic

1. $\overline{v_1'^2} = \overline{v_2'^2} = \overline{v_3'^2}$.

¶See Section 11.7.3, Dissipation term, ε_{ij}

$$\underbrace{\bar{v}_k \frac{\partial \overline{v'_i v'_j}}{\partial x_k}}_{C_{ij}} = \underbrace{-\overline{v'_i v'_k} \frac{\partial \bar{v}_j}{\partial x_k} - \overline{v'_j v'_k} \frac{\partial \bar{v}_i}{\partial x_k}}_{P_{ij}} + \underbrace{\overline{\frac{p'}{\rho}\left(\frac{\partial v'_i}{\partial x_j} + \frac{\partial v'_j}{\partial x_i}\right)}}_{\Pi_{ij}}$$

$$\underbrace{-\frac{\partial}{\partial x_k}\left[\overline{v'_i v'_j v'_k} + \frac{\overline{p' v'_j}}{\rho}\delta_{ik} + \frac{\overline{p' v'_i}}{\rho}\delta_{jk}\right]}_{D_{ij,t}} + \underbrace{\nu \frac{\partial^2 \overline{v'_i v'_j}}{\partial x_k \partial x_k}}_{D_{ij,\nu}}$$

$$\underbrace{-g_i \beta \overline{v'_j \theta'} - g_j \beta \overline{v'_i \theta'}}_{G_{ij}} - \underbrace{2\nu \overline{\frac{\partial v'_i}{\partial x_k} \frac{\partial v'_j}{\partial x_k}}}_{\varepsilon_{ij}}$$

►The dissipation term, ε_{ij}, in the $\overline{v'_i v'_j}$ equation (eq. 30.2), is modeled as follows: Small-scale turbulence is isotropic

1. $\overline{v_1'^2} = \overline{v_2'^2} = \overline{v_3'^2}$.
2. All shear stresses are zero (if we flip one coordinate axis the sign will change: hence not isotropic) $\Rightarrow$

¶See Section 11.7.3, Dissipation term, ε_{ij}

$$\underbrace{\bar{v}_k \frac{\partial \overline{v'_i v'_j}}{\partial x_k}}_{C_{ij}} = \underbrace{-\overline{v'_i v'_k}\frac{\partial \bar{v}_j}{\partial x_k} - \overline{v'_j v'_k}\frac{\partial \bar{v}_i}{\partial x_k}}_{P_{ij}} + \underbrace{\overline{\frac{p'}{\rho}\left(\frac{\partial v'_i}{\partial x_j} + \frac{\partial v'_j}{\partial x_i}\right)}}_{\Pi_{ij}}$$

$$\underbrace{-\frac{\partial}{\partial x_k}\left[\overline{v'_i v'_j v'_k} + \frac{\overline{p' v'_j}}{\rho}\delta_{ik} + \frac{\overline{p' v'_i}}{\rho}\delta_{jk}\right]}_{D_{ij,t}} + \underbrace{\nu \frac{\partial^2 \overline{v'_i v'_j}}{\partial x_k \partial x_k}}_{D_{ij,\nu}}$$

$$\underbrace{-g_i \beta \overline{v'_j \theta'} - g_j \beta \overline{v'_i \theta'}}_{G_{ij}} - \underbrace{2\nu \overline{\frac{\partial v'_i}{\partial x_k}\frac{\partial v'_j}{\partial x_k}}}_{\varepsilon_{ij}}$$

►The dissipation term, ε_{ij}, in the $\overline{v'_i v'_j}$ equation (eq. 30.2), is modeled as follows: Small-scale turbulence is isotropic

1. $\overline{v_1'^2} = \overline{v_2'^2} = \overline{v_3'^2}$.
2. All shear stresses are zero (if we flip one coordinate axis the sign will change: hence not isotropic) $\Rightarrow$

$$\varepsilon_{ij} = \frac{2}{3}\varepsilon\delta_{ij} \tag{31.1}$$

¶See Section 11.7.2, Diffusion terms

$$
\underbrace{\bar{v}_k \frac{\partial \overline{v_i' v_j'}}{\partial x_k}}_{C_{ij}} = \underbrace{-\overline{v_i' v_k'} \frac{\partial \bar{v}_j}{\partial x_k} - \overline{v_j' v_k'} \frac{\partial \bar{v}_i}{\partial x_k}}_{P_{ij}} + \underbrace{\overline{\frac{p'}{\rho} \left(\frac{\partial v_i'}{\partial x_j} + \frac{\partial v_j'}{\partial x_i} \right)}}_{\Pi_{ij}}
$$

$$
\underbrace{- \frac{\partial}{\partial x_k} \left[\overline{v_i' v_j' v_k'} + \frac{\overline{p' v_j'}}{\rho} \delta_{ik} + \frac{\overline{p' v_i'}}{\rho} \delta_{jk} \right]}_{D_{ij,t}} + \underbrace{\nu \frac{\partial^2 \overline{v_i' v_j'}}{\partial x_k \partial x_k}}_{D_{ij,\nu}}
$$

$$
\underbrace{-g_i \beta \overline{v_j' \theta'} - g_j \beta \overline{v_i' \theta'}}_{G_{ij}} - \underbrace{2\nu \overline{\frac{\partial v_i'}{\partial x_k} \frac{\partial v_j'}{\partial x_k}}}_{\varepsilon_{ij}}
$$

Flux of $\overline{v_i' v_j'}$:

¶See Section 11.7.2, Diffusion terms

$$\underbrace{\bar{v}_k \frac{\partial \overline{v_i' v_j'}}{\partial x_k}}_{C_{ij}} = \underbrace{-\overline{v_i' v_k'} \frac{\partial \bar{v}_j}{\partial x_k} - \overline{v_j' v_k'} \frac{\partial \bar{v}_i}{\partial x_k}}_{P_{ij}} + \underbrace{\overline{\frac{p'}{\rho}\left(\frac{\partial v_i'}{\partial x_j} + \frac{\partial v_j'}{\partial x_i}\right)}}_{\Pi_{ij}}$$

$$\underbrace{-\frac{\partial}{\partial x_k}\left[\overline{v_i' v_j' v_k'} + \frac{\overline{p' v_j'}}{\rho}\delta_{ik} + \frac{\overline{p' v_i'}}{\rho}\delta_{jk}\right]}_{D_{ij,t}} + \underbrace{\nu \frac{\partial^2 \overline{v_i' v_j'}}{\partial x_k \partial x_k}}_{D_{ij,\nu}}$$

$$\underbrace{-g_i \beta \overline{v_j' \theta'} - g_j \beta \overline{v_i' \theta'}}_{G_{ij}} - \underbrace{2\nu \overline{\frac{\partial v_i'}{\partial x_k}\frac{\partial v_j'}{\partial x_k}}}_{\varepsilon_{ij}}$$

Flux of $\overline{v_i' v_j'}$:

$$D_{ij,t} = \overline{v_i' v_j' v_k'}$$

¶See Section 11.7.2, Diffusion terms

$$\underbrace{\bar{v}_k \frac{\partial \overline{v_i' v_j'}}{\partial x_k}}_{C_{ij}} = \underbrace{-\overline{v_i' v_k'} \frac{\partial \bar{v}_j}{\partial x_k} - \overline{v_j' v_k'} \frac{\partial \bar{v}_i}{\partial x_k}}_{P_{ij}} + \underbrace{\overline{\frac{p'}{\rho} \left(\frac{\partial v_i'}{\partial x_j} + \frac{\partial v_j'}{\partial x_i} \right)}}_{\Pi_{ij}}$$

$$\underbrace{-\frac{\partial}{\partial x_k} \left[\overline{v_i' v_j' v_k'} + \frac{\overline{p' v_j'}}{\rho} \delta_{ik} + \frac{\overline{p' v_i'}}{\rho} \delta_{jk} \right]}_{D_{ij,t}} + \underbrace{\nu \frac{\partial^2 \overline{v_i' v_j'}}{\partial x_k \partial x_k}}_{D_{ij,\nu}}$$

$$\underbrace{-g_i \beta \overline{v_j' \theta'} - g_j \beta \overline{v_i' \theta'}}_{G_{ij}} - \underbrace{2\nu \overline{\frac{\partial v_i'}{\partial x_k} \frac{\partial v_j'}{\partial x_k}}}_{\varepsilon_{ij}}$$

Flux of $\overline{v_i' v_j'}$:

$$D_{ij,t} = \overline{v_i' v_j' v_k'} \ =$$

¶See Section 11.7.2, Diffusion terms

$$\underbrace{\bar{v}_k \frac{\partial \overline{v_i' v_j'}}{\partial x_k}}_{C_{ij}} = \underbrace{-\overline{v_i' v_k'} \frac{\partial \bar{v}_j}{\partial x_k} - \overline{v_j' v_k'} \frac{\partial \bar{v}_i}{\partial x_k}}_{P_{ij}} + \underbrace{\overline{\frac{p'}{\rho}\left(\frac{\partial v_i'}{\partial x_j} + \frac{\partial v_j'}{\partial x_i}\right)}}_{\Pi_{ij}}$$

$$\underbrace{-\frac{\partial}{\partial x_k}\left[\overline{v_i' v_j' v_k'} + \frac{\overline{p' v_j'}}{\rho}\delta_{ik} + \frac{\overline{p' v_i'}}{\rho}\delta_{jk}\right]}_{D_{ij,t}} + \underbrace{\nu \frac{\partial^2 \overline{v_i' v_j'}}{\partial x_k \partial x_k}}_{D_{ij,\nu}}$$

$$\underbrace{-g_i \beta \overline{v_j' \theta'} - g_j \beta \overline{v_i' \theta'}}_{G_{ij}} - \underbrace{2\nu \overline{\frac{\partial v_i'}{\partial x_k} \frac{\partial v_j'}{\partial x_k}}}_{\varepsilon_{ij}}$$

Flux of $\overline{v_i' v_j'}$:

$$D_{ij,t} = \overline{v_i' v_j' v_k'} = -\frac{\nu_t}{\sigma_k} \frac{\partial \overline{v_i' v_j'}}{\partial x_k}$$

¶See Section 11.7.2, Diffusion terms

$$\underbrace{\bar{v}_k \frac{\partial \overline{v_i' v_j'}}{\partial x_k}}_{C_{ij}} = \underbrace{-\overline{v_i' v_k'} \frac{\partial \bar{v}_j}{\partial x_k} - \overline{v_j' v_k'} \frac{\partial \bar{v}_i}{\partial x_k}}_{P_{ij}} + \underbrace{\overline{\frac{p'}{\rho}\left(\frac{\partial v_i'}{\partial x_j} + \frac{\partial v_j'}{\partial x_i}\right)}}_{\Pi_{ij}}$$

$$\underbrace{-\frac{\partial}{\partial x_k}\left[\overline{v_i' v_j' v_k'} + \frac{\overline{p' v_j'}}{\rho}\delta_{ik} + \frac{\overline{p' v_i'}}{\rho}\delta_{jk}\right]}_{D_{ij,t}} + \underbrace{\nu \frac{\partial^2 \overline{v_i' v_j'}}{\partial x_k \partial x_k}}_{D_{ij,\nu}}$$

$$\underbrace{-g_i \beta \overline{v_j' \theta'} - g_j \beta \overline{v_i' \theta'}}_{G_{ij}} - \underbrace{2\nu \overline{\frac{\partial v_i'}{\partial x_k} \frac{\partial v_j'}{\partial x_k}}}_{\varepsilon_{ij}}$$

Flux of $\overline{v_i' v_j'}$:

$$D_{ij,t} = \overline{v_i' v_j' v_k'} \;=\; -\frac{\nu_t}{\sigma_k} \frac{\partial \overline{v_i' v_j'}}{\partial x_k}$$

$$\Rightarrow -\frac{\partial \overline{v_i' v_j' v_k'}}{\partial x_k} = \frac{\partial}{\partial x_k}\left(\frac{\nu_t}{\sigma_k} \frac{\partial \overline{v_i' v_j'}}{\partial x_k}\right)$$

¶See Section 11.7.2, Diffusion terms

$$
\underbrace{\bar{v}_k \frac{\partial \overline{v_i' v_j'}}{\partial x_k}}_{C_{ij}} = \underbrace{-\overline{v_i' v_k'} \frac{\partial \bar{v}_j}{\partial x_k} - \overline{v_j' v_k'} \frac{\partial \bar{v}_i}{\partial x_k}}_{P_{ij}} + \underbrace{\overline{\frac{p'}{\rho}\left(\frac{\partial v_i'}{\partial x_j} + \frac{\partial v_j'}{\partial x_i}\right)}}_{\Pi_{ij}}
$$

$$
\underbrace{-\frac{\partial}{\partial x_k}\left[\overline{v_i' v_j' v_k'} + \frac{\overline{p' v_j'}}{\rho}\delta_{ik} + \frac{\overline{p' v_i'}}{\rho}\delta_{jk}\right]}_{D_{ij,t}} + \underbrace{\nu \frac{\partial^2 \overline{v_i' v_j'}}{\partial x_k \partial x_k}}_{D_{ij,\nu}}
$$

$$
\underbrace{-g_i \beta \overline{v_j' \theta'} - g_j \beta \overline{v_i' \theta'}}_{G_{ij}} \underbrace{- 2\nu \overline{\frac{\partial v_i'}{\partial x_k}\frac{\partial v_j'}{\partial x_k}}}_{\varepsilon_{ij}}
$$

Flux of $\overline{v_i' v_j'}$:

$$
D_{ij,t} = \overline{v_i' v_j' v_k'} \; = \; -\frac{\nu_t}{\sigma_k}\frac{\partial \overline{v_i' v_j'}}{\partial x_k}
$$

$$
\Rightarrow -\frac{\partial \overline{v_i' v_j' v_k'}}{\partial x_k} = \frac{\partial}{\partial x_k}\left(\frac{\nu_t}{\sigma_k}\frac{\partial \overline{v_i' v_j'}}{\partial x_k}\right)
$$

¶See Section 11.7.4, Slow pressure-strain term

¶See Section 11.7.4, Slow pressure-strain term

$$\underbrace{\bar{v}_k\frac{\partial\overline{v'_iv'_j}}{\partial x_k}}_{C_{ij}} = \underbrace{-\overline{v'_iv'_k}\frac{\partial\bar{v}_j}{\partial x_k} - \overline{v'_jv'_k}\frac{\partial\bar{v}_i}{\partial x_k}}_{P_{ij}} + \underbrace{\overline{\frac{p'}{\rho}\left(\frac{\partial v'_i}{\partial x_j}+\frac{\partial v'_j}{\partial x_i}\right)}}_{\Pi_{ij}}$$

$$\underbrace{-\frac{\partial}{\partial x_k}\left[\overline{v'_iv'_jv'_k}+\frac{\overline{p'v'_j}}{\rho}\delta_{ik}+\frac{\overline{p'v'_i}}{\rho}\delta_{jk}\right]}_{D_{ij,t}} + \underbrace{\nu\frac{\partial^2\overline{v'_iv'_j}}{\partial x_k\partial x_k}}_{D_{ij,\nu}}$$

$$\underbrace{-g_i\beta\overline{v'_j\theta'} - g_j\beta\overline{v'_i\theta'}}_{G_{ij}} - \underbrace{2\nu\overline{\frac{\partial v'_i}{\partial x_k}\frac{\partial v'_j}{\partial x_k}}}_{\varepsilon_{ij}}$$

►The pressure-strain term is the last unknown term that needs to be modelled in the $\overline{v'_iv'_j}$ equation, Eq. 11.11.

¶See Section 11.7.4, Slow pressure-strain term

$$\underbrace{\bar{v}_k \frac{\partial \overline{v_i' v_j'}}{\partial x_k}}_{C_{ij}} = \underbrace{-\overline{v_i' v_k'} \frac{\partial \bar{v}_j}{\partial x_k} - \overline{v_j' v_k'} \frac{\partial \bar{v}_i}{\partial x_k}}_{P_{ij}} + \underbrace{\overline{\frac{p'}{\rho}\left(\frac{\partial v_i'}{\partial x_j} + \frac{\partial v_j'}{\partial x_i}\right)}}_{\Pi_{ij}}$$

$$\underbrace{-\frac{\partial}{\partial x_k}\left[\overline{v_i' v_j' v_k'} + \frac{\overline{p' v_j'}}{\rho}\delta_{ik} + \frac{\overline{p' v_i'}}{\rho}\delta_{jk}\right]}_{D_{ij,t}} + \underbrace{\nu \frac{\partial^2 \overline{v_i' v_j'}}{\partial x_k \partial x_k}}_{D_{ij,\nu}}$$

$$\underbrace{-g_i \beta \overline{v_j' \theta'} - g_j \beta \overline{v_i' \theta'}}_{G_{ij}} - \underbrace{2\nu \overline{\frac{\partial v_i'}{\partial x_k} \frac{\partial v_j'}{\partial x_k}}}_{\varepsilon_{ij}}$$

►The pressure-strain term is the last unknown term that needs to be modelled in the $\overline{v_i' v_j'}$ equation, Eq. 11.11.

►When modeled, it is divided into two parts:

•

¶See Section 11.7.4, Slow pressure-strain term

$$
\underbrace{\bar{v}_k \frac{\partial \overline{v'_i v'_j}}{\partial x_k}}_{C_{ij}} = \underbrace{-\overline{v'_i v'_k} \frac{\partial \bar{v}_j}{\partial x_k} - \overline{v'_j v'_k} \frac{\partial \bar{v}_i}{\partial x_k}}_{P_{ij}} + \underbrace{\overline{\frac{p'}{\rho}\left(\frac{\partial v'_i}{\partial x_j} + \frac{\partial v'_j}{\partial x_i}\right)}}_{\Pi_{ij}}
$$

$$
\underbrace{-\frac{\partial}{\partial x_k}\left[\overline{v'_i v'_j v'_k} + \frac{\overline{p' v'_j}}{\rho}\delta_{ik} + \frac{\overline{p' v'_i}}{\rho}\delta_{jk}\right]}_{D_{ij,t}} + \underbrace{\nu \frac{\partial^2 \overline{v'_i v'_j}}{\partial x_k \partial x_k}}_{D_{ij,\nu}}
$$

$$
\underbrace{-g_i \beta \overline{v'_j \theta'} - g_j \beta \overline{v'_i \theta'}}_{G_{ij}} - \underbrace{2\nu \overline{\frac{\partial v'_i}{\partial x_k} \frac{\partial v'_j}{\partial x_k}}}_{\varepsilon_{ij}}
$$

►The pressure-strain term is the last unknown term that needs to be modelled in the $\overline{v'_i v'_j}$ equation, Eq. 11.11.

►When modeled, it is divided into two parts:

- a slow part which depends on turbulence

¶See Section 11.7.4, Slow pressure-strain term

$$
\underbrace{\bar{v}_k \frac{\partial \overline{v'_i v'_j}}{\partial x_k}}_{C_{ij}} = \underbrace{-\overline{v'_i v'_k} \frac{\partial \bar{v}_j}{\partial x_k} - \overline{v'_j v'_k} \frac{\partial \bar{v}_i}{\partial x_k}}_{P_{ij}} + \underbrace{\overline{\frac{p'}{\rho}\left(\frac{\partial v'_i}{\partial x_j} + \frac{\partial v'_j}{\partial x_i}\right)}}_{\Pi_{ij}}
$$

$$
\underbrace{-\frac{\partial}{\partial x_k}\left[\overline{v'_i v'_j v'_k} + \frac{\overline{p' v'_j}}{\rho}\delta_{ik} + \frac{\overline{p' v'_i}}{\rho}\delta_{jk}\right]}_{D_{ij,t}} + \underbrace{\nu \frac{\partial^2 \overline{v'_i v'_j}}{\partial x_k \partial x_k}}_{D_{ij,\nu}}
$$

$$
\underbrace{-g_i \beta \overline{v'_j \theta'} - g_j \beta \overline{v'_i \theta'}}_{G_{ij}} - \underbrace{2\nu \overline{\frac{\partial v'_i}{\partial x_k} \frac{\partial v'_j}{\partial x_k}}}_{\varepsilon_{ij}}
$$

►The pressure-strain term is the last unknown term that needs to be modelled in the $\overline{v'_i v'_j}$ equation, Eq. 11.11.

►When modeled, it is divided into two parts:

- a slow part which depends on turbulence
- a rapid part which depends on mean flow gradients and turbulence

¶See Section 11.7.4, Slow pressure-strain term

$$
\underbrace{\bar{v}_k\frac{\partial \overline{v'_i v'_j}}{\partial x_k}}_{C_{ij}} = \underbrace{-\overline{v'_i v'_k}\frac{\partial \bar{v}_j}{\partial x_k} - \overline{v'_j v'_k}\frac{\partial \bar{v}_i}{\partial x_k}}_{P_{ij}} + \underbrace{\overline{\frac{p'}{\rho}\left(\frac{\partial v'_i}{\partial x_j} + \frac{\partial v'_j}{\partial x_i}\right)}}_{\Pi_{ij}}
$$

$$
\underbrace{-\frac{\partial}{\partial x_k}\left[\overline{v'_i v'_j v'_k} + \frac{\overline{p' v'_j}}{\rho}\delta_{ik} + \frac{\overline{p' v'_i}}{\rho}\delta_{jk}\right]}_{D_{ij,t}} + \underbrace{\nu\frac{\partial^2 \overline{v'_i v'_j}}{\partial x_k \partial x_k}}_{D_{ij,\nu}}
$$

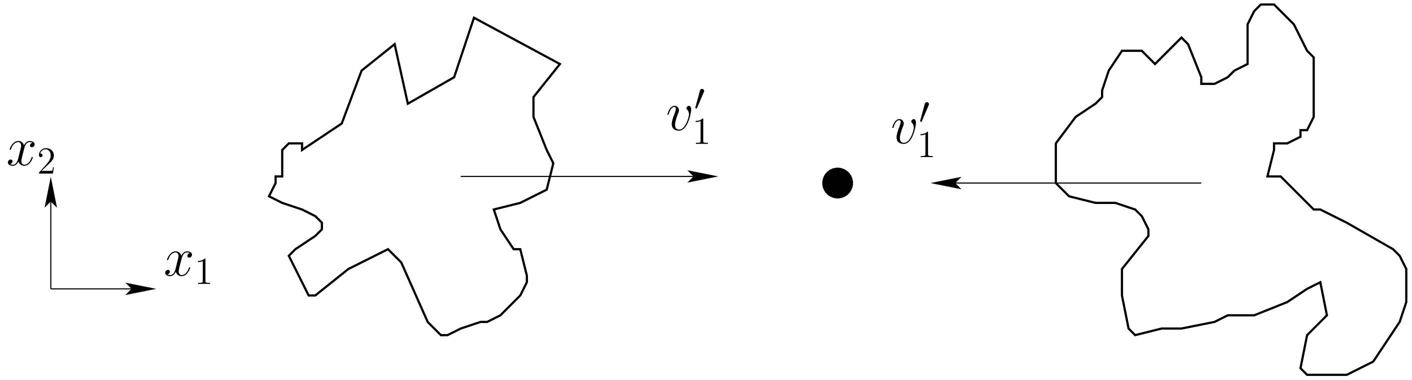
$$
\underbrace{-g_i\beta\overline{v'_j\theta'} - g_j\beta\overline{v'_i\theta'}}_{G_{ij}} - \underbrace{2\nu\overline{\frac{\partial v'_i}{\partial x_k}\frac{\partial v'_j}{\partial x_k}}}_{\varepsilon_{ij}}
$$

►The pressure-strain term is the last unknown term that needs to be modelled in the $\overline{v'_i v'_j}$ equation, Eq. 11.11.

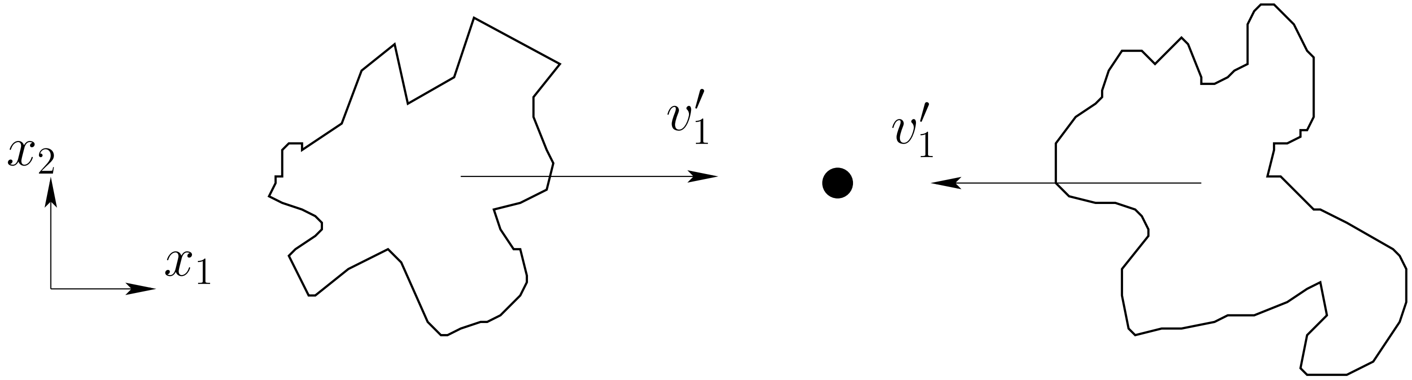
►When modeled, it is divided into two parts:

- a slow part which depends on turbulence
- a rapid part which depends on mean flow gradients and turbulence

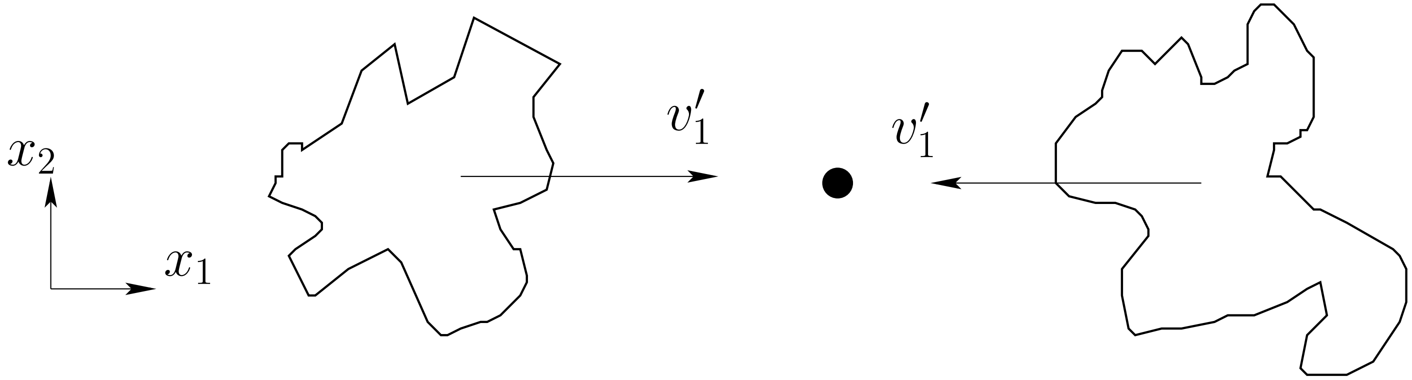
►We start with the slow part of the pressure-strain term



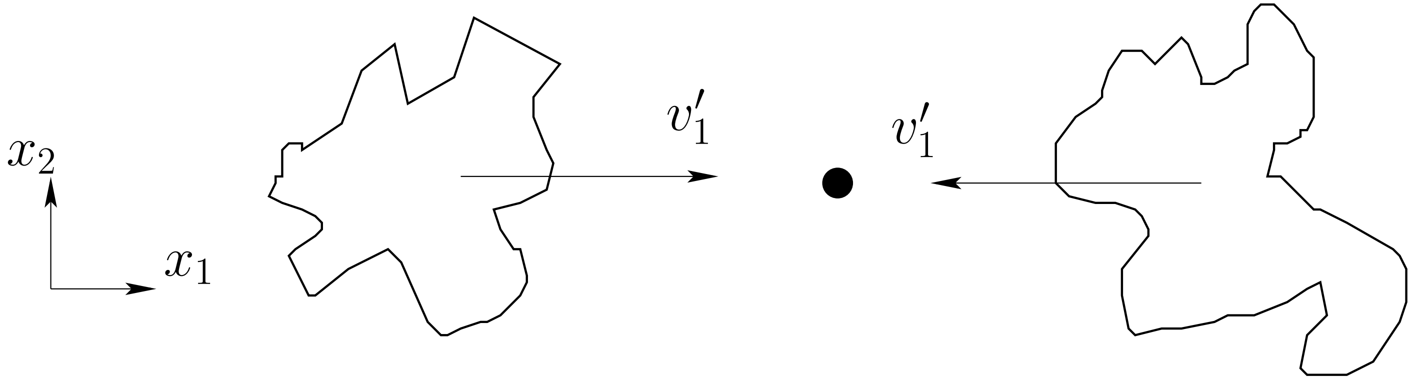
x_2
x_1
v_1'
v_1'



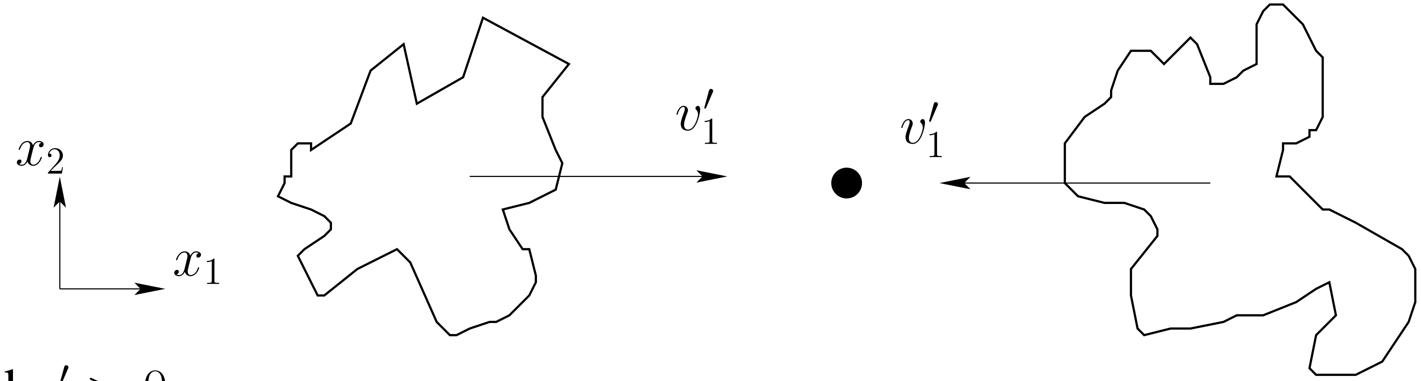
x_2
x_1
v_1'
v_1'



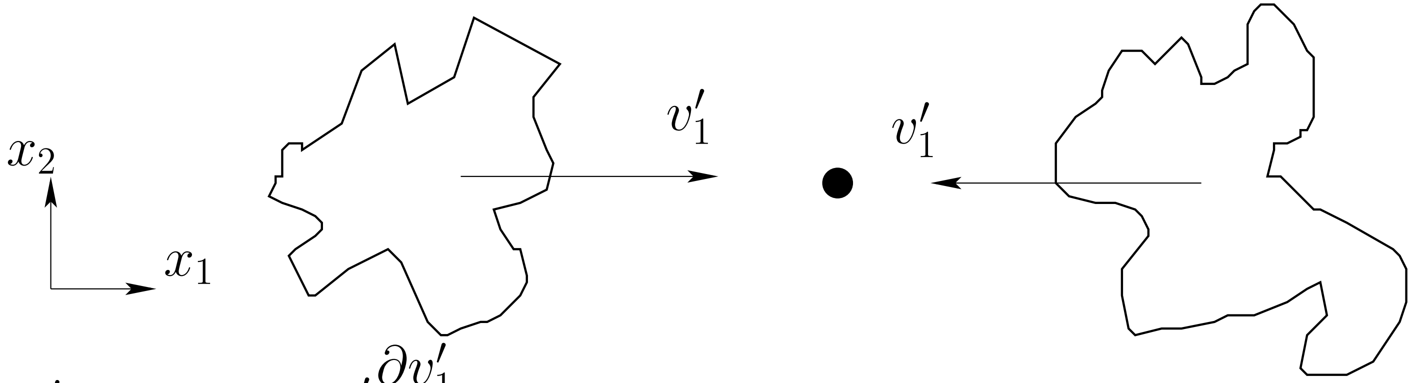
x_2
x_1
v'_1
v'_1



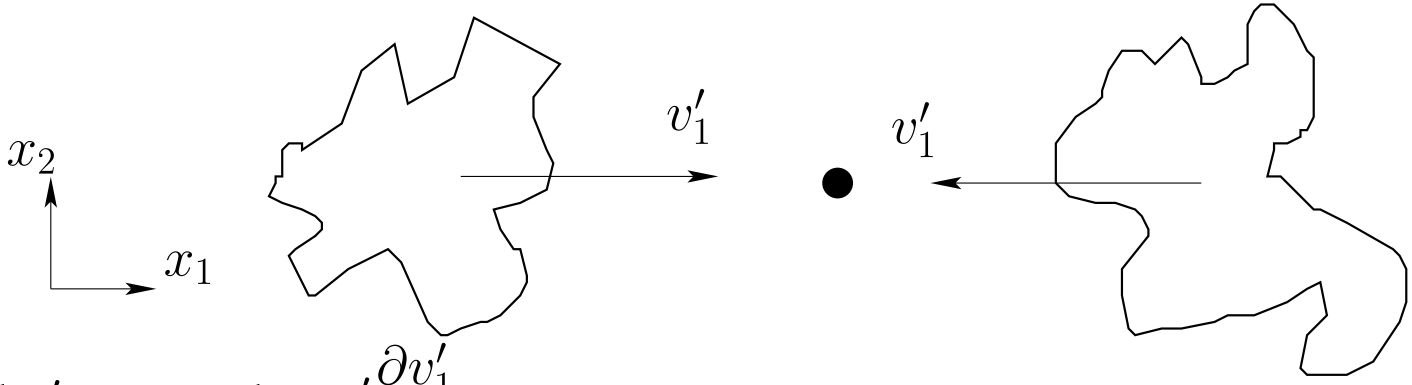
$\partial v_1'/\partial x_1 < 0$



$\partial v_1'/\partial x_1 < 0$ and $p' > 0$

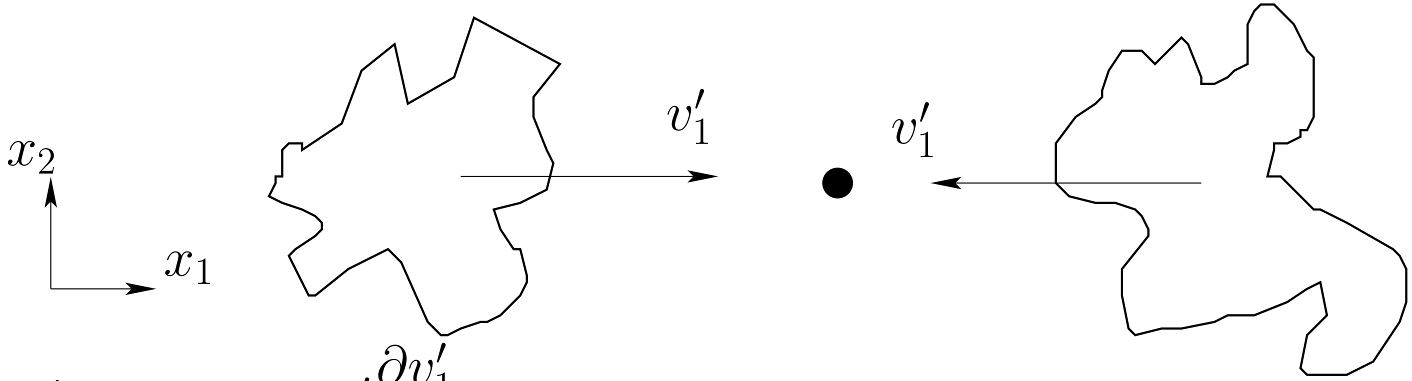


$\partial v_1'/\partial x_1 < 0$ and $p' > 0$ so that $p'\dfrac{\partial v_1'}{\partial x_1} < 0$



$\partial v_1'/\partial x_1 < 0$ **and** $p' > 0$ **so that** $p'\dfrac{\partial v_1'}{\partial x_1} < 0$

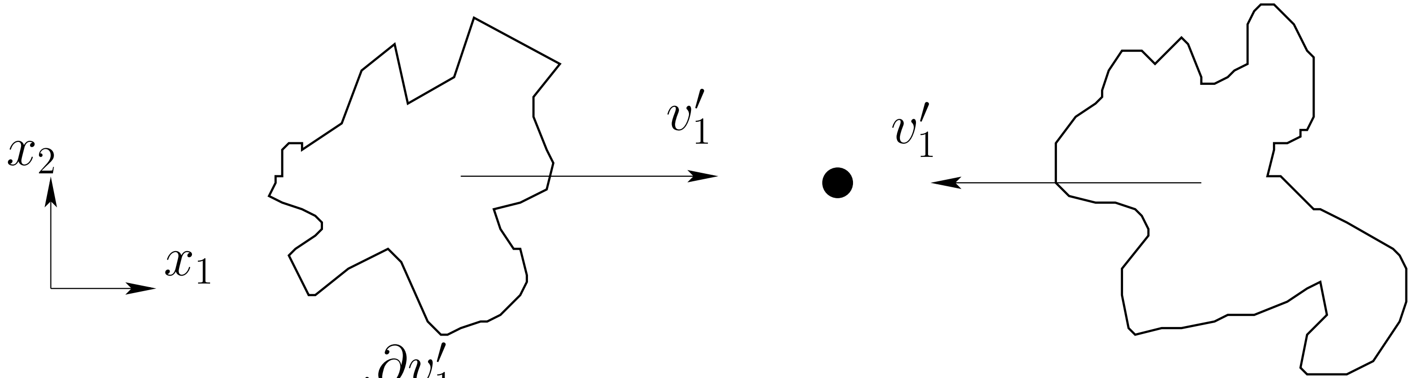
$$\frac{\partial v_2'}{\partial x_2} > 0, \quad \frac{\partial v_3'}{\partial x_3} > 0$$



$\partial v_1'/\partial x_1 < 0$ and $p' > 0$ so that $p'\dfrac{\partial v_1'}{\partial x_1} < 0$

$$\frac{\partial v_2'}{\partial x_2} > 0, \quad \frac{\partial v_3'}{\partial x_3} > 0$$

If this happens then



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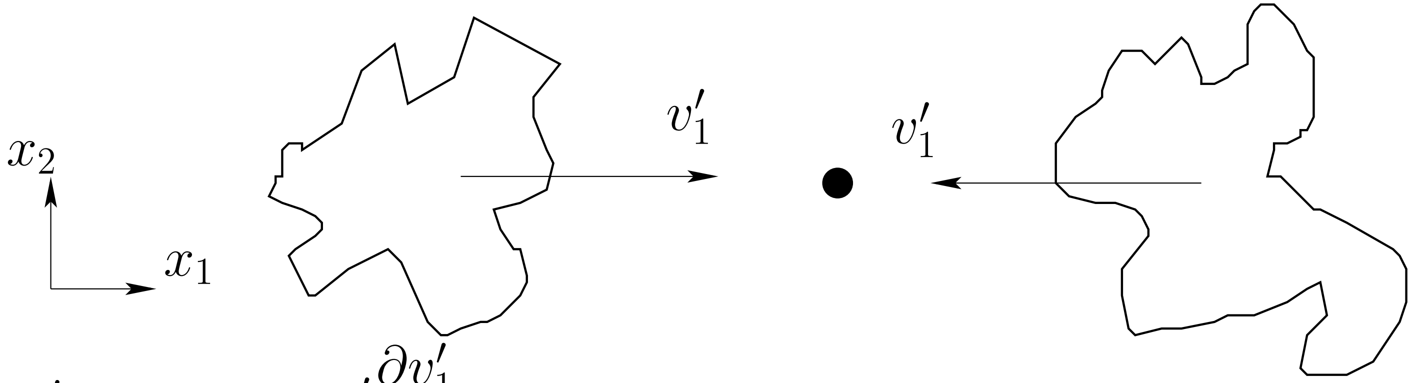
If this happens then

$$\overline{v_1'^2} > \overline{v_2'^2}, \overline{v_1'^2} > \overline{v_3'^2}$$



$\partial v'_1/\partial x_1 < 0$ and $p' > 0$ so that $p'\dfrac{\partial v'_1}{\partial x_1} < 0$

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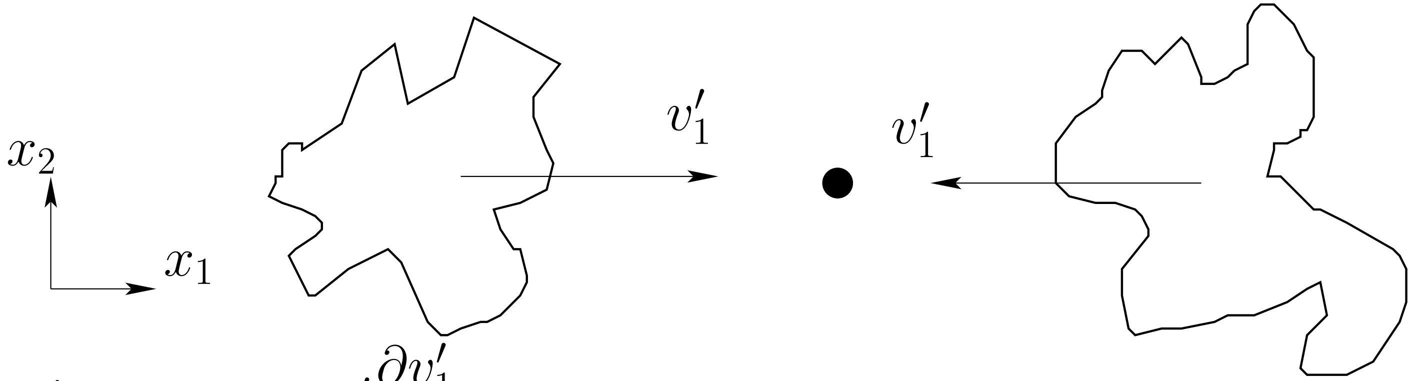
$$\overline{v'^2_1} > \overline{v'^2_2}, \overline{v'^2_1} > \overline{v'^2_3}$$



$\partial v_1'/\partial x_1 < 0$ and $p' > 0$ so that $p'\dfrac{\partial v_1'}{\partial x_1} < 0$

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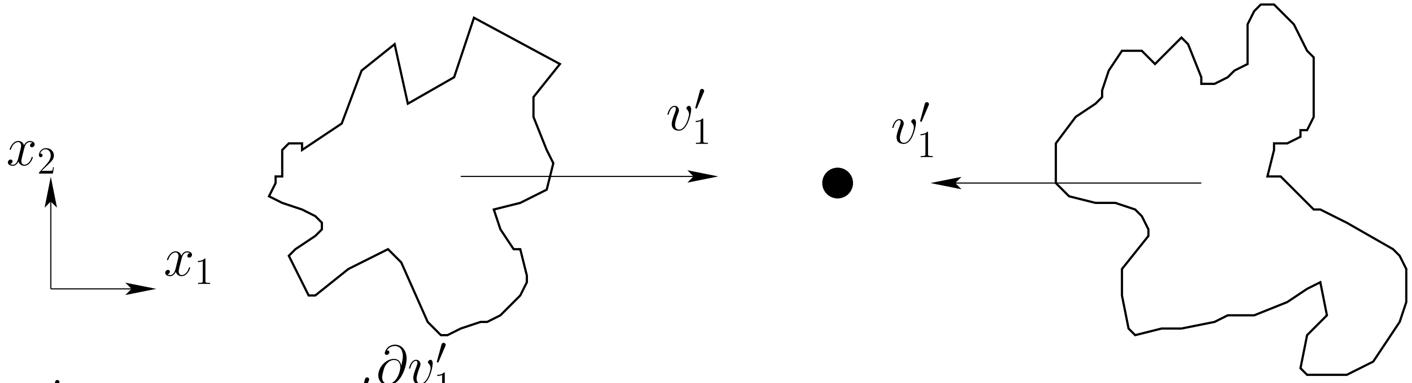
If this happens then

$$\overline{v_1'^2} > \overline{v_2'^2}, \overline{v_1'^2} > \overline{v_3'^2}$$

$$\overline{p'\frac{\partial v_1'}{\partial x_1}}$$



$\partial v'_1/\partial x_1 < 0$ and $p' > 0$ so that $p'\dfrac{\partial v'_1}{\partial x_1} < 0$

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If this happens then

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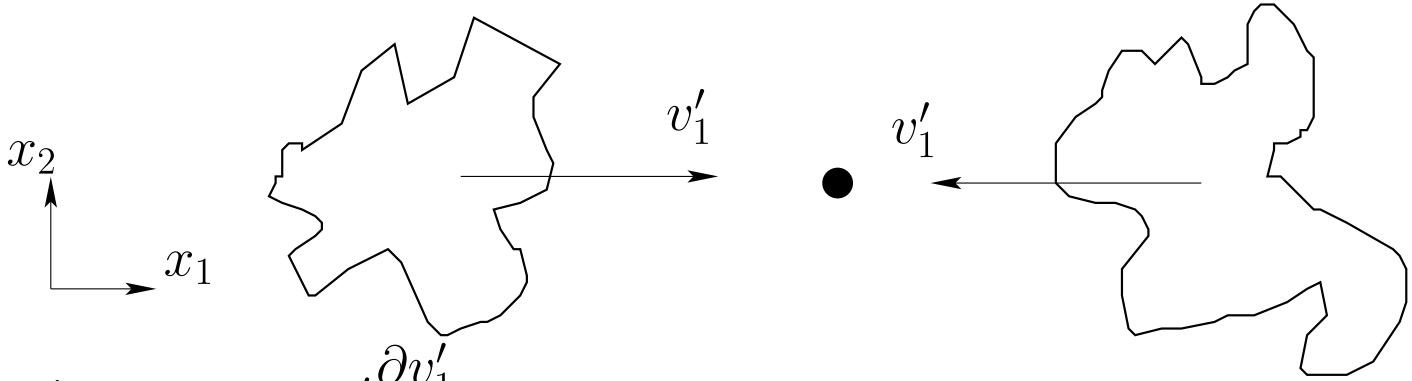
$$\overline{p'\frac{\partial v'_1}{\partial x_1}} \propto$$



$\partial v'_1/\partial x_1 < 0$ and $p' > 0$ so that $p'\dfrac{\partial v'_1}{\partial x_1} < 0$

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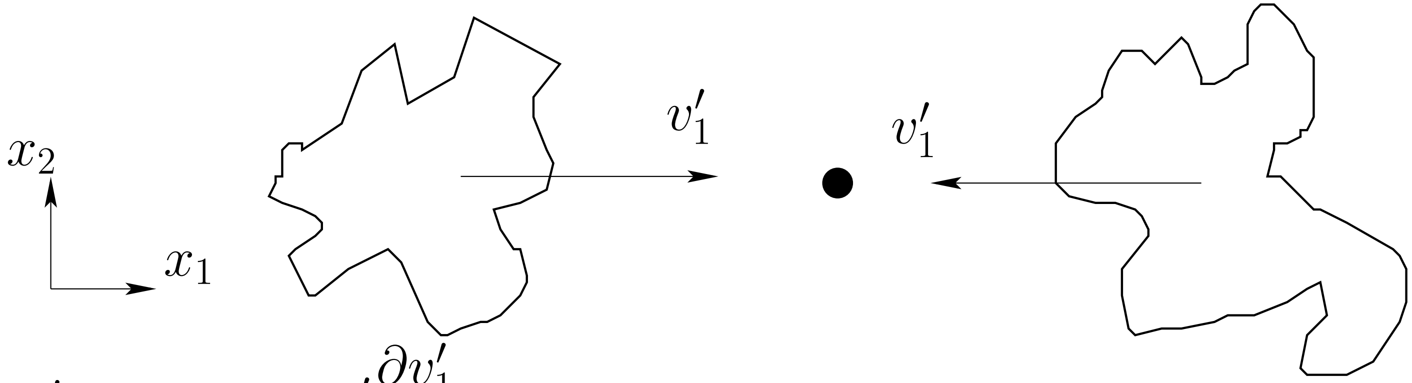
$$\overline{v'^2_1} > \overline{v'^2_2}, \overline{v'^2_1} > \overline{v'^2_3}$$

$$\overline{p'\frac{\partial v'_1}{\partial x_1}} \ \propto \ -\frac{\rho}{2t}\left[\left(\overline{v'^2_1} - \overline{v'^2_2}\right) + \left(\overline{v'^2_1} - \overline{v'^2_3}\right)\right]$$



$\partial v_1'/\partial x_1 < 0$ and $p' > 0$ so that $p'\dfrac{\partial v_1'}{\partial x_1} < 0$

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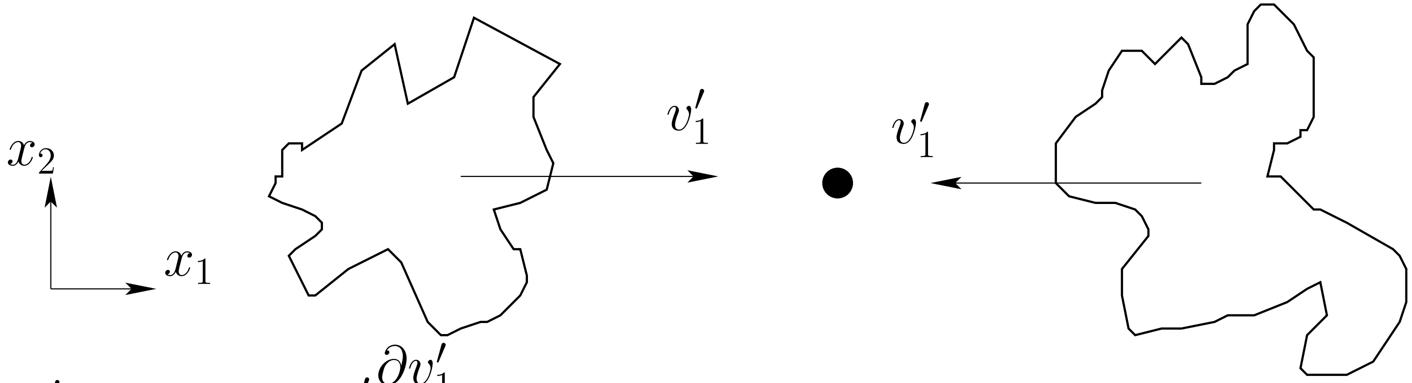
$$\overline{p'\frac{\partial v_1'}{\partial x_1}} \propto -\frac{\rho}{2t}\left[\left(\overline{v_1'^2} - \overline{v_2'^2}\right) + \left(\overline{v_1'^2} - \overline{v_3'^2}\right)\right] = -\frac{\rho}{t}\left[\overline{v_1'^2} - \frac{1}{2}\left(\overline{v_2'^2} + \overline{v_3'^2}\right)\right]$$



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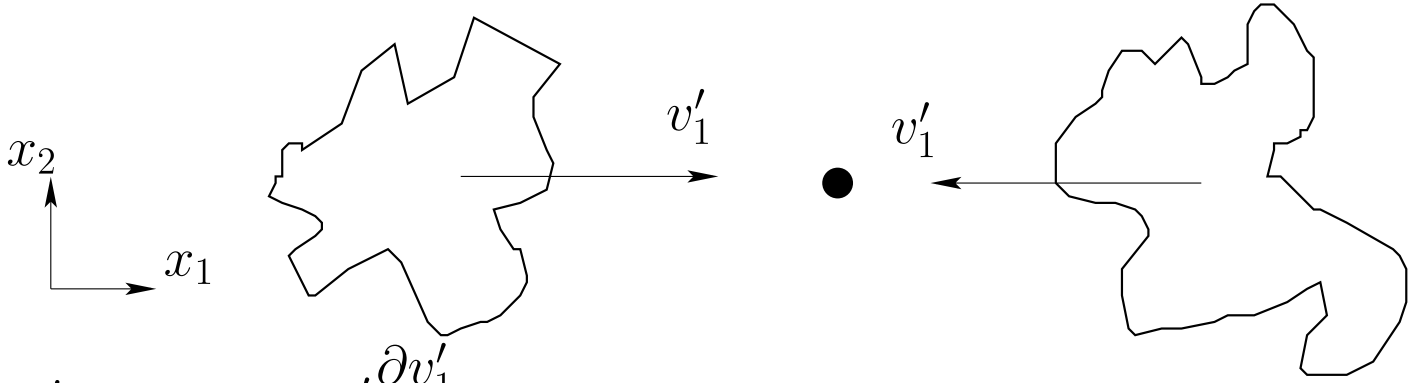
$$=$$



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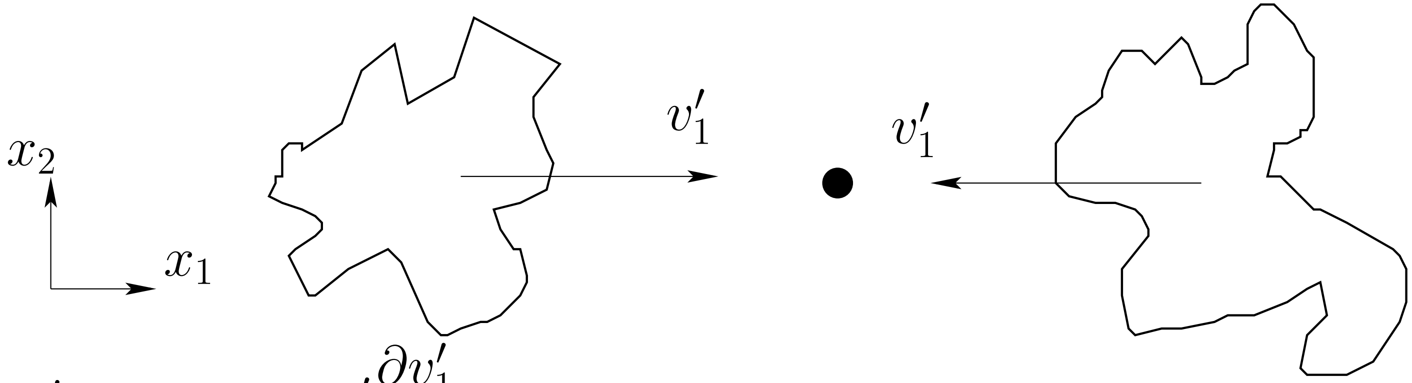
If this happens then

$$\overline{v_1'^2} > \overline{v_2'^2}, \overline{v_1'^2} > \overline{v_3'^2}$$

$$\begin{aligned}
\overline{p'\frac{\partial v_1'}{\partial x_1}} &\propto -\frac{\rho}{2t}\left[\left(\overline{v_1'^2} - \overline{v_2'^2}\right) + \left(\overline{v_1'^2} - \overline{v_3'^2}\right)\right] = -\frac{\rho}{t}\left[\overline{v_1'^2} - \frac{1}{2}\left(\overline{v_2'^2} + \overline{v_3'^2}\right)\right] \\
&= -\frac{\rho}{t}\left[\frac{3}{2}\overline{v_1'^2} - \frac{1}{2}\left(\overline{v_1'^2} + \overline{v_2'^2} + \overline{v_3'^2}\right)\right]
\end{aligned}$$



$\partial v'_1/\partial x_1 < 0$ and $p' > 0$ so that $p'\dfrac{\partial v'_1}{\partial x_1} < 0$

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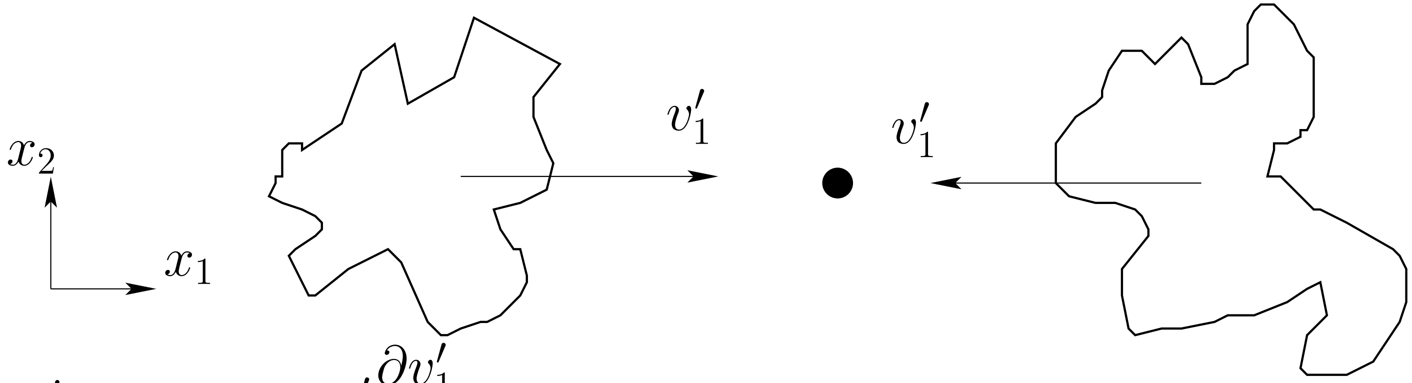
$$\overline{v'^2_1} > \overline{v'^2_2}, \overline{v'^2_1} > \overline{v'^2_3}$$

$$\begin{aligned}
\overline{p'\frac{\partial v'_1}{\partial x_1}} &\propto -\frac{\rho}{2t}\left[\left(\overline{v'^2_1} - \overline{v'^2_2}\right) + \left(\overline{v'^2_1} - \overline{v'^2_3}\right)\right] = -\frac{\rho}{t}\left[\overline{v'^2_1} - \frac{1}{2}\left(\overline{v'^2_2} + \overline{v'^2_3}\right)\right] \\
&= -\frac{\rho}{t}\left[\frac{3}{2}\overline{v'^2_1} - \frac{1}{2}\left(\overline{v'^2_1} + \overline{v'^2_2} + \overline{v'^2_3}\right)\right] = -\frac{\rho}{t}\left(\frac{3}{2}\overline{v'^2_1} - k\right)
\end{aligned}$$



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$$\begin{aligned}
\overline{p'\frac{\partial v'_1}{\partial x_1}} &\propto -\frac{\rho}{2t}\left[\left(\overline{v_1'^2}-\overline{v_2'^2}\right)+\left(\overline{v_1'^2}-\overline{v_3'^2}\right)\right]=-\frac{\rho}{t}\left[\overline{v_1'^2}-\frac{1}{2}\left(\overline{v_2'^2}+\overline{v_3'^2}\right)\right]\\
&= -\frac{\rho}{t}\left[\frac{3}{2}\overline{v_1'^2}-\frac{1}{2}\left(\overline{v_1'^2}+\overline{v_2'^2}+\overline{v_3'^2}\right)\right]=-\frac{\rho}{t}\left(\frac{3}{2}\overline{v_1'^2}-k\right)
\end{aligned}$$

$$\Phi_{ij,1} \equiv \overline{p'\left(\frac{\partial v'_i}{\partial x_j}+\frac{\partial v'_j}{\partial x_i}\right)} = -c_1\rho\frac{\varepsilon}{k}\left(\overline{v'_i v'_j}-\frac{2}{3}\delta_{ij}k\right) \tag{31.2}$$

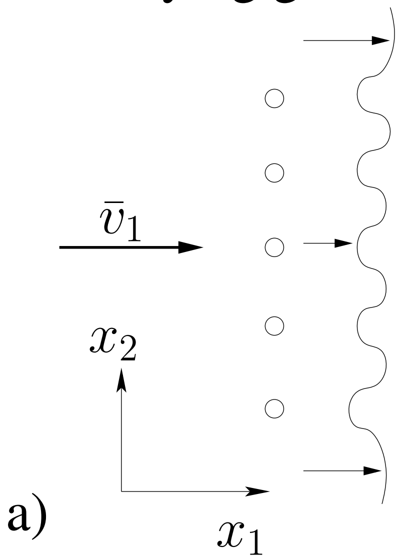
see Eq. 11.57

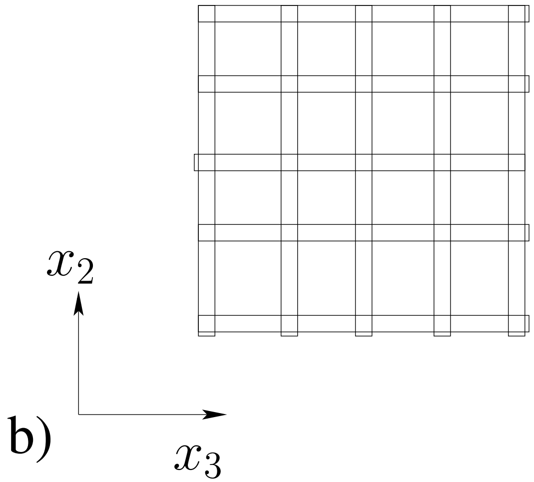
¶See Eq. 11.2

►Decaying grid turbulence

¶See Eq. 11.2

▶Decaying grid turbulence





¶See Eq. 11.2

▶Decaying grid turbulence

▶$\overline{v_i'v_j'}$ equation

$$\bar{v}_1 \frac{d\overline{v_i'v_j'}}{dx_1} = \overline{\frac{p'}{\rho}\left(\frac{\partial v_i'}{\partial x_j} + \frac{\partial v_j'}{\partial x_i}\right)} - \varepsilon_{ij} \tag{31.3}$$

¶See Eq. 11.2

▶Decaying grid turbulence

▶$\overline{v_i' v_j'}$ equation

$$\bar{v}_1 \frac{d\overline{v_i' v_j'}}{dx_1} = \overline{\frac{p'}{\rho}\left(\frac{\partial v_i'}{\partial x_j} + \frac{\partial v_j'}{\partial x_i}\right)} - \varepsilon_{ij} \tag{31.3}$$

An anisotropy stress tensor is defined as

(31.4)

¶See Eq. 11.2

▶Decaying grid turbulence

▶$\overline{v_i' v_j'}$ equation

$$\bar{v}_1 \frac{d\overline{v_i' v_j'}}{dx_1} = \overline{\frac{p'}{\rho}\left(\frac{\partial v_i'}{\partial x_j} + \frac{\partial v_j'}{\partial x_i}\right)} - \varepsilon_{ij} \tag{31.3}$$

An anisotropy stress tensor is defined as

$$a_{ij} = \frac{\overline{v_i' v_j'}}{k} - \frac{2}{3}\delta_{ij} \tag{31.4}$$

¶See Eq. 11.2

▶Decaying grid turbulence

▶$\overline{v_i'v_j'}$ equation

$$\bar{v}_1 \frac{d\overline{v_i'v_j'}}{dx_1} = \overline{\frac{p'}{\rho}\left(\frac{\partial v_i'}{\partial x_j} + \frac{\partial v_j'}{\partial x_i}\right)} - \varepsilon_{ij} \tag{31.3}$$

An anisotropy stress tensor is defined as

$$a_{ij} = \frac{\overline{v_i'v_j'}}{k} - \frac{2}{3}\delta_{ij} \quad \Rightarrow \tag{31.4}$$

¶See Eq. 11.2

▶Decaying grid turbulence

▶$\overline{v_i'v_j'}$ equation

$$\bar{v}_1 \frac{d\overline{v_i'v_j'}}{dx_1} = \overline{\frac{p'}{\rho}\left(\frac{\partial v_i'}{\partial x_j} + \frac{\partial v_j'}{\partial x_i}\right)} - \varepsilon_{ij} \tag{31.3}$$

An anisotropy stress tensor is defined as

$$a_{ij} = \frac{\overline{v_i'v_j'}}{k} - \frac{2}{3}\delta_{ij} \quad \Rightarrow \quad \overline{v_i'v_j'} = k a_{ij} + \frac{2k}{3}\delta_{ij} \tag{31.4}$$

¶See Eq. 11.2

▶Decaying grid turbulence

▶$\overline{v_i'v_j'}$ equation

$$\bar{v}_1 \frac{d\overline{v_i'v_j'}}{dx_1} = \overline{\frac{p'}{\rho}\left(\frac{\partial v_i'}{\partial x_j} + \frac{\partial v_j'}{\partial x_i}\right)} - \varepsilon_{ij} \tag{31.3}$$

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$$a_{ij} = \frac{\overline{v_i'v_j'}}{k} - \frac{2}{3}\delta_{ij} \quad \Rightarrow \quad \overline{v_i'v_j'} = k a_{ij} + \frac{2k}{3}\delta_{ij} \tag{31.4}$$

In isotropic turbulence, $a_{ij} = 0$.

¶See Eq. 11.2

▶Decaying grid turbulence

▶$\overline{v'_i v'_j}$ equation

$$\bar{v}_1 \frac{d\overline{v'_i v'_j}}{dx_1} = \overline{\frac{p'}{\rho}\left(\frac{\partial v'_i}{\partial x_j} + \frac{\partial v'_j}{\partial x_i}\right)} - \varepsilon_{ij} \tag{31.3}$$

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$$a_{ij} = \frac{\overline{v'_i v'_j}}{k} - \frac{2}{3}\delta_{ij} \quad \Rightarrow \quad \overline{v'_i v'_j} = k a_{ij} + \frac{2k}{3}\delta_{ij} \tag{31.4}$$

In isotropic turbulence, $a_{ij} = 0$.

We insert Eq. 31.4 into Eq. 31.3 and use the models for the pressure strain term, $\phi_{ij,1}$ (Eq. 31.2)

¶See Eq. 11.2

▶Decaying grid turbulence

▶$\overline{v_i'v_j'}$ equation

$$\bar v_1 \frac{d\overline{v_i'v_j'}}{dx_1} = \overline{\frac{p'}{\rho}\left(\frac{\partial v_i'}{\partial x_j}+\frac{\partial v_j'}{\partial x_i}\right)} - \varepsilon_{ij} \tag{31.3}$$

An anisotropy stress tensor is defined as

$$a_{ij} = \frac{\overline{v_i'v_j'}}{k} - \frac{2}{3}\delta_{ij} \quad \Rightarrow \quad \overline{v_i'v_j'} = k a_{ij} + \frac{2k}{3}\delta_{ij} \tag{31.4}$$

In isotropic turbulence, $a_{ij} = 0$.

We insert Eq. 31.4 into Eq. 31.3 and use the models for the pressure strain term, $\phi_{ij,1}$ (Eq. 31.2) and dissipation, $\varepsilon_{ij} = (2/3)\delta_{ij}$ (Eq. 31.1) so that

¶See Eq. 11.2

▶Decaying grid turbulence

▶$\overline{v_i' v_j'}$ equation

$$\bar{v}_1 \frac{d\overline{v_i' v_j'}}{dx_1} = \overline{\frac{p'}{\rho}\left(\frac{\partial v_i'}{\partial x_j} + \frac{\partial v_j'}{\partial x_i}\right)} - \varepsilon_{ij} \tag{31.3}$$

An anisotropy stress tensor is defined as

$$a_{ij} = \frac{\overline{v_i' v_j'}}{k} - \frac{2}{3}\delta_{ij} \quad \Rightarrow \quad \overline{v_i' v_j'} = k a_{ij} + \frac{2k}{3}\delta_{ij} \tag{31.4}$$

In isotropic turbulence, $a_{ij} = 0$.

We insert Eq. 31.4 into Eq. 31.3 and use the models for the pressure strain term, $\phi_{ij,1}$ (Eq. 31.2) and dissipation, $\varepsilon_{ij} = (2/3)\delta_{ij}$ (Eq. 31.1) so that

$$\bar{v}_1 \left(\frac{d(k a_{ij})}{dx_1} + \frac{2}{3}\delta_{ij}\frac{dk}{dx_1} \right) = -c_1 \varepsilon a_{ij} - \frac{2}{3}\delta_{ij}\varepsilon$$

$$\bar{v}_1 \left(\frac{d(k a_{ij})}{dx_1} + \frac{2}{3}\delta_{ij}\frac{dk}{dx_1} \right) = \underline{-c_1 \varepsilon a_{ij} - \frac{2}{3}\delta_{ij}\varepsilon}$$

$$\bar{v}_1 \left(\frac{d(k a_{ij})}{dx_1} + \frac{2}{3} \delta_{ij} \frac{dk}{dx_1} \right) = \underline{-c_1 \varepsilon a_{ij} - \frac{2}{3} \delta_{ij} \varepsilon}$$

$$\bar{v}_1 \left(k \frac{d a_{ij}}{dx_1} + a_{ij} \frac{dk}{dx_1} + \frac{2}{3} \delta_{ij} \frac{dk}{dx_1} \right) = \underline{-c_1 \varepsilon a_{ij} - \frac{2}{3} \delta_{ij} \varepsilon}$$

$$\bar{v}_1 \left(\frac{d(k a_{ij})}{dx_1} + \frac{2}{3}\delta_{ij}\frac{dk}{dx_1} \right) = \underline{-c_1 \varepsilon a_{ij} - \frac{2}{3}\delta_{ij}\varepsilon}$$

$$\bar{v}_1 \left(k\frac{da_{ij}}{dx_1} + a_{ij}\frac{dk}{dx_1} + \frac{2}{3}\delta_{ij}\frac{dk}{dx_1} \right) = \underline{-c_1 \varepsilon a_{ij} - \frac{2}{3}\delta_{ij}\varepsilon}$$

► Using the k equation, $\bar{v}_1 \frac{dk}{dx_1} = -\varepsilon$, and dividing by k give

$$\bar{v}_1\left(\frac{d(ka_{ij})}{dx_1}+\frac{2}{3}\delta_{ij}\frac{dk}{dx_1}\right)=\underline{-c_1\varepsilon a_{ij}-\frac{2}{3}\delta_{ij}\varepsilon}$$

$$\bar{v}_1\left(k\frac{da_{ij}}{dx_1}+a_{ij}\frac{dk}{dx_1}+\frac{2}{3}\delta_{ij}\frac{dk}{dx_1}\right)=\underline{-c_1\varepsilon a_{ij}-\frac{2}{3}\delta_{ij}\varepsilon}$$

▶Using the k equation, $\bar{v}_1\frac{dk}{dx_1}=-\varepsilon$, and dividing by k give

$$\bar{v}_1\frac{da_{ij}}{dx_1}=\underline{-c_1\frac{\varepsilon}{k}a_{ij}-\frac{2}{3}\delta_{ij}\frac{\varepsilon}{k}}+\frac{\varepsilon}{k}a_{ij}+\frac{2}{3}\delta_{ij}\frac{\varepsilon}{k}=\frac{\varepsilon}{k}a_{ij}(1-c_1)$$

$$\bar{v}_1\left(\frac{d(ka_{ij})}{dx_1}+\frac{2}{3}\delta_{ij}\frac{dk}{dx_1}\right)=\underline{-c_1\varepsilon a_{ij}-\frac{2}{3}\delta_{ij}\varepsilon}$$

$$\bar{v}_1\left(k\frac{da_{ij}}{dx_1}+a_{ij}\frac{dk}{dx_1}+\frac{2}{3}\delta_{ij}\frac{dk}{dx_1}\right)=\underline{-c_1\varepsilon a_{ij}-\frac{2}{3}\delta_{ij}\varepsilon}$$

►Using the k equation, $\bar{v}_1\frac{dk}{dx_1}=-\varepsilon$, and dividing by k give

$$\bar{v}_1\frac{da_{ij}}{dx_1}=\underline{-c_1\frac{\varepsilon}{k}a_{ij}-\frac{2}{3}\delta_{ij}}\frac{\varepsilon}{k}+\frac{\varepsilon}{k}a_{ij}+\frac{2}{3}\delta_{ij}\frac{\varepsilon}{k}=\frac{\varepsilon}{k}a_{ij}(1-c_1)$$

$da_{ij}/dx<0$ (the turbulence becomes isotropic). Hence we find that c_1 must be larger than one.

On-line Lecture 3

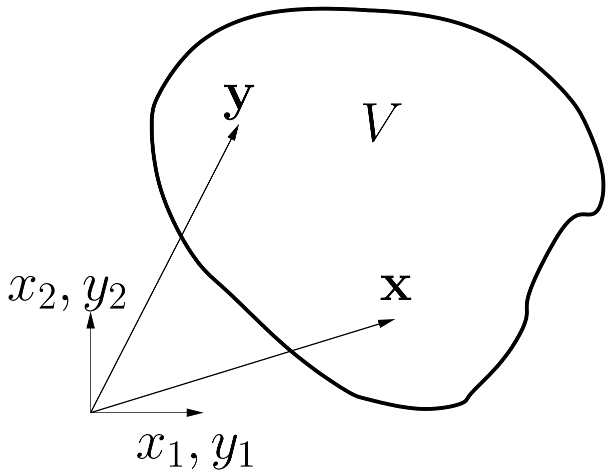
¶See Section 11.7.5, Rapid pressure-strain term

▶Pressure strain: rapid part

On-line Lecture 3

¶See Section 11.7.5, Rapid pressure-strain term

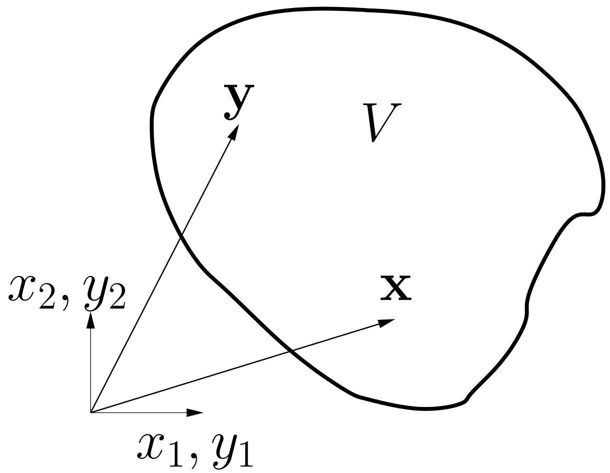
►Pressure strain: rapid part



On-line Lecture 3

¶See Section 11.7.5, Rapid pressure-strain term

▶Pressure strain: rapid part

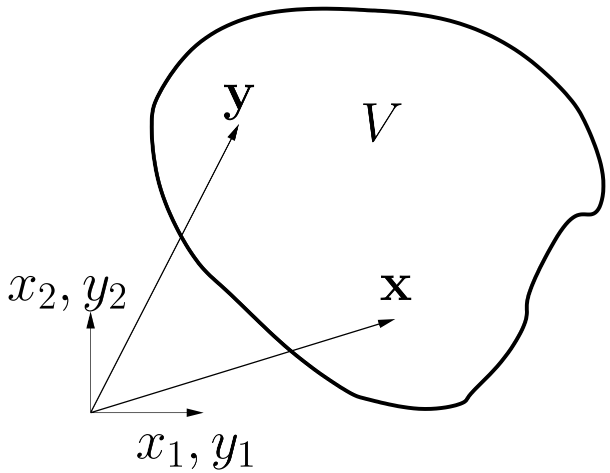


1.

On-line Lecture 3

¶See Section 11.7.5, Rapid pressure-strain term

▶Pressure strain: rapid part

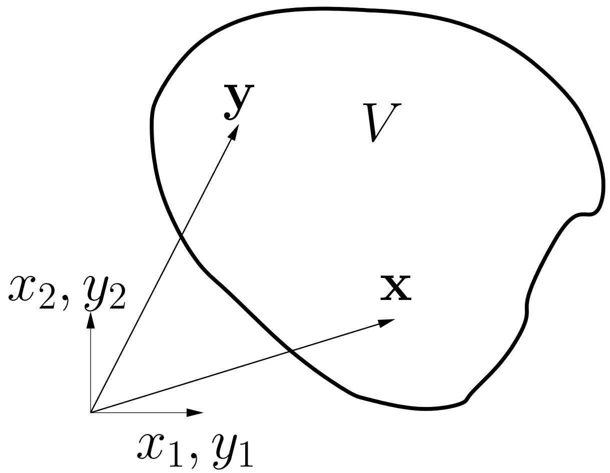


1. Take the divergence of the incompressible Navier-Stokes equation assuming constant viscosity (see Eq. 6.6)

On-line Lecture 3

¶See Section 11.7.5, Rapid pressure-strain term

►Pressure strain: rapid part

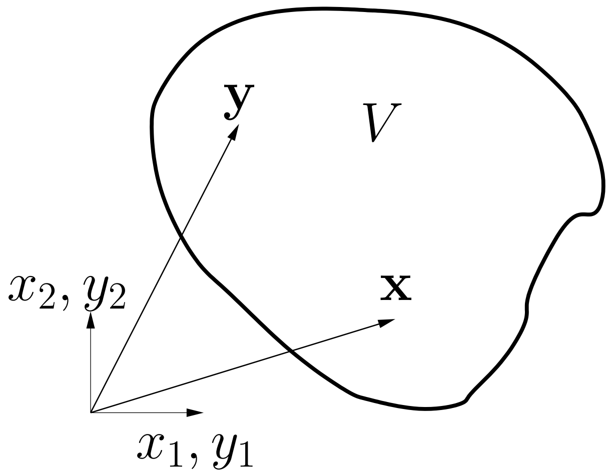


1. Take the divergence of the incompressible Navier-Stokes equation assuming constant viscosity (see Eq. 6.6) i.e. $\frac{\partial}{\partial x_i}\left(v_j\frac{\partial v_i}{\partial x_j}\right) = \frac{\partial}{\partial x_i}\left(-\frac{\partial p}{\partial x_i} + \nu\frac{\partial^2 v_i}{\partial x_j \partial x_j}\right) \ldots \Rightarrow$ Equation **A**.

On-line Lecture 3

¶See Section 11.7.5, Rapid pressure-strain term

▶Pressure strain: rapid part



1. Take the divergence of the incompressible Navier-Stokes equation assuming constant viscosity (see Eq. 6.6) i.e. $\frac{\partial}{\partial x_i}\left(v_j\frac{\partial v_i}{\partial x_j}\right) = \frac{\partial}{\partial x_i}\left(-\frac{\partial p}{\partial x_i} + \nu\frac{\partial^2 v_i}{\partial x_j \partial x_j}\right)\ldots \Rightarrow$ Equation **A**.
2. Take the divergence of the incompressible time-averaged Navier-Stokes equation assuming constant viscosity (see Eq. 6.10)

On-line Lecture 3

¶See Section 11.7.5, Rapid pressure-strain term

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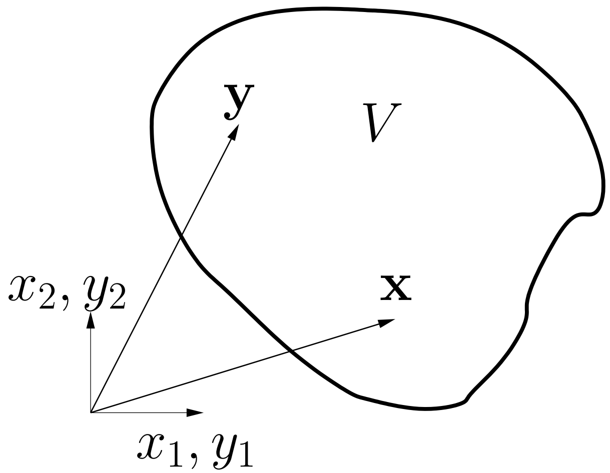

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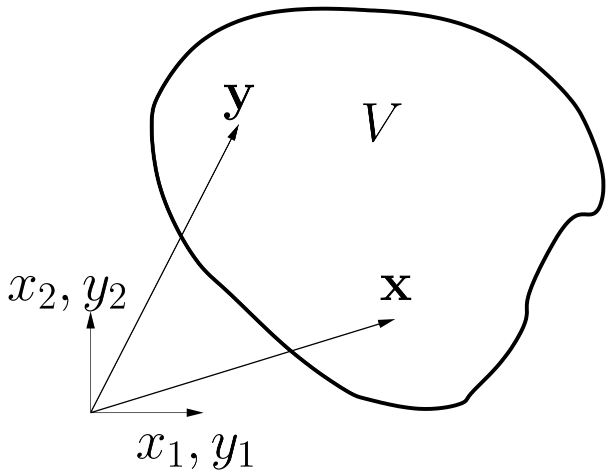
On-line Lecture 3

¶See Section 11.7.5, Rapid pressure-strain term

▶Pressure strain: rapid part



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Eq. A - Eq. B gives

On-line Lecture 3

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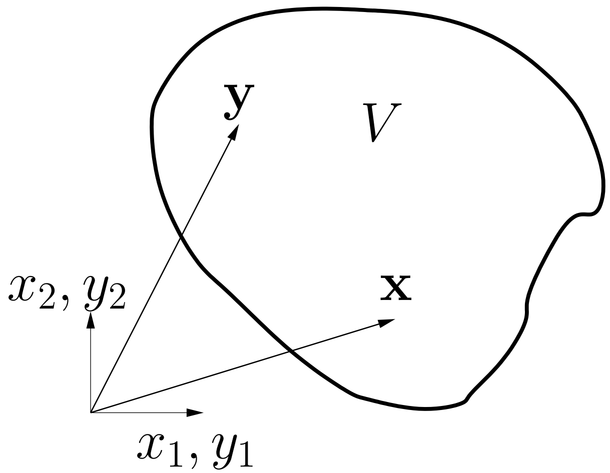

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$$\frac{1}{\rho}\frac{\partial^2 p'}{\partial x_j \partial x_j} = -\underbrace{2\frac{\partial \bar{v}_i}{\partial x_j}\frac{\partial v'_j}{\partial x_i}}_{\text{rapid term}} - \underbrace{\frac{\partial^2}{\partial x_i \partial x_j}\left(v'_i v'_j - \overline{v'_i v'_j}\right)}_{\text{slow term}} \tag{32.1}$$

On-line Lecture 3

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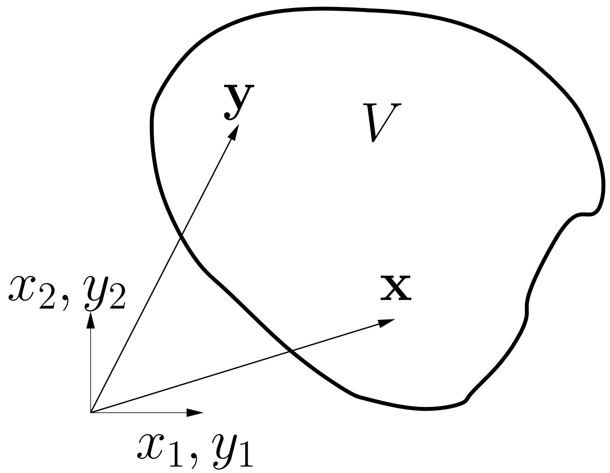

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►Multiply Eq. 32.2 with $\partial v'_i/\partial x_j + \partial v'_j/\partial x_i$ and average:

$$p'(\mathbf{x}) = \frac{\rho}{2\pi}\frac{\partial \bar{v}_i(\mathbf{x})}{\partial x_j}\int_V \frac{\partial v'_j(\mathbf{y})}{\partial y_i}\frac{d\mathbf{y}^3}{|\mathbf{y}-\mathbf{x}|} - \frac{\rho}{4\pi}\int_V \frac{\partial^2}{\partial y_i \partial y_j}\left(v'_i(\mathbf{y})v'_j(\mathbf{y})\right)\frac{d\mathbf{y}^3}{|\mathbf{y}-\mathbf{x}|}$$

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$$\begin{aligned}\overline{\frac{p'(\mathbf{x})}{\rho}\left(\frac{\partial v'_i(\mathbf{x})}{\partial x_j}+\frac{\partial v'_j(\mathbf{x})}{\partial x_i}\right)} &= \frac{\partial \bar{v}_k(\mathbf{x})}{\partial x_\ell}\underbrace{\frac{1}{2\pi}\int_V \overline{\left(\frac{\partial v'_i(\mathbf{x})}{\partial x_j}+\frac{\partial v'_j(\mathbf{x})}{\partial x_i}\right)\frac{\partial v'_\ell(\mathbf{y})}{\partial y_k}}\frac{d\mathbf{y}^3}{|\mathbf{y}-\mathbf{x}|}}_{M_{ijk\ell}} \\ &+ \underbrace{\frac{1}{4\pi}\int_V \overline{\left(\frac{\partial v'_i(\mathbf{x})}{\partial x_j}+\frac{\partial v'_j(\mathbf{x})}{\partial x_i}\right)\frac{\partial^2}{\partial y_k \partial y_\ell}\left(v'_k(\mathbf{y})v'_\ell(\mathbf{y})\right)}\frac{d\mathbf{y}^3}{|\mathbf{y}-\mathbf{x}|}}_{A_{ij}}\end{aligned} \tag{32.3}$$

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Short form of Eq. 32.3:

$$p'(\mathbf{x}) = \frac{\rho}{2\pi}\frac{\partial \bar{v}_i(\mathbf{x})}{\partial x_j}\int_V \frac{\partial v'_j(\mathbf{y})}{\partial y_i}\frac{d\mathbf{y}^3}{|\mathbf{y}-\mathbf{x}|} - \frac{\rho}{4\pi}\int_V \frac{\partial^2}{\partial y_i \partial y_j}\left(v'_i(\mathbf{y})v'_j(\mathbf{y})\right)\frac{d\mathbf{y}^3}{|\mathbf{y}-\mathbf{x}|}$$

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Short form of Eq. 32.3:

$$\overline{\frac{p'}{\rho}\left(\frac{\partial v'_i}{\partial x_j} + \frac{\partial v'_j}{\partial x_i}\right)} = A_{ij} + M_{ijk\ell}\frac{\partial \bar{v}_k}{\partial x_\ell} = \Phi_{ij,1} + \Phi_{ij,2}$$

•

$$p'(\mathbf{x}) = \frac{\rho}{2\pi}\frac{\partial \bar{v}_i(\mathbf{x})}{\partial x_j}\int_V \frac{\partial v'_j(\mathbf{y})}{\partial y_i}\frac{d\mathbf{y}^3}{|\mathbf{y}-\mathbf{x}|} - \frac{\rho}{4\pi}\int_V \frac{\partial^2}{\partial y_i \partial y_j}\left(v'_i(\mathbf{y})v'_j(\mathbf{y})\right)\frac{d\mathbf{y}^3}{|\mathbf{y}-\mathbf{x}|}$$

►Multiply Eq. 32.2 with $\partial v'_i/\partial x_j + \partial v'_j/\partial x_i$ and average:

$$\begin{aligned}
\overline{\frac{p'(\mathbf{x})}{\rho}\left(\frac{\partial v'_i(\mathbf{x})}{\partial x_j} + \frac{\partial v'_j(\mathbf{x})}{\partial x_i}\right)} &= \frac{\partial \bar{v}_k(\mathbf{x})}{\partial x_\ell}\underbrace{\frac{1}{2\pi}\int_V \overline{\left(\frac{\partial v'_i(\mathbf{x})}{\partial x_j} + \frac{\partial v'_j(\mathbf{x})}{\partial x_i}\right)\frac{\partial v'_\ell(\mathbf{y})}{\partial y_k}}\frac{d\mathbf{y}^3}{|\mathbf{y}-\mathbf{x}|}}_{M_{ijk\ell}} \\
&+ \underbrace{\frac{1}{4\pi}\int_V \overline{\left(\frac{\partial v'_i(\mathbf{x})}{\partial x_j} + \frac{\partial v'_j(\mathbf{x})}{\partial x_i}\right)\frac{\partial^2}{\partial y_k \partial y_\ell}\left(v'_k(\mathbf{y})v'_\ell(\mathbf{y})\right)}\frac{d\mathbf{y}^3}{|\mathbf{y}-\mathbf{x}|}}_{A_{ij}}
\end{aligned} \tag{32.3}$$

Short form of Eq. 32.3:

$$\overline{\frac{p'}{\rho}\left(\frac{\partial v'_i}{\partial x_j} + \frac{\partial v'_j}{\partial x_i}\right)} = A_{ij} + M_{ijk\ell}\frac{\partial \bar{v}_k}{\partial x_\ell} = \Phi_{ij,1} + \Phi_{ij,2}$$

- First term=slow term, $\Phi_{ij,1}$,

$$p'(\mathbf{x}) = \frac{\rho}{2\pi}\frac{\partial \bar{v}_i(\mathbf{x})}{\partial x_j}\int_V \frac{\partial v'_j(\mathbf{y})}{\partial y_i}\frac{d\mathbf{y}^3}{|\mathbf{y}-\mathbf{x}|} - \frac{\rho}{4\pi}\int_V \frac{\partial^2}{\partial y_i \partial y_j}\left(v'_i(\mathbf{y})v'_j(\mathbf{y})\right)\frac{d\mathbf{y}^3}{|\mathbf{y}-\mathbf{x}|}$$

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- First term=slow term, $\Phi_{ij,1}$,
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$$\Phi_{ij,2} = -c_2\rho\left(P_{ij} - \frac{2}{3}\delta_{ij}P^k\right) \quad \text{IP model}$$

¶See Section 11.7.6, Wall model of the pressure-strain term

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►Wall models of pressure-strain:

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$$\Phi_{ij} = \Phi_{ij,1} + \Phi_{ij,2} + \Phi_{ij,1w} + \Phi_{ij,2w}$$

$$\Phi_{22,1w} = -2c_{1w}\frac{\varepsilon}{k}\overline{v_2'^2}f, \quad f \propto \frac{L_t}{|x_i - x_{i,wall}|} = \frac{k^{\frac{3}{2}}}{2.55|n_{i,w}(x_i - x_{i,w})|\varepsilon}, \quad 0 < f < 1$$

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Traceless $\Rightarrow$

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Traceless ⇒

$$\Phi_{11,1w} = \Phi_{33,1w} = c_{1w}\frac{\varepsilon}{k}\overline{v_2'^2}f$$

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$$\Phi_{12,1w} = -\frac{3}{2}c_{1w}\frac{\varepsilon}{k}\overline{v_1'v_2'}f$$

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$$\Phi_{ij,1w} = c_{1w}\frac{\varepsilon}{k}\left(\overline{v_k'v_m'}n_{k,w}n_{m,w}\delta_{ij} - \frac{3}{2}\overline{v_k'v_i'}n_{k,w}n_{j,w} - \frac{3}{2}\overline{v_k'v_j'}n_{i,w}n_{k,w}\right)f$$

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The analogous wall model for the rapid part reads

¶See Section 11.7.6, Wall model of the pressure-strain term

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The analogous wall model for the rapid part reads

$$\Phi_{ij,2w} = c_{2w}\left(\Phi_{km,2}n_{k,w}n_{m,w}\delta_{ij} - \frac{3}{2}\Phi_{ki,2}n_{k,w}n_{j,w} - \frac{3}{2}\Phi_{kj,2}n_{i,w}n_{k,w}\right)f$$

¶See Section 11.9, The modeled $\overline{v_i'v_j'}$ equation with IP model

►We can finally formulate the **modelled** $\overline{v_i'v_j'}$ equation (with IP model), the Reynolds Stress Model

(RSM)

$$
\begin{aligned}
&\frac{\partial \overline{v_i' v_j'}}{\partial t} + && \text{(unsteady term)}\\
&\bar{v}_k \frac{\partial \overline{v_i' v_j'}}{\partial x_k} = && \text{(convection)}\\
&-\overline{v_i' v_k'} \frac{\partial \bar{v}_j}{\partial x_k} - \overline{v_j' v_k'} \frac{\partial \bar{v}_i}{\partial x_k} && \text{(production)}\\
&-c_1 \frac{\varepsilon}{k} \left(\overline{v_i' v_j'} - \frac{2}{3} \delta_{ij} k \right) && \text{(slow part)}\\
&-c_2 \left(P_{ij} - \frac{2}{3} \delta_{ij} P^k \right) && \text{(rapid part, IP model)}\\
&+c_{1w} \rho \frac{\varepsilon}{k} \left[\overline{v_k' v_m'} n_k n_m \delta_{ij} - \frac{3}{2} \overline{v_i' v_k'} n_k n_j - \frac{3}{2} \overline{v_j' v_k'} n_k n_i \right] f && \text{(wall, slow part)}\\
&+c_{2w} \left[\Phi_{km,2} n_k n_m \delta_{ij} - \frac{3}{2} \Phi_{ik,2} n_k n_j - \frac{3}{2} \Phi_{jk,2} n_k n_i \right] f && \text{(wall, rapid part, IP model))}\\
&+\nu \frac{\partial^2 \overline{v_i' v_j'}}{\partial x_k \partial x_k} && \text{(viscous diffusion)}\\
&+\frac{\partial}{\partial x_k} \left[\frac{\nu_t}{\sigma_k} \frac{\partial \overline{v_i' v_j'}}{\partial x_m} \right] && \text{(turbulent diffusion)}\\
&-g_i \beta \overline{v_j' \theta'} - g_j \beta \overline{v_i' \theta'} && \text{(buoyancy production)}\\
&-\frac{2}{3} \varepsilon \delta_{ij} && \text{(dissipation)}
\end{aligned}
\tag{32.4}
$$

¶See Section 11.10, Algebraic Reynolds Stress Model (ASM)

►The Algebraic Reynolds Stress Model (ASM) is a simplified Reynolds Stress Model (RSM)

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$$\mathrm{RSM}: C_{ij} - D_{ij} \;=\; P_{ij} + \Phi_{ij} - \varepsilon_{ij}$$

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$$\begin{aligned}
\mathrm{RSM}: C_{ij} - D_{ij} &= P_{ij} + \Phi_{ij} - \varepsilon_{ij} \\
k - \varepsilon: \; C^k - D^k &= P^k - \varepsilon
\end{aligned}$$

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Assumption in ASM:

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Assumption in ASM:

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$$C_{ij} - D_{ij} = \left(C^k - D^k\right)\frac{\overline{v_i'v_j'}}{k}$$

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$$C_{ij} - D_{ij} = \left(C^k - D^k\right)\frac{\overline{v_i' v_j'}}{k}$$

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This gives

$$\overline{v_i' v_j'} = \frac{2}{3}\delta_{ij} k + \frac{k}{\varepsilon}\frac{(1 - c_2)\left(P_{ij} - \frac{2}{3}\delta_{ij} P^k\right) + \Phi_{ij,1w} + \Phi_{ij,2w}}{c_1 + P^k/\varepsilon - 1}$$

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¶See Section 11.13, Boundary layer flow

►We will consider source terms in the modelled $\overline{v_i' v_j'}$ equation for boundary layer flow.

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We have $\bar{v}_2 \simeq 0$, $\partial\bar{v}_1/\partial x_2 \gg \partial\bar{v}_1/\partial x_1$. The production term reads:

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In this special case we get (all velocity gradients except $\partial\bar{v}_1/\partial x_2$ are negligible):

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$$P_{11} = -2\overline{v_1'v_2'}\frac{\partial\bar{v}_1}{\partial x_2}$$

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$$P_{ij} = -\overline{v_i'v_k'}\frac{\partial \bar{v}_j}{\partial x_k} - \overline{v_j'v_k'}\frac{\partial \bar{v}_i}{\partial x_k}$$

In this special case we get (all velocity gradients except $\partial\bar{v}_1/\partial x_2$ are negligible):

$$P_{11} = -2\overline{v_1'v_2'}\frac{\partial \bar{v}_1}{\partial x_2}$$

$$P_{12} = -\overline{v_2'^2}\frac{\partial \bar{v}_1}{\partial x_2}$$

¶See Section 11.13, Boundary layer flow

▶We will consider source terms in the modelled $\overline{v_i'v_j'}$ equation for boundary layer flow.

We have $\bar{v}_2 \simeq 0,\ \partial\bar{v}_1/\partial x_2 \gg \partial\bar{v}_1/\partial x_1$. The production term reads:

$$P_{ij} = -\overline{v_i'v_k'}\frac{\partial\bar{v}_j}{\partial x_k} - \overline{v_j'v_k'}\frac{\partial\bar{v}_i}{\partial x_k}$$

In this special case we get (all velocity gradients except $\partial\bar{v}_1/\partial x_2$ are negligible):

$$\begin{aligned}
P_{11} &= -2\overline{v_1'v_2'}\frac{\partial\bar{v}_1}{\partial x_2} \\
P_{12} &= -\overline{v_2'^2}\frac{\partial\bar{v}_1}{\partial x_2} \\
P_{22} &= 0
\end{aligned}$$

¶See Section 11.13, Boundary layer flow

▶We will consider source terms in the modelled $\overline{v_i'v_j'}$ equation for boundary layer flow.
We have $\bar{v}_2 \simeq 0,\ \partial\bar{v}_1/\partial x_2 \gg \partial\bar{v}_1/\partial x_1$. The production term reads:

$$P_{ij} = -\overline{v_i'v_k'}\frac{\partial\bar{v}_j}{\partial x_k} - \overline{v_j'v_k'}\frac{\partial\bar{v}_i}{\partial x_k}$$

In this special case we get (all velocity gradients except $\partial\bar{v}_1/\partial x_2$ are negligible):

$$\begin{aligned} P_{11} &= -2\overline{v_1'v_2'}\frac{\partial\bar{v}_1}{\partial x_2} \\ P_{12} &= -\overline{v_2'^2}\frac{\partial\bar{v}_1}{\partial x_2} \\ P_{22} &= 0, \quad P_{33} = 0 \end{aligned}$$

▶

¶See Section 11.13, Boundary layer flow

►We will consider source terms in the modelled $\overline{v_i'v_j'}$ equation for boundary layer flow. We have $\bar{v}_2 \simeq 0,\ \partial\bar{v}_1/\partial x_2 \gg \partial\bar{v}_1/\partial x_1$. The production term reads:

$$P_{ij} = -\overline{v_i'v_k'}\frac{\partial\bar{v}_j}{\partial x_k} - \overline{v_j'v_k'}\frac{\partial\bar{v}_i}{\partial x_k}$$

In this special case we get (all velocity gradients except $\partial\bar{v}_1/\partial x_2$ are negligible):

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►No production term in $\overline{v_2'^2}$ and $\overline{v_3'^2}$! How are they produced?

¶See Section 11.13, Boundary layer flow

►We will consider source terms in the modelled $\overline{v_i'v_j'}$ equation for boundary layer flow.
We have $\bar{v}_2 \simeq 0,\ \partial\bar{v}_1/\partial x_2 \gg \partial\bar{v}_1/\partial x_1$. The production term reads:

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P_{11} &= -2\overline{v_1'v_2'}\frac{\partial\bar{v}_1}{\partial x_2} \\
P_{12} &= -\overline{v_2'^2}\frac{\partial\bar{v}_1}{\partial x_2} \\
P_{22} &= 0, \quad P_{33} = 0
\end{aligned}$$

►No production term in $\overline{v_2'^2}$ and $\overline{v_3'^2}$! How are they produced? Answer: the pressure strain (Robin Hood)

¶See Section 11.13, Boundary layer flow

▶We will consider source terms in the modelled $\overline{v'_i v'_j}$ equation for boundary layer flow.
We have $\bar{v}_2 \simeq 0,\ \partial\bar{v}_1/\partial x_2 \gg \partial\bar{v}_1/\partial x_1$. The production term reads:

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In this special case we get (all velocity gradients except $\partial\bar{v}_1/\partial x_2$ are negligible):

$$\begin{aligned} P_{11} &= -2\overline{v'_1 v'_2}\frac{\partial\bar{v}_1}{\partial x_2} \\ P_{12} &= -\overline{v'^2_2}\frac{\partial\bar{v}_1}{\partial x_2} \\ P_{22} &= 0, \quad P_{33} = 0 \end{aligned}$$

▶No production term in $\overline{v'^2_2}$ and $\overline{v'^2_3}$! How are they produced? Answer: the pressure strain (Robin Hood)

$$\Phi_{22,1} = c_1\frac{\varepsilon}{k}\left(\frac{2}{3}k - \overline{v'^2_2}\right) > 0$$

¶See Section 11.13, Boundary layer flow

►We will consider source terms in the modelled $\overline{v_i' v_j'}$ equation for boundary layer flow.
We have $\bar{v}_2 \simeq 0$, $\partial \bar{v}_1/\partial x_2 \gg \partial \bar{v}_1/\partial x_1$. The production term reads:

$$P_{ij} = -\overline{v_i' v_k'}\frac{\partial \bar{v}_j}{\partial x_k} - \overline{v_j' v_k'}\frac{\partial \bar{v}_i}{\partial x_k}$$

In this special case we get (all velocity gradients except $\partial \bar{v}_1/\partial x_2$ are negligible):

$$\begin{aligned} P_{11} &= -2\overline{v_1' v_2'}\frac{\partial \bar{v}_1}{\partial x_2} \\ P_{12} &= -\overline{v_2'^2}\frac{\partial \bar{v}_1}{\partial x_2} \\ P_{22} &= 0, \quad P_{33} = 0 \end{aligned}$$

►No production term in $\overline{v_2'^2}$ and $\overline{v_3'^2}$! How are they produced? Answer: the pressure strain (Robin Hood)

$$\Phi_{22,1} = c_1 \frac{\varepsilon}{k}\left(\frac{2}{3}k - \overline{v_2'^2}\right) > 0, \quad \frac{2}{3}k = (\overline{v_1'^2} + \overline{v_2'^2} + \overline{v_3'^2})/3$$

¶See Section 11.13, Boundary layer flow

►We will consider source terms in the modelled $\overline{v_i'v_j'}$ equation for boundary layer flow.
We have $\bar{v}_2 \simeq 0,\ \partial\bar{v}_1/\partial x_2 \gg \partial\bar{v}_1/\partial x_1$. The production term reads:

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$$\begin{aligned} \Phi_{22,1} &= c_1\frac{\varepsilon}{k}\left(\frac{2}{3}k - \overline{v_2'^2}\right) > 0, \quad \frac{2}{3}k = (\overline{v_1'^2} + \overline{v_2'^2} + \overline{v_3'^2})/3 \\ \Phi_{22,2} &= c_2\frac{1}{3}P_{11} = -c_2\frac{2}{3}\overline{v_1'v_2'}\frac{\partial \bar{v}_1}{\partial x_2} > 0 \end{aligned}$$

¶See Section 11.13, Boundary layer flow

▶We will consider source terms in the modelled $\overline{v_i'v_j'}$ equation for boundary layer flow.
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▶No production term in $\overline{v_2'^2}$ and $\overline{v_3'^2}$! How are they produced? Answer: the pressure strain (Robin Hood)

$$\begin{aligned} \Phi_{22,1} &= c_1\frac{\varepsilon}{k}\left(\frac{2}{3}k - \overline{v_2'^2}\right) > 0, \quad \frac{2}{3}k = (\overline{v_1'^2} + \overline{v_2'^2} + \overline{v_3'^2})/3 \\ \Phi_{22,2} &= c_2\frac{1}{3}P_{11} = -c_2\frac{2}{3}\overline{v_1'v_2'}\frac{\partial\bar{v}_1}{\partial x_2} > 0 \end{aligned}$$

¶See Section 11.13, Boundary layer flow

▶We will consider source terms in the modelled $\overline{v'_i v'_j}$ equation for boundary layer flow.
We have $\bar{v}_2 \simeq 0,\ \partial \bar{v}_1/\partial x_2 \gg \partial \bar{v}_1/\partial x_1$. The production term reads:

$$P_{ij} = -\overline{v'_i v'_k}\frac{\partial \bar{v}_j}{\partial x_k} - \overline{v'_j v'_k}\frac{\partial \bar{v}_i}{\partial x_k}$$

In this special case we get (all velocity gradients except $\partial \bar{v}_1/\partial x_2$ are negligible):

$$\begin{aligned} P_{11} &= -2\overline{v'_1 v'_2}\frac{\partial \bar{v}_1}{\partial x_2} \\ P_{12} &= -\overline{v'^2_2}\frac{\partial \bar{v}_1}{\partial x_2} \\ P_{22} &= 0, \quad P_{33} = 0 \end{aligned}$$

▶No production term in $\overline{v'^2_2}$ and $\overline{v'^2_3}$! How are they produced? Answer: the pressure strain (Robin Hood)

$$\begin{aligned} \Phi_{22,1} &= c_1\frac{\varepsilon}{k}\left(\frac{2}{3}k - \overline{v'^2_2}\right) > 0, \quad \frac{2}{3}k = (\overline{v'^2_1} + \overline{v'^2_2} + \overline{v'^2_3})/3 \\ \Phi_{22,2} &= c_2\frac{1}{3}P_{11} = -c_2\frac{2}{3}\overline{v'_1 v'_2}\frac{\partial \bar{v}_1}{\partial x_2} > 0 \end{aligned}$$

$\varepsilon_{12} = 0$: No sink term in $\overline{v'_1 v'_2}$ eq?

¶See Section 11.13, Boundary layer flow

▶We will consider source terms in the modelled $\overline{v_i'v_j'}$ equation for boundary layer flow.
We have $\bar{v}_2 \simeq 0,\ \partial\bar{v}_1/\partial x_2 \gg \partial\bar{v}_1/\partial x_1$. The production term reads:

$$P_{ij} = -\overline{v_i'v_k'}\frac{\partial\bar{v}_j}{\partial x_k} - \overline{v_j'v_k'}\frac{\partial\bar{v}_i}{\partial x_k}$$

In this special case we get (all velocity gradients except $\partial\bar{v}_1/\partial x_2$ are negligible):

$$\begin{aligned} P_{11} &= -2\overline{v_1'v_2'}\frac{\partial\bar{v}_1}{\partial x_2} \\ P_{12} &= -\overline{v_2'^2}\frac{\partial\bar{v}_1}{\partial x_2} \\ P_{22} &= 0, \quad P_{33} = 0 \end{aligned}$$

▶No production term in $\overline{v_2'^2}$ and $\overline{v_3'^2}$! How are they produced? Answer: the pressure strain (Robin Hood)

$$\begin{aligned} \Phi_{22,1} &= c_1\frac{\varepsilon}{k}\left(\frac{2}{3}k - \overline{v_2'^2}\right) > 0, \quad \frac{2}{3}k = (\overline{v_1'^2} + \overline{v_2'^2} + \overline{v_3'^2})/3 \\ \Phi_{22,2} &= c_2\frac{1}{3}P_{11} = -c_2\frac{2}{3}\overline{v_1'v_2'}\frac{\partial\bar{v}_1}{\partial x_2} > 0 \end{aligned}$$

$\varepsilon_{12} = 0$: No sink term in $\overline{v_1'v_2'}$ eq? Answer:

¶See Section 11.13, Boundary layer flow

►We will consider source terms in the modelled $\overline{v_i'v_j'}$ equation for boundary layer flow.
We have $\bar{v}_2 \simeq 0,\ \partial\bar{v}_1/\partial x_2 \gg \partial\bar{v}_1/\partial x_1$. The production term reads:

$$P_{ij} = -\overline{v_i'v_k'}\frac{\partial\bar{v}_j}{\partial x_k} - \overline{v_j'v_k'}\frac{\partial\bar{v}_i}{\partial x_k}$$

In this special case we get (all velocity gradients except $\partial\bar{v}_1/\partial x_2$ are negligible):

$$\begin{aligned} P_{11} &= -2\overline{v_1'v_2'}\frac{\partial\bar{v}_1}{\partial x_2} \\ P_{12} &= -\overline{v_2'^2}\frac{\partial\bar{v}_1}{\partial x_2} \\ P_{22} &= 0, \quad P_{33} = 0 \end{aligned}$$

►No production term in $\overline{v_2'^2}$ and $\overline{v_3'^2}$! How are they produced? Answer: the pressure strain (Robin Hood)

$$\begin{aligned} \Phi_{22,1} &= c_1\frac{\varepsilon}{k}\left(\frac{2}{3}k - \overline{v_2'^2}\right) > 0, \quad \frac{2}{3}k = (\overline{v_1'^2} + \overline{v_2'^2} + \overline{v_3'^2})/3 \\ \Phi_{22,2} &= c_2\frac{1}{3}P_{11} = -c_2\frac{2}{3}\overline{v_1'v_2'}\frac{\partial\bar{v}_1}{\partial x_2} > 0 \end{aligned}$$

$\varepsilon_{12} = 0$: No sink term in $\overline{v_1'v_2'}$ eq? Answer: the pressure strain terms $\Phi_{12,1}$ and $\Phi_{12,2}$ act as sink terms.

¶See Section 12.1, Stable and unstable stratification

¶See Section 12.1, Stable and unstable stratification

►Assume there is a non-constant temperature field and that the natural convection is important (no forced convection). We have then two different flow conditions, stable or unstable conditions.

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►Assume there is a non-constant temperature field and that the natural convection is important (no forced convection). We have then two different flow conditions, stable or unstable conditions.

►We start with stable stratification for which $\partial\bar{\theta}/\partial x_3 > 0$.

¶See Section 12.1, Stable and unstable stratification

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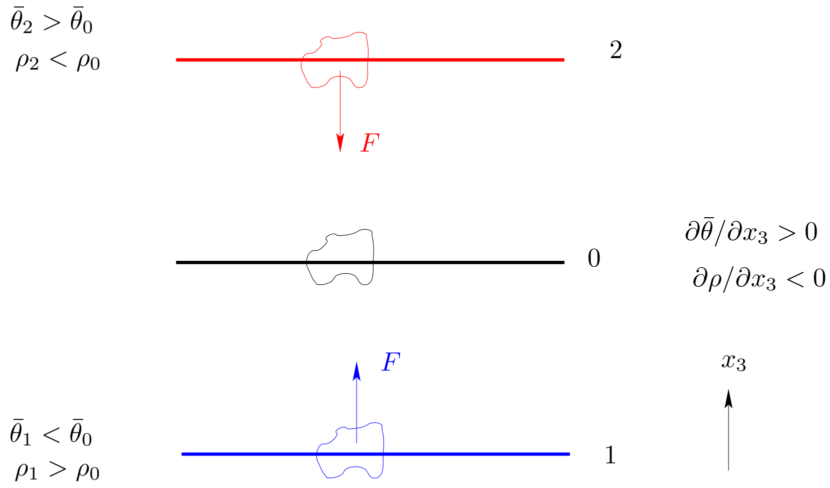

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¶See Section 12.1, Stable and unstable stratification

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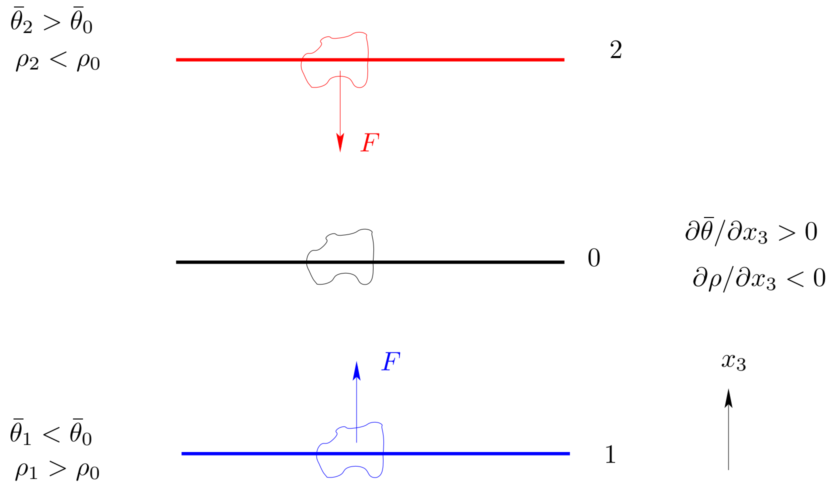
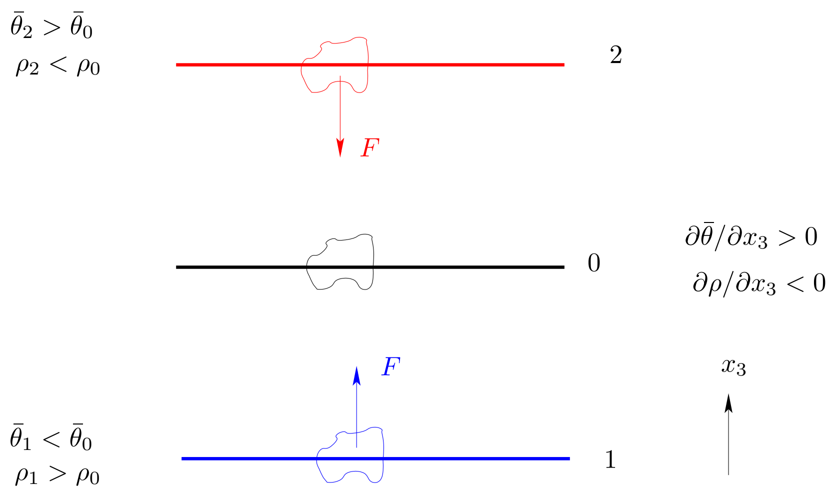


$$G_{ij} = -g_i\beta\overline{v_j'\theta'} - g_j\beta\overline{v_i'\theta'},$$

¶See Section 12.1, Stable and unstable stratification

▶Assume there is a non-constant temperature field and that the natural convection is important (no forced convection). We have then two different flow conditions, stable or unstable conditions.

▶We start with stable stratification for which $\partial\bar{\theta}/\partial x_3 > 0$.



$$G_{ij} = -g_i\beta\overline{v'_j\theta'} - g_j\beta\overline{v'_i\theta'}, \quad g_i = (0, 0, -g)$$

¶See Section 12.1, Stable and unstable stratification

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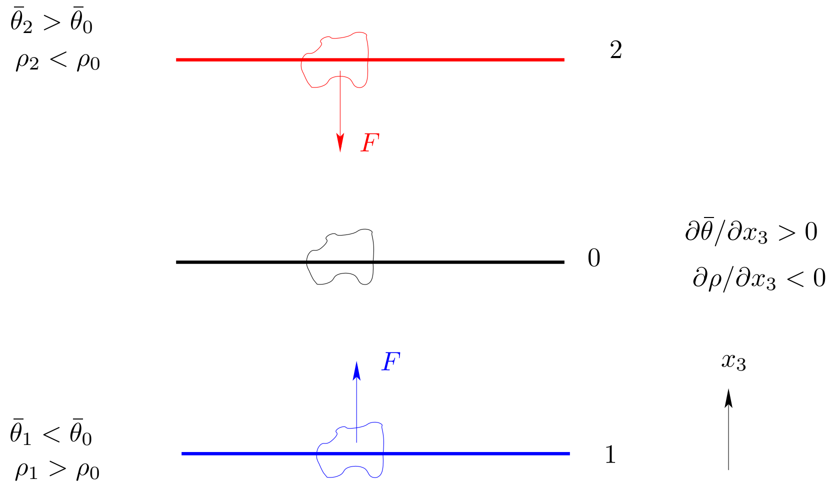
▶We start with stable stratification for which $\partial\bar{\theta}/\partial x_3 > 0$.

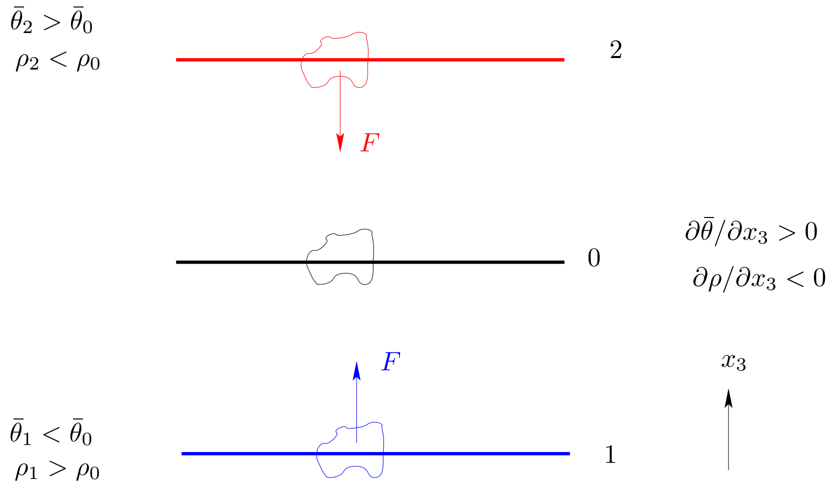


$$G_{ij} = -g_i\beta\overline{v_j'\theta'} - g_j\beta\overline{v_i'\theta'}, \quad g_i = (0, 0, -g) \qquad \Rightarrow \overline{v_3'^2} \text{ eq.:} \quad G_{33} = 2g\beta\overline{v_3'\theta'}$$

¶See Section 12.1, Stable and unstable stratification

►Assume there is a non-constant temperature field and that the natural convection is important (no forced convection). We have then two different flow conditions, stable or unstable conditions.

►We start with stable stratification for which $\partial\bar{\theta}/\partial x_3 > 0$.



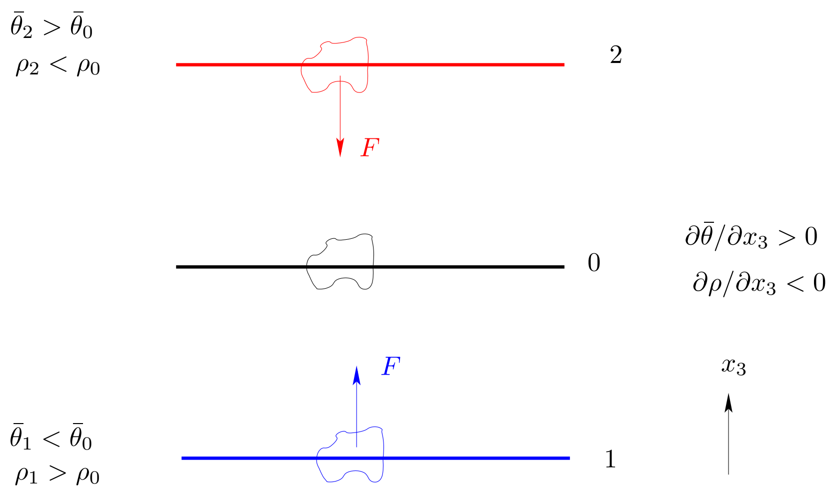
$$G_{ij} = -g_i\beta\overline{v'_j\theta'} - g_j\beta\overline{v'_i\theta'}, \quad g_i = (0,0,-g) \qquad \Rightarrow \overline{v'^2_3}\text{ eq.:} \quad G_{33} = 2g\beta\overline{v'_3\theta'}$$

which is the source term in the $\overline{v'^2_3}$ eq due to buoyancy.

¶See Section 12.1, Stable and unstable stratification

▶Assume there is a non-constant temperature field and that the natural convection is important (no forced convection). We have then two different flow conditions, stable or unstable conditions.

▶We start with stable stratification for which $\partial\bar{\theta}/\partial x_3 > 0$.



$$G_{ij} = -g_i\beta\overline{v'_j\theta'} - g_j\beta\overline{v'_i\theta'}, \quad g_i = (0,0,-g) \qquad \Rightarrow \overline{v'^2_3}\ \text{eq.:} \quad G_{33} = 2g\beta\overline{v'_3\theta'}$$

which is the source term in the $\overline{v'^2_3}$ eq due to buoyancy.

Now we need $\overline{v'_3\theta'}$.

¶See Section 12.1, Stable and unstable stratification

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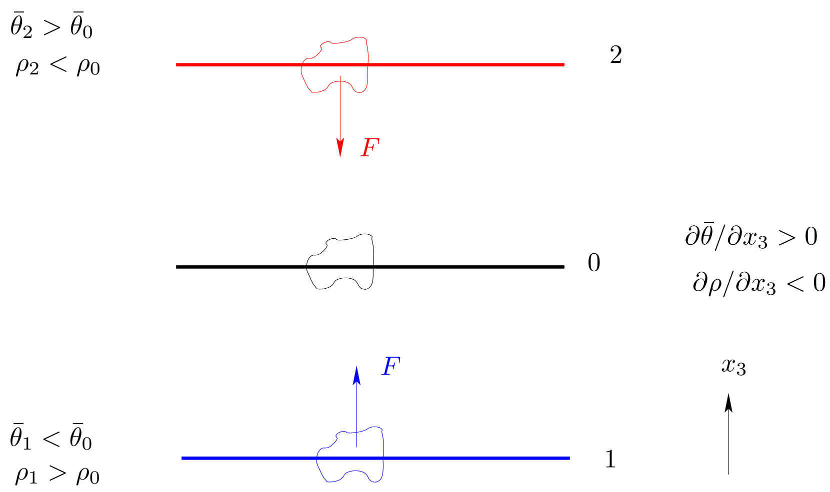


$$G_{ij} = -g_i\beta\overline{v'_j\theta'} - g_j\beta\overline{v'_i\theta'}, \quad g_i = (0, 0, -g) \quad \Rightarrow \overline{v'^2_3} \text{ eq.:} \quad G_{33} = 2g\beta\overline{v'_3\theta'}$$

which is the source term in the $\overline{v'^2_3}$ eq due to buoyancy.

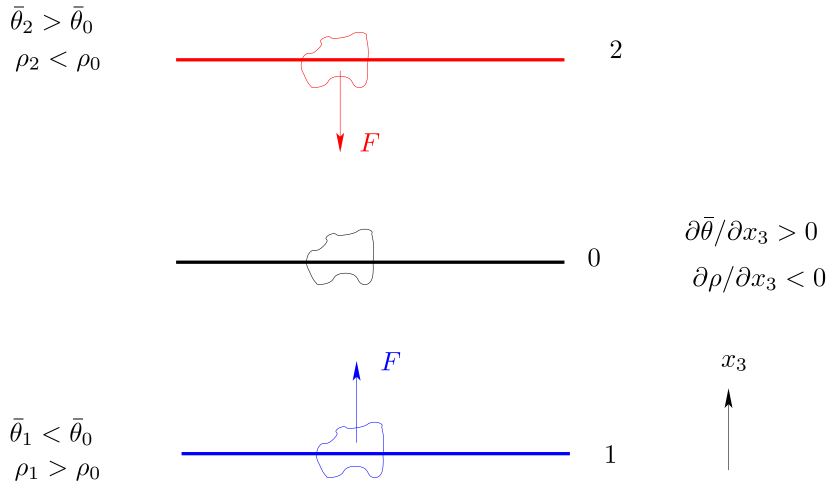
Now we need $\overline{v'_3\theta'}$.
The main source term in this equation is (see Eq 30.3)

$$P_{3\theta} = -\overline{v'_3v'_k}\frac{\partial\bar{\theta}}{\partial x_k} - \overline{v'_k\theta'}\frac{\partial\bar{v}_3}{\partial x_k}$$

¶See Section 12.1, Stable and unstable stratification

▶Assume there is a non-constant temperature field and that the natural convection is important (no forced convection). We have then two different flow conditions, stable or unstable conditions.

▶We start with stable stratification for which $\partial\bar{\theta}/\partial x_3 > 0$.



$$G_{ij} = -g_i\beta\overline{v_j'\theta'} - g_j\beta\overline{v_i'\theta'}, \quad g_i = (0,0,-g) \qquad \Rightarrow \overline{v_3'^2} \text{ eq.:} \quad G_{33} = 2g\beta\overline{v_3'\theta'}$$

which is the source term in the $\overline{v_3'^2}$ eq due to buoyancy.

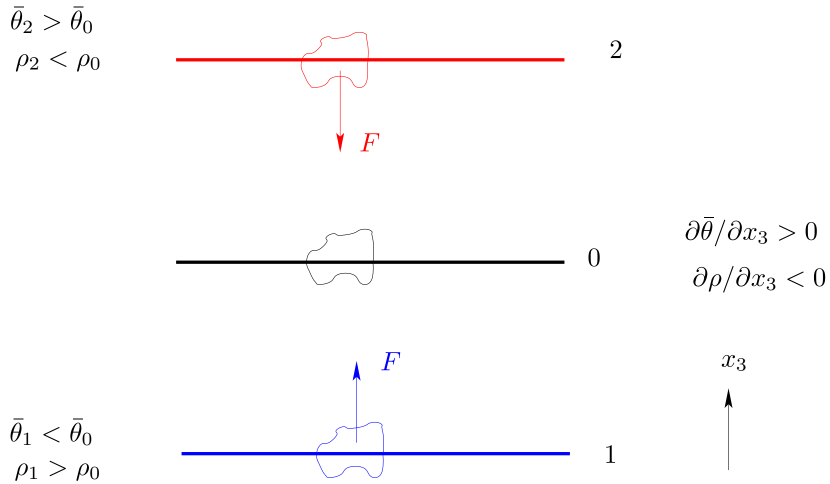
Now we need $\overline{v_3'\theta'}$.
The main source term in this equation is (see Eq 30.3)

$$P_{3\theta} = -\overline{v_3'v_k'}\frac{\partial\bar{\theta}}{\partial x_k} - \overline{v_k'\theta'}\frac{\partial\bar{v}_3}{\partial x_k} =$$

¶See Section 12.1, Stable and unstable stratification

▶Assume there is a non-constant temperature field and that the natural convection is important (no forced convection). We have then two different flow conditions, stable or unstable conditions.

▶We start with stable stratification for which $\partial\bar{\theta}/\partial x_3 > 0$.



$$G_{ij} = -g_i\beta\overline{v'_j\theta'} - g_j\beta\overline{v'_i\theta'}, \quad g_i = (0,0,-g) \qquad \Rightarrow \overline{v'^2_3} \text{ eq.:} \quad G_{33} = 2g\beta\overline{v'_3\theta'}$$

which is the source term in the $\overline{v'^2_3}$ eq due to buoyancy.

Now we need $\overline{v'_3\theta'}$.

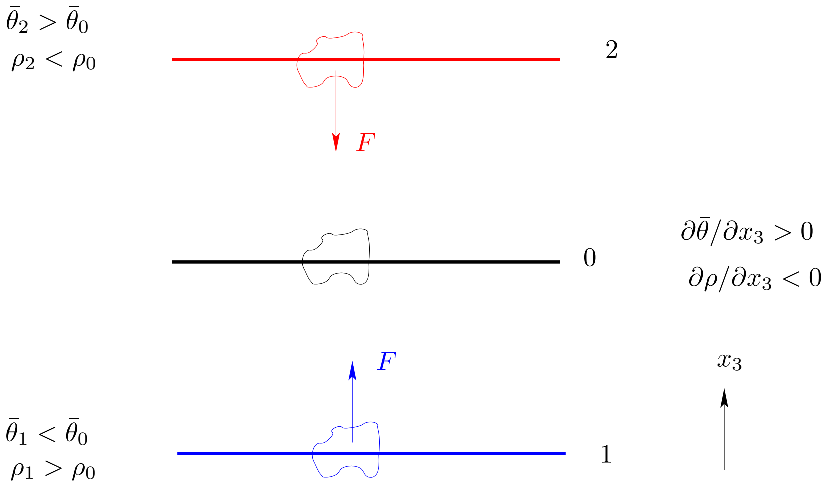
The main source term in this equation is (see Eq 30.3)

$$P_{3\theta} = -\overline{v'_3v'_k}\frac{\partial\bar{\theta}}{\partial x_k} - \overline{v'_k\theta'}\frac{\partial\bar{v}_3}{\partial x_k} = -\overline{v'_3v'_k}\frac{\partial\bar{\theta}}{\partial x_k} - \cancelto{0}{\overline{v'_k\theta'}\frac{\partial\bar{v}_3}{\partial x_k}}$$

¶See Section 12.1, Stable and unstable stratification

▶Assume there is a non-constant temperature field and that the natural convection is important (no forced convection). We have then two different flow conditions, stable or unstable conditions.

▶We start with stable stratification for which $\partial\bar{\theta}/\partial x_3 > 0$.



$$G_{ij} = -g_i\beta\overline{v'_j\theta'} - g_j\beta\overline{v'_i\theta'}, \quad g_i = (0,0,-g) \qquad \Rightarrow \overline{v'^2_3} \text{ eq.:} \quad G_{33} = 2g\beta\overline{v'_3\theta'}$$

which is the source term in the $\overline{v'^2_3}$ eq due to buoyancy.

Now we need $\overline{v'_3\theta'}$.
The main source term in this equation is (see Eq 30.3)

$$P_{3\theta} = -\overline{v'_3v'_k}\frac{\partial\bar{\theta}}{\partial x_k} - \overline{v'_k\theta'}\frac{\partial\bar{v}_3}{\partial x_k} = -\overline{v'_3v'_k}\frac{\partial\bar{\theta}}{\partial x_k} - \cancelto{0}{\overline{v'_k\theta'}\frac{\partial\bar{v}_3}{\partial x_k}} = -\overline{v'^2_3}\frac{\partial\bar{\theta}}{\partial x_3} < 0$$

So

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So

- $P_{3\theta} < 0$

So

- $P_{3\theta} < 0$
- $\Rightarrow \overline{v_3'\theta'} < 0$

So

- $P_{3\theta} < 0$
- $\Rightarrow \overline{v_3'\theta'} < 0$
- $\Rightarrow G_{33} = 2g\beta\overline{v_3'\theta'} < 0$

So

- $P_{3\theta} < 0$
- $\Rightarrow \overline{v_3'\theta'} < 0$
- $\Rightarrow G_{33} = 2g\beta\overline{v_3'\theta'} < 0$
- which dampens $\overline{v_3'^2}$ (but not $\overline{v_1'^2}$, $\overline{v_2'^2}$) as it should.

So

- $P_{3\theta} < 0$
- $\Rightarrow \overline{v_3'\theta'} < 0$
- $\Rightarrow G_{33} = 2g\beta\overline{v_3'\theta'} < 0$
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►Note that the $k-\varepsilon$ model incorrecly dampens all normal stress, not only the vertical one

On-line Lecture 4

¶See Section 12.2, Curvature effects

►Streamline curvature affects the turbulence.

On-line Lecture 4

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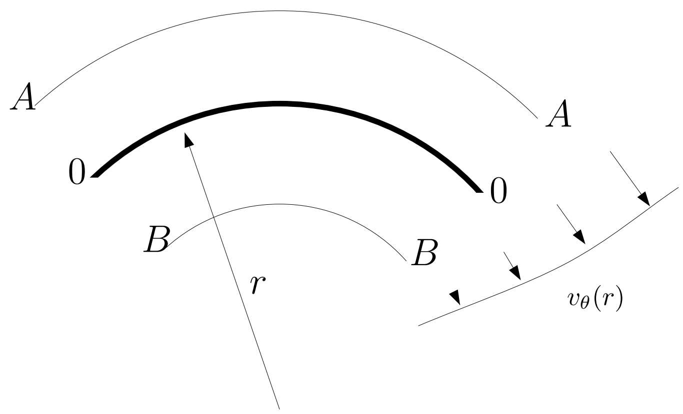

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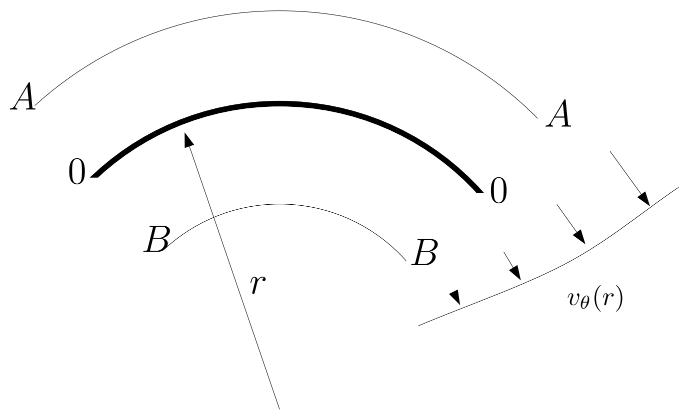

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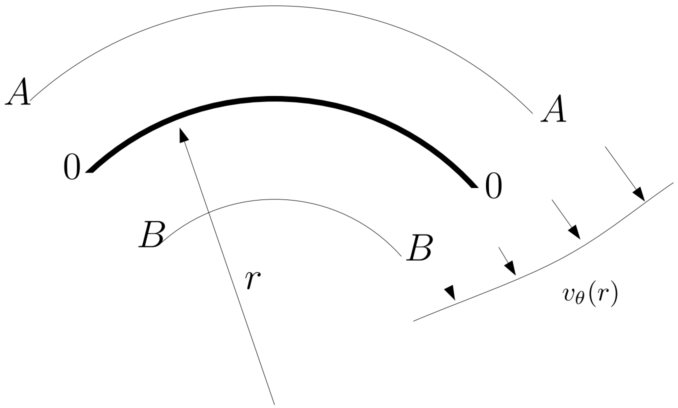

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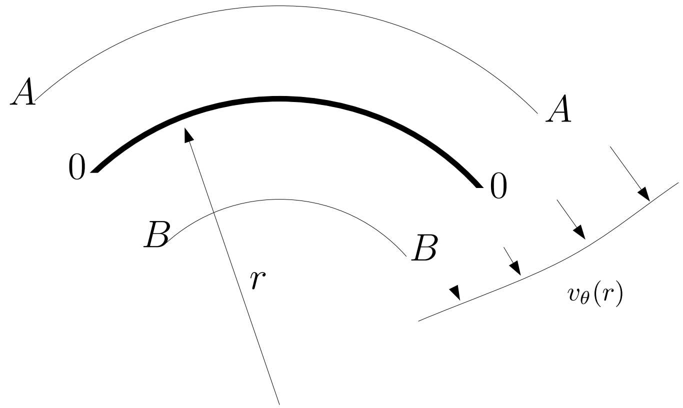

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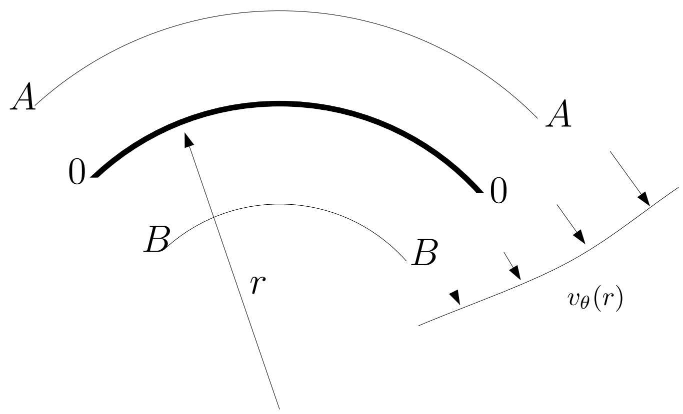

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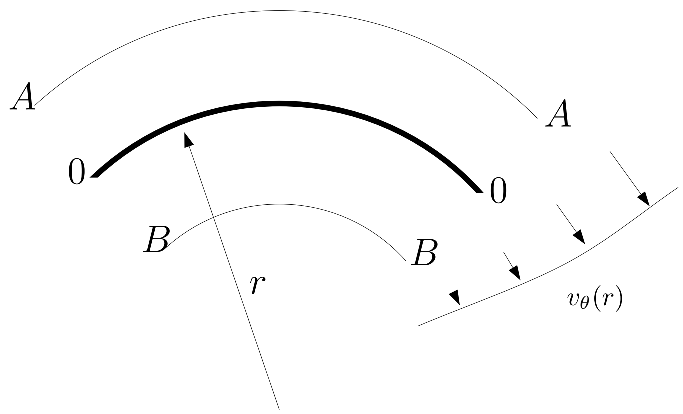

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▶The streamline curvature stabilized (decreases) the turbulence

▶Change sign of velocity gradient $\partial v_\theta/\partial r < 0$: now streamline curvature will increase the turbulence

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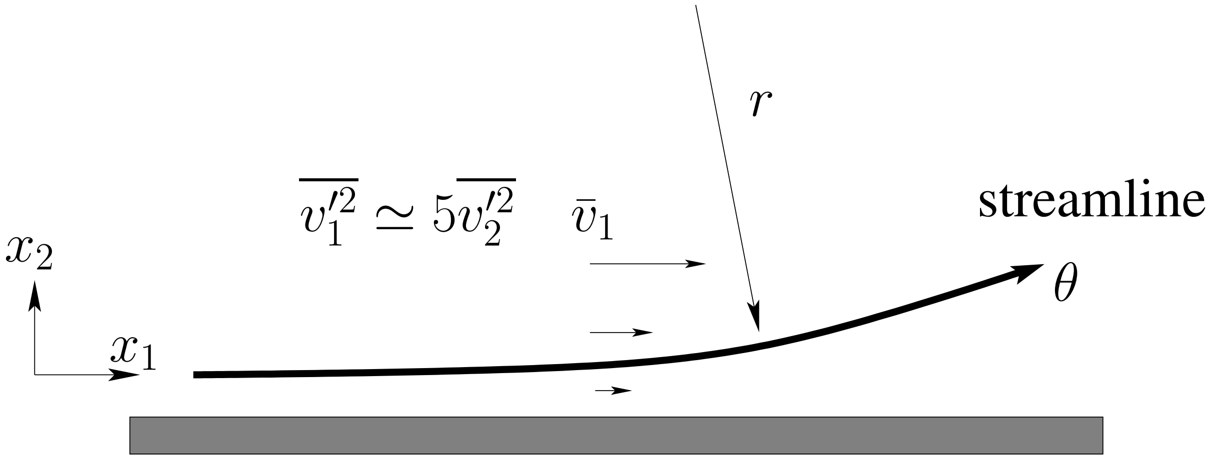
►We choose a boundary layer flow as below



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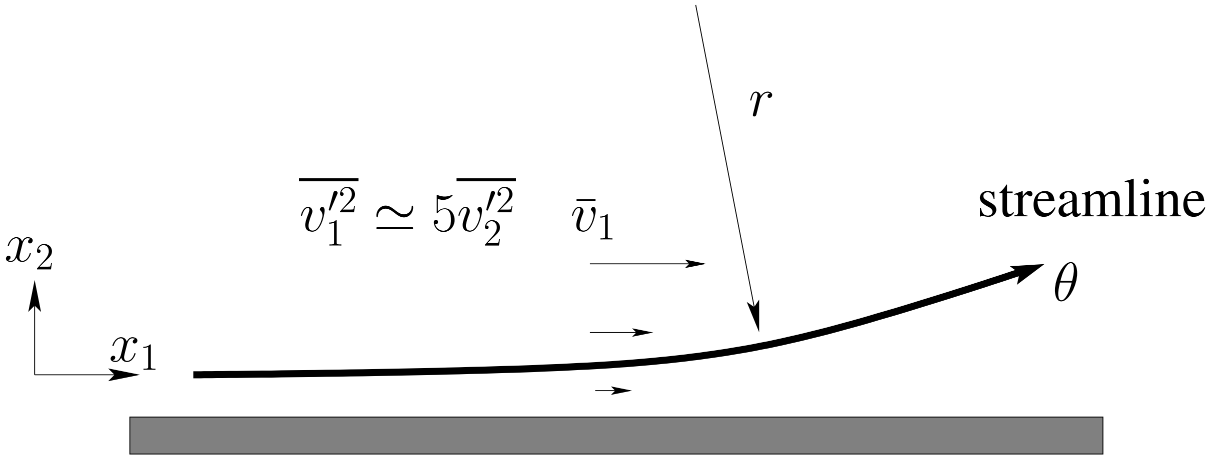

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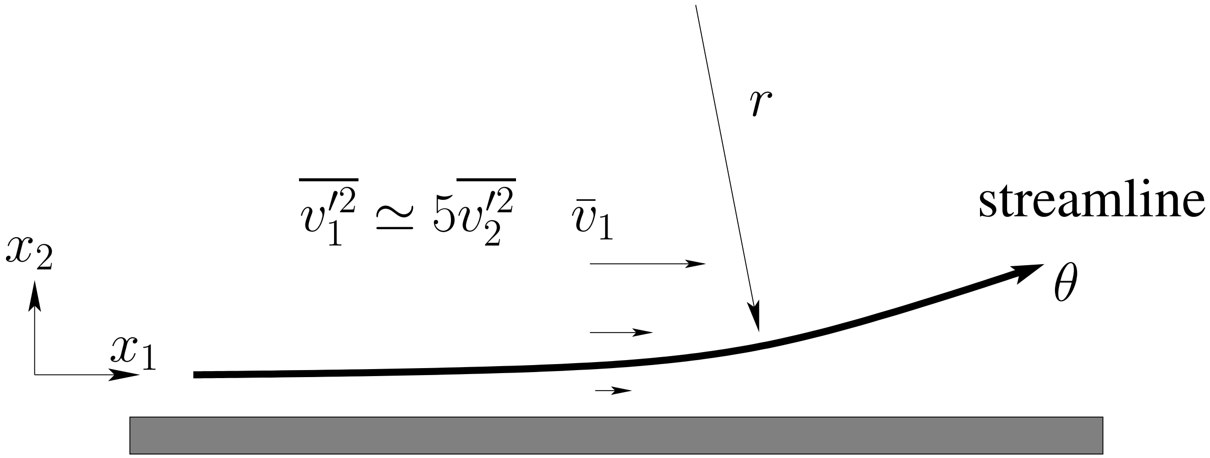

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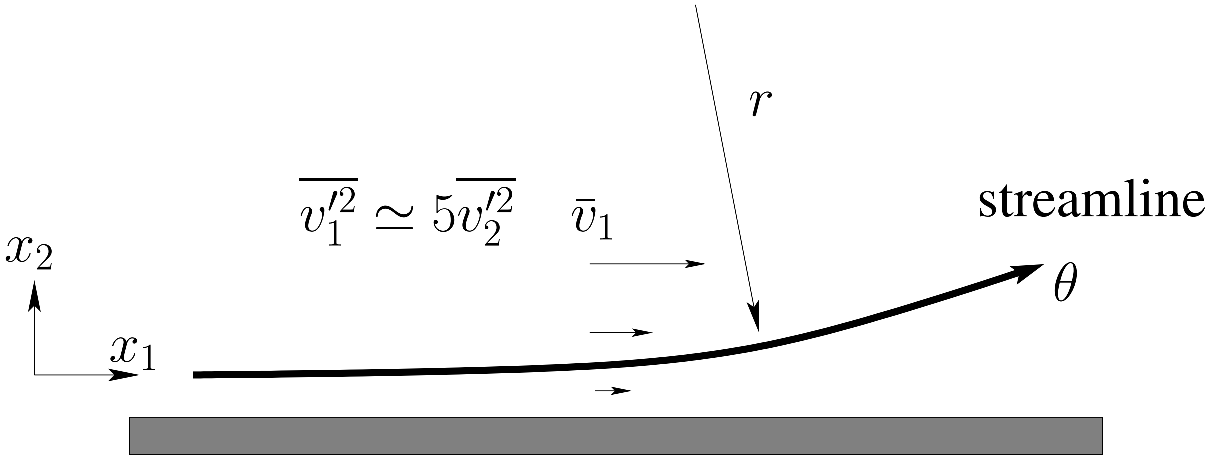

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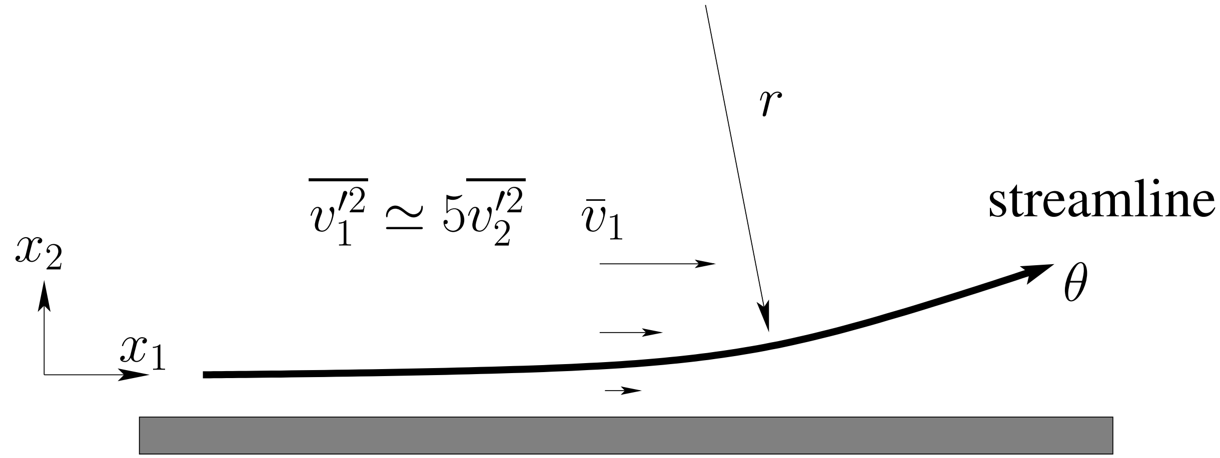

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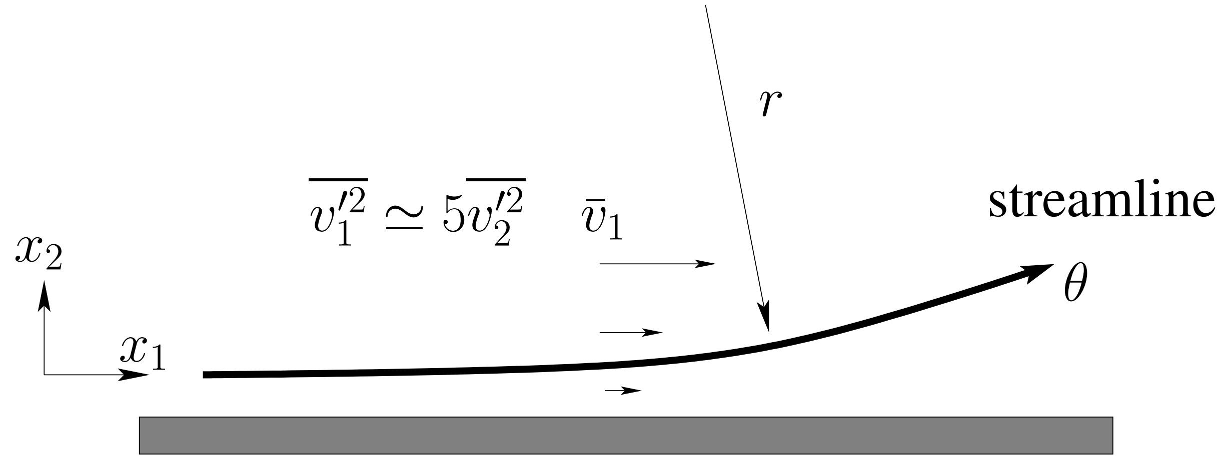

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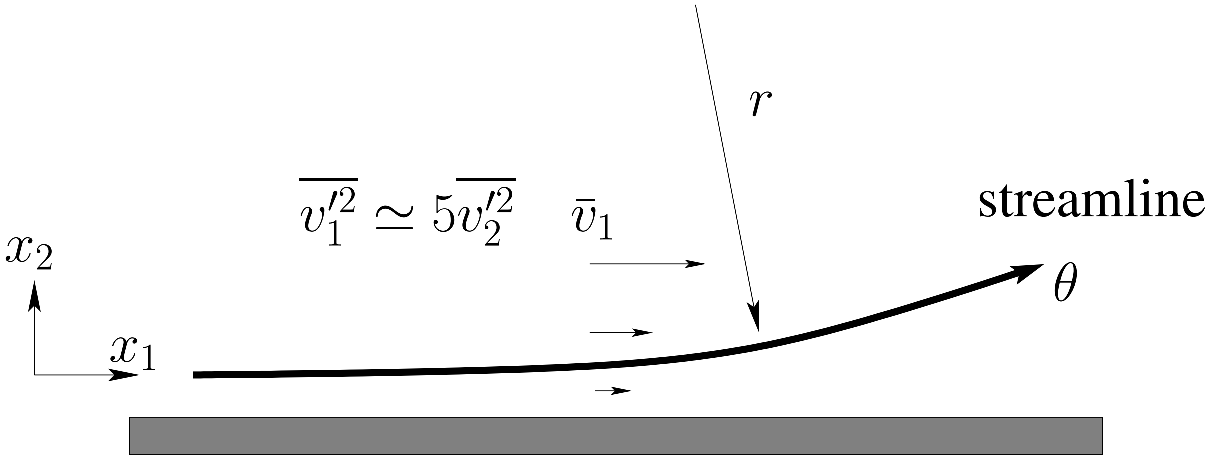

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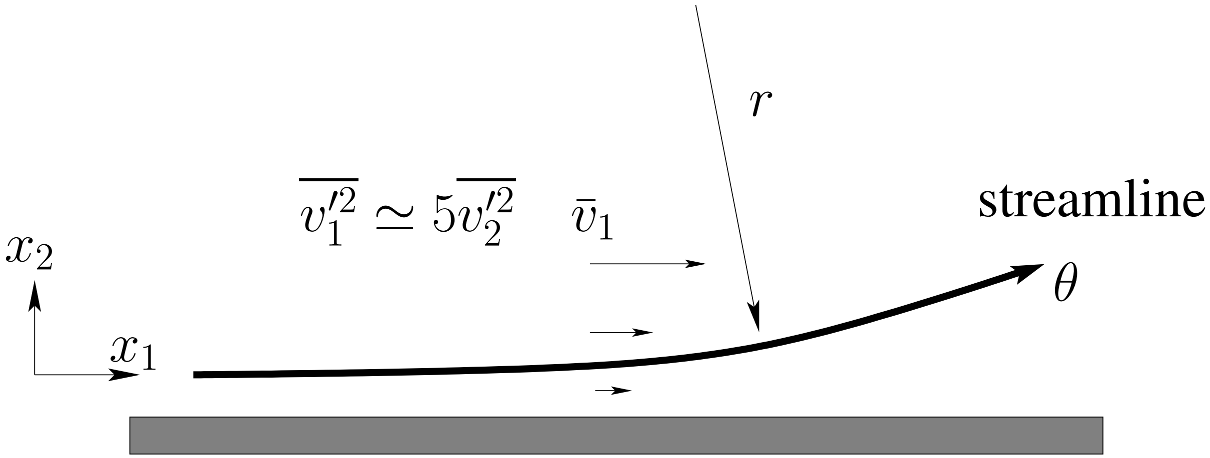

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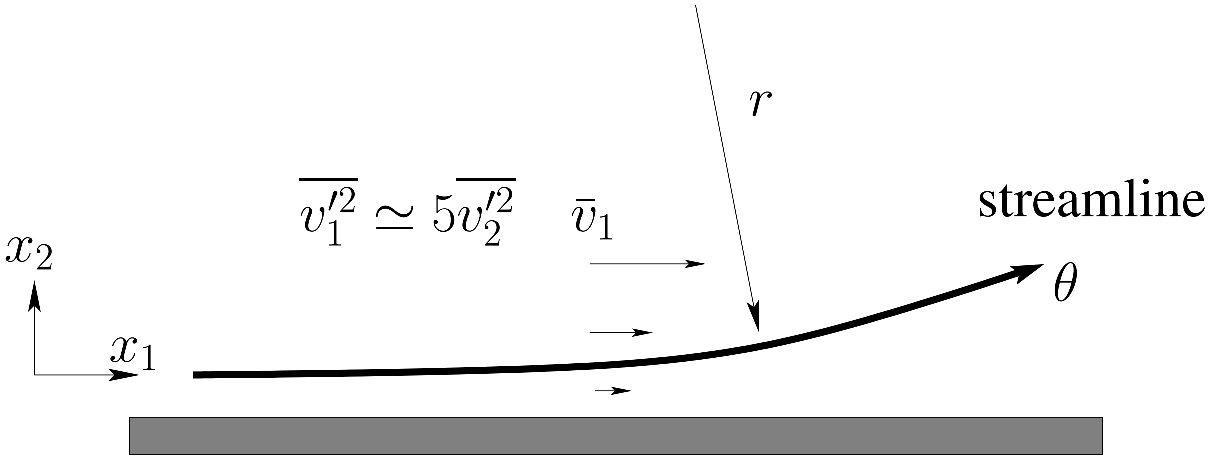

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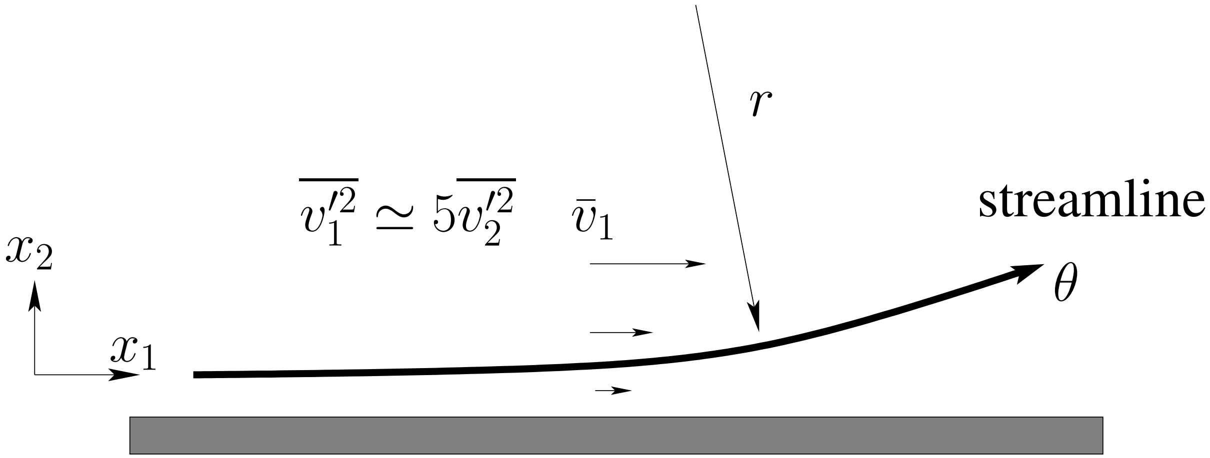

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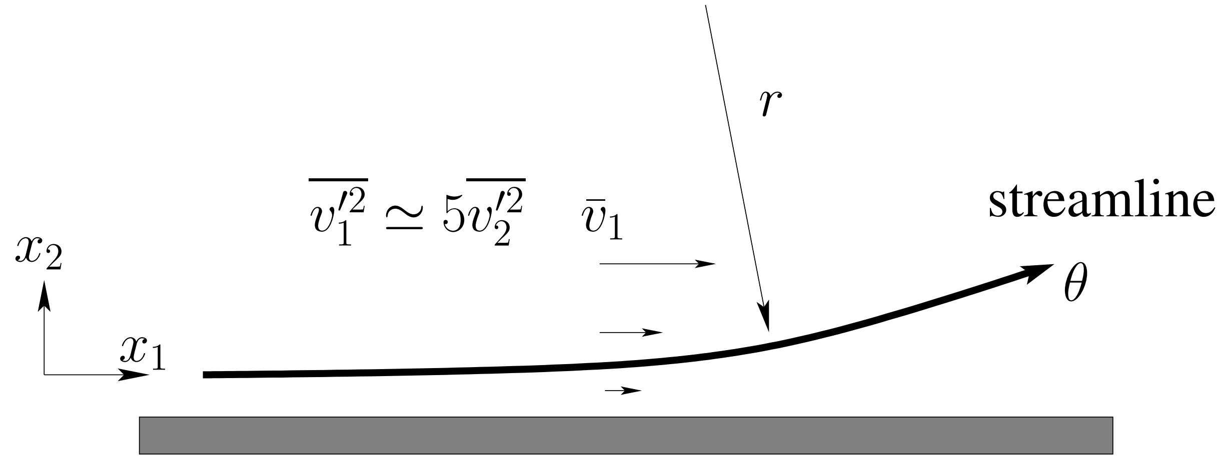

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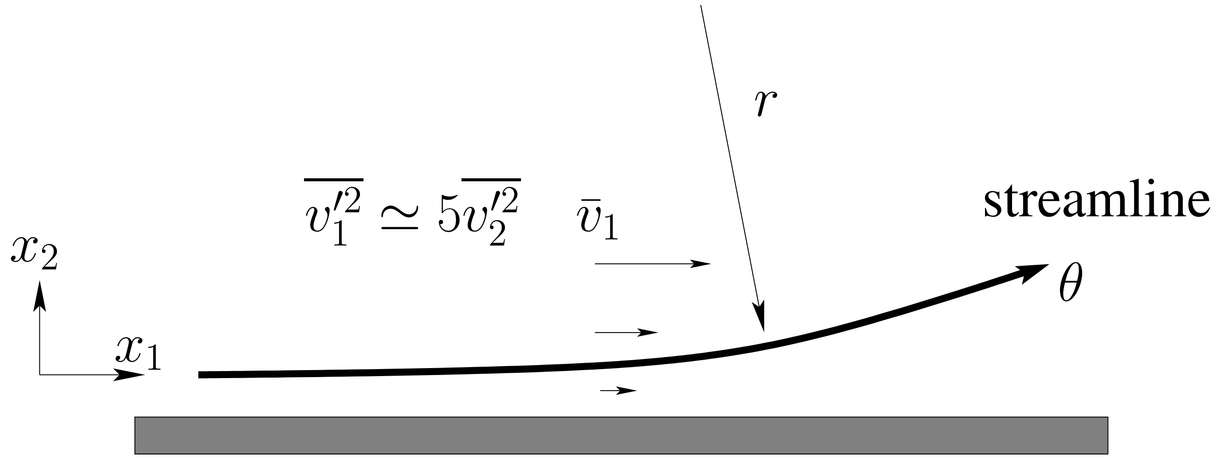

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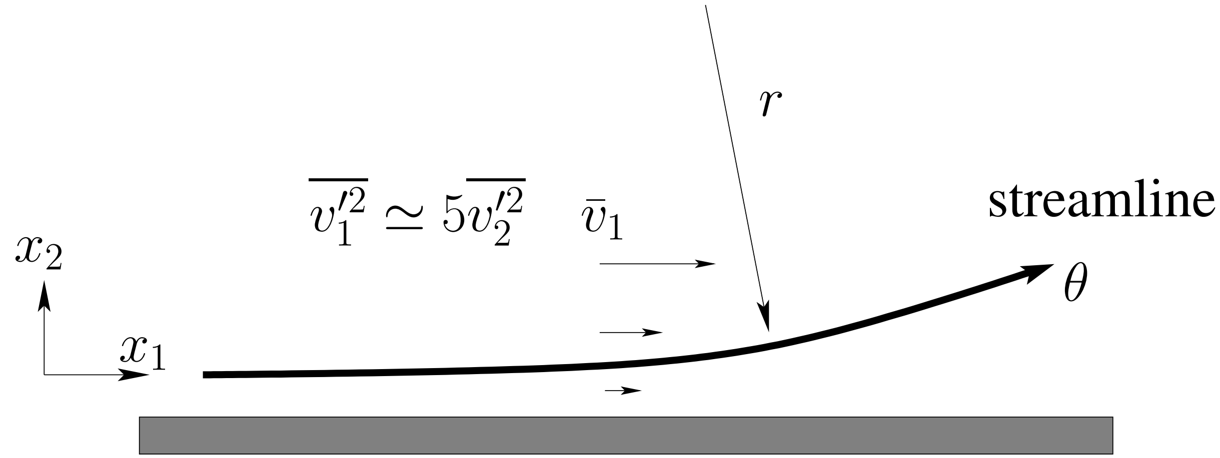

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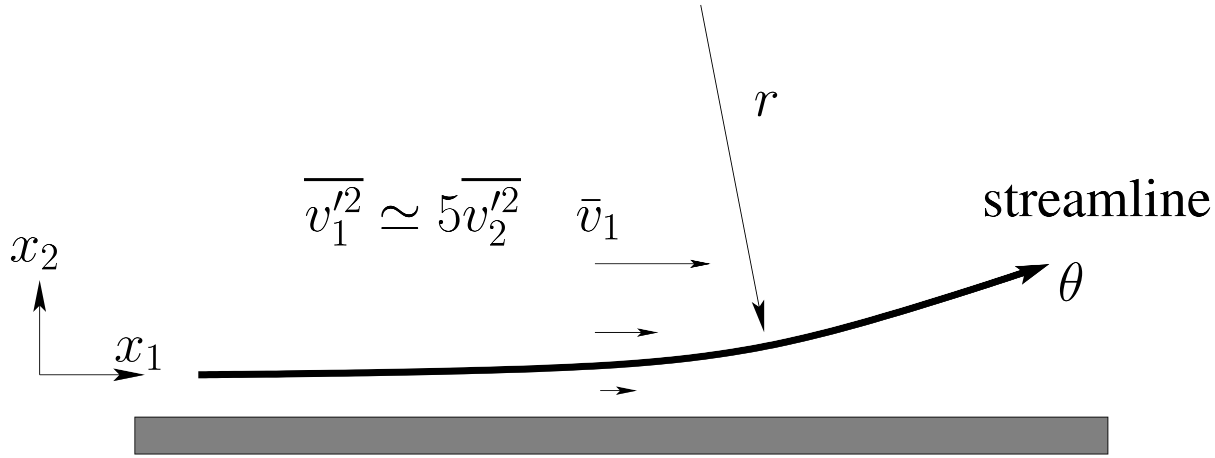

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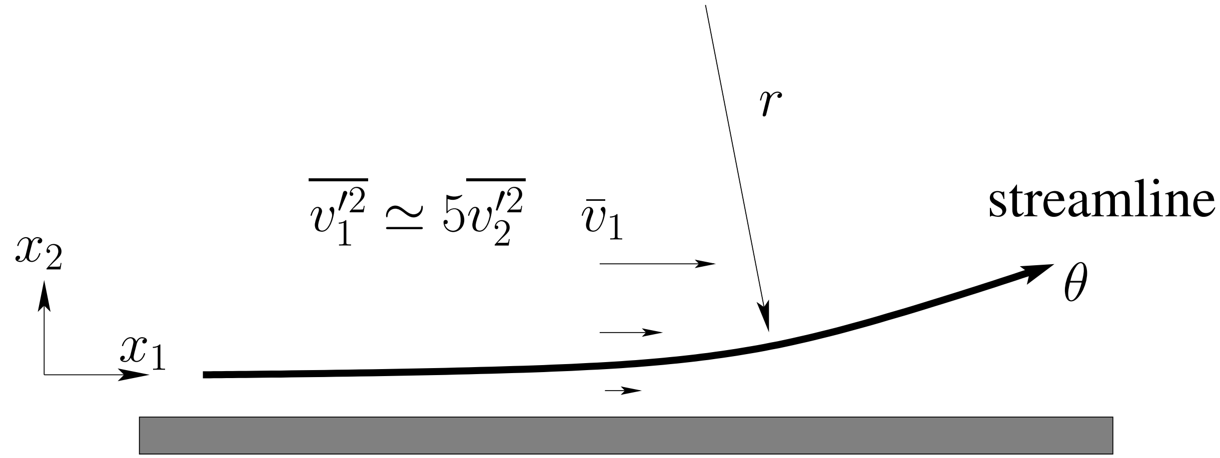

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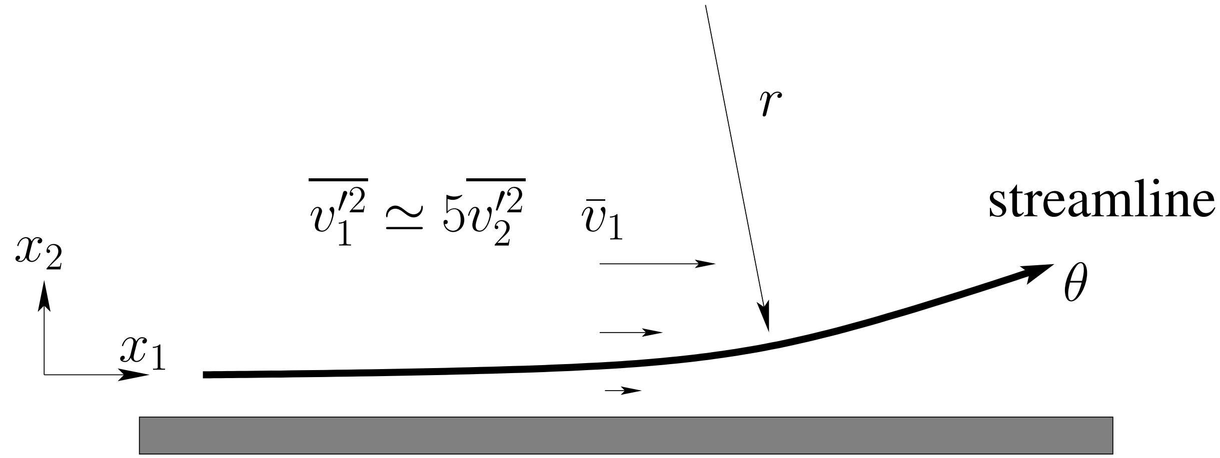

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k - \varepsilon \quad P^k &= \nu_t \left\{ \left(\frac{\partial \bar{v}_1}{\partial x_2}\right) + \boxed{\left(\frac{\partial \bar{v}_2}{\partial x_1}\right)} \right\}^2
\end{aligned}
$$

$$
\begin{aligned}
RSM,\ \overline{v_1'^2}-eq.: \quad P_{11} &= -2\overline{v_1'v_2'}\frac{\partial \bar{v}_1}{\partial x_2}\\
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\end{aligned}
$$

► $\frac{\partial \bar{v}_1}{\partial x_2} > 0,\ \frac{\partial \bar{v}_2}{\partial x_1} > 0$

$$
\begin{aligned}
RSM,\ \overline{v_1'^2}-eq.: \quad P_{11} &= -2\overline{v_1'v_2'}\frac{\partial \bar{v}_1}{\partial x_2} \\
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\end{aligned}
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▶ $\frac{\partial \bar{v}_1}{\partial x_2} > 0$, $\frac{\partial \bar{v}_2}{\partial x_1} > 0$, ▶ $-\overline{v_1'^2}\frac{\partial \bar{v}_2}{\partial x_1}$ increases $|P_{12}|$ (recall that $\overline{v_1'^2} \gg \overline{v_2'^2}$)

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RSM,\ \overline{v_1'^2}-eq.: \quad P_{11} &= -2\overline{v_1'v_2'}\frac{\partial \bar{v}_1}{\partial x_2}\\
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RSM,\ \overline{v_1'^2}-eq.:\quad P_{11} &= -2\overline{v_1'v_2'}\frac{\partial \bar v_1}{\partial x_2}\\
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▶ Hence: the turbulence increases (as it should).

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▶ $\frac{\partial \bar{v}_1}{\partial x_2} > 0, \frac{\partial \bar{v}_2}{\partial x_1} > 0$, ▶ $-\overline{v_1'^2}\frac{\partial \bar{v}_2}{\partial x_1}$ increases $|P_{12}|$ (recall that $\overline{v_1'^2} \gg \overline{v_2'^2}$) , ▶ $\Rightarrow |\overline{v_1'v_2'}|$ increases

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▶ Hence: the turbulence increases (as it should). ▶ We have a de-stabilizing streamline curvature.

$$
\begin{aligned}
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▶ $\dfrac{\partial \bar{v}_1}{\partial x_2} > 0, \dfrac{\partial \bar{v}_2}{\partial x_1} > 0$, ▶ $-\overline{v_1'^2}\dfrac{\partial \bar{v}_2}{\partial x_1}$ increases $|P_{12}|$ (recall that $\overline{v_1'^2} \gg \overline{v_2'^2}$) , ▶ $\Rightarrow |\overline{v_1'v_2'}|$ increases ▶ $\Rightarrow \overline{v_1'^2}$ and $\overline{v_2'^2}$ increase , ▶ $\Rightarrow |P_{12}|$ increases even more (a positive feedback loop)

▶ Hence: the turbulence increases (as it should). ▶ We have a de-stabilizing streamline curvature.

▶ Change the sign of $\dfrac{\partial \bar{v}_1}{\partial x_2}$ gives decreased turbulence (stabilizing streamline curvature)

$$RSM,\ \overline{v_1'^2}-eq.: \quad P_{11} = -2\overline{v_1'v_2'}\frac{\partial \bar{v}_1}{\partial x_2}$$

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► $\dfrac{\partial \bar{v}_1}{\partial x_2} > 0, \dfrac{\partial \bar{v}_2}{\partial x_1} > 0$, ► $-\overline{v_1'^2}\dfrac{\partial \bar{v}_2}{\partial x_1}$ increases $|P_{12}|$ (recall that $\overline{v_1'^2} \gg \overline{v_2'^2}$) , ► $\Rightarrow |\overline{v_1'v_2'}|$ increases ► $\Rightarrow \overline{v_1'^2}$ and $\overline{v_2'^2}$ increase , ► $\Rightarrow |P_{12}|$ increases even more (a positive feedback loop)

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► $k-\varepsilon$ model:

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RSM,\ \overline{v_1'^2}-eq.: \quad P_{11} &= -2\overline{v_1'v_2'}\frac{\partial \bar v_1}{\partial x_2}\\
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RSM,\ \overline{v_2'^2}-eq.: \quad P_{22} &= \boxed{-2\overline{v_1'v_2'}\frac{\partial \bar v_2}{\partial x_1}}\\
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▶ $\dfrac{\partial \bar v_1}{\partial x_2}>0,\ \dfrac{\partial \bar v_2}{\partial x_1}>0$, ▶ $-\overline{v_1'^2}\dfrac{\partial \bar v_2}{\partial x_1}$ increases $|P_{12}|$ (recall that $\overline{v_1'^2}\gg\overline{v_2'^2}$) , ▶ $\Rightarrow |\overline{v_1'v_2'}|$ increases ▶ $\Rightarrow \overline{v_1'^2}$ and $\overline{v_2'^2}$ increase , ▶ $\Rightarrow |P_{12}|$ increases even more (a positive feedback loop)

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▶ $k-\varepsilon$ model: it does react to streamline curvaturve but much less

$$RSM,\ \overline{v_1'^2} - eq.: \quad P_{11} = -2\overline{v_1'v_2'}\frac{\partial \bar{v}_1}{\partial x_2}$$

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▶ $\frac{\partial \bar{v}_1}{\partial x_2} > 0$, $\frac{\partial \bar{v}_2}{\partial x_1} > 0$, ▶ $-\overline{v_1'^2}\frac{\partial \bar{v}_2}{\partial x_1}$ increases $|P_{12}|$ (recall that $\overline{v_1'^2} \gg \overline{v_2'^2}$) , ▶ $\Rightarrow |\overline{v_1'v_2'}|$ increases ▶ $\Rightarrow \overline{v_1'^2}$ and $\overline{v_2'^2}$ increase , ▶ $\Rightarrow |P_{12}|$ increases even more (a positive feedback loop)

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▶ $k-\varepsilon$ model: it does react to streamline curvaturve but much less ▶ Why?

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\begin{aligned}
RSM,\ \overline{v_1'^2}-eq.: \quad P_{11} &= -2\overline{v_1'v_2'}\frac{\partial \bar v_1}{\partial x_2}\\
RSM,\ \overline{v_1'v_2'}-eq.: \quad P_{12} &= \boxed{-\overline{v_1'^2}\frac{\partial \bar v_2}{\partial x_1}} - \overline{v_2'^2}\frac{\partial \bar v_1}{\partial x_2}\\
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k-\varepsilon \quad P^k &= \nu_t\left\{\left(\frac{\partial \bar v_1}{\partial x_2}\right)+\boxed{\left(\frac{\partial \bar v_2}{\partial x_1}\right)}\right\}^2
\end{aligned}
$$

►$\frac{\partial \bar v_1}{\partial x_2}>0, \frac{\partial \bar v_2}{\partial x_1}>0$, ►$-\overline{v_1'^2}\frac{\partial \bar v_2}{\partial x_1}$ increases $|P_{12}|$ (recall that $\overline{v_1'^2} \gg \overline{v_2'^2}$) , ►$\Rightarrow |\overline{v_1'v_2'}|$ increases ►$\Rightarrow \overline{v_1'^2}$ and $\overline{v_2'^2}$ increase , ►$\Rightarrow |P_{12}|$ increases even more (a positive feedback loop)

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►$k-\varepsilon$ model: it does react to streamline curvaturve but much less ►Why?

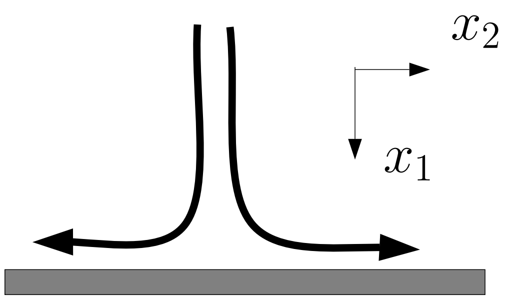
Contrary to RSM, the two velocity gradients are multiplied by the same coefficient

¶See Section 12.3, Stagnation flow

▶Stagnation 2D flow

¶See Section 12.3, Stagnation flow

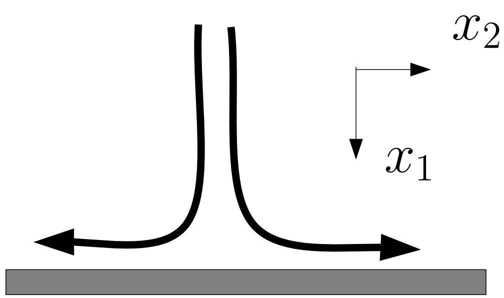
►Stagnation 2D flow



The flow pattern for stagnation flow.

¶See Section 12.3, Stagnation flow

▶Stagnation 2D flow

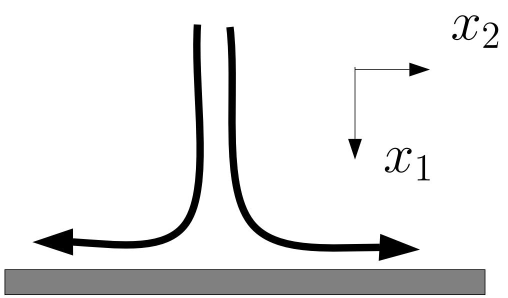


The flow pattern for stagnation flow.

▶Near the plate, strong deceleration, i.e. large $\dfrac{\partial \bar{v}_1}{\partial x_1}$.

¶See Section 12.3, Stagnation flow

▶Stagnation 2D flow

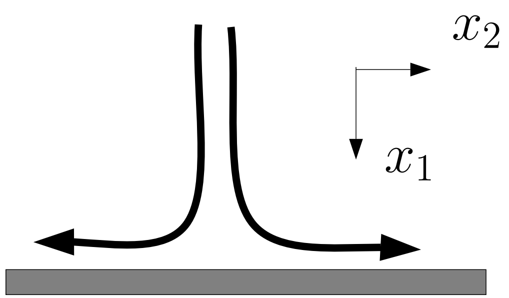


The flow pattern for stagnation flow.

▶Near the plate, strong deceleration, i.e. large $\dfrac{\partial \bar{v}_1}{\partial x_1}$. ▶Continuity equation $\Rightarrow$ large $\dfrac{\partial \bar{v}_2}{\partial x_2}$

¶See Section 12.3, Stagnation flow

▶Stagnation 2D flow



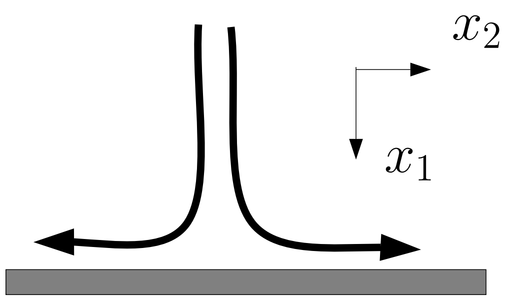
The flow pattern for stagnation flow.

▶Near the plate, strong deceleration, i.e. large $\dfrac{\partial \bar{v}_1}{\partial x_1}$. ▶Continuity equation ⇒ large $\dfrac{\partial \bar{v}_2}{\partial x_2}$

▶The velocity gradient $\dfrac{\partial \bar{v}_1}{\partial x_2}$ and $\dfrac{\partial \bar{v}_2}{\partial x_1}$ are in this flow negligible.

¶See Section 12.3, Stagnation flow

▶Stagnation 2D flow



The flow pattern for stagnation flow.

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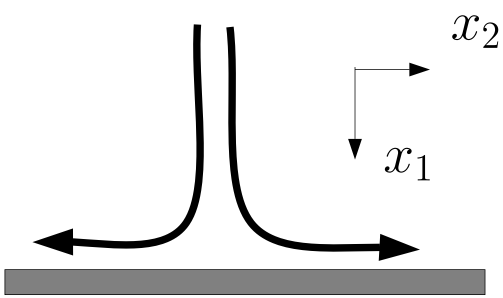
$$RSM/ASM \;:$$

¶See Section 12.3, Stagnation flow

▶Stagnation 2D flow



The flow pattern for stagnation flow.

▶Near the plate, strong deceleration, i.e. large $\dfrac{\partial \bar{v}_1}{\partial x_1}$. ▶Continuity equation $\Rightarrow$ large $\dfrac{\partial \bar{v}_2}{\partial x_2}$

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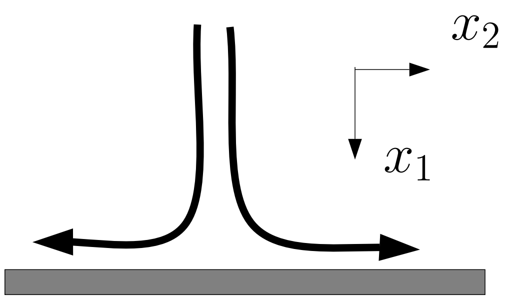
$$RSM/ASM \; : \; 0.5\,(P_{11} + P_{22})$$

¶See Section 12.3, Stagnation flow

▶Stagnation 2D flow



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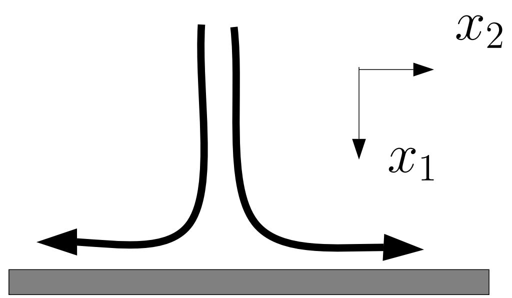
$$RSM/ASM\ :\ 0.5\,(P_{11}+P_{22}) = -\overline{v_1'^2}\frac{\partial \bar{v}_1}{\partial x_1} - \overline{v_2'^2}\frac{\partial \bar{v}_2}{\partial x_2}$$

¶See Section 12.3, Stagnation flow

▶Stagnation 2D flow



The flow pattern for stagnation flow.

▶Near the plate, strong deceleration, i.e. large $\dfrac{\partial \bar{v}_1}{\partial x_1}$. ▶Continuity equation $\Rightarrow$ large $\dfrac{\partial \bar{v}_2}{\partial x_2}$

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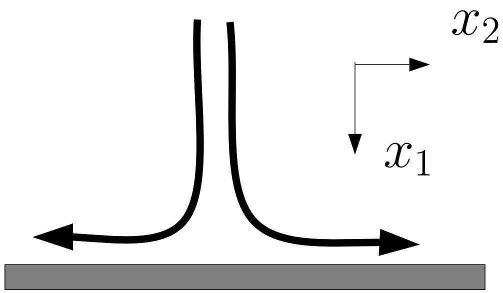
$$RSM/ASM\ :\ 0.5\left(P_{11}+P_{22}\right)= -\overline{v_1'^2}\frac{\partial \bar{v}_1}{\partial x_1} - \overline{v_2'^2}\frac{\partial \bar{v}_2}{\partial x_2} = -\frac{\partial \bar{v}_1}{\partial x_1}\left(\overline{v_1'^2} - \overline{v_2'^2}\right)$$

¶See Section 12.3, Stagnation flow

▶Stagnation 2D flow



The flow pattern for stagnation flow.

▶Near the plate, strong deceleration, i.e. large $\frac{\partial \bar{v}_1}{\partial x_1}$. ▶Continuity equation ⇒ large $\frac{\partial \bar{v}_2}{\partial x_2}$

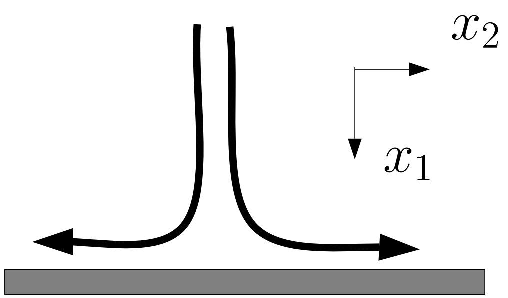
▶The velocity gradient $\frac{\partial \bar{v}_1}{\partial x_2}$ and $\frac{\partial \bar{v}_2}{\partial x_1}$ are in this flow negligible.

$$RSM/ASM \;:\; 0.5\,(P_{11}+P_{22}) = -\overline{v_1'^2}\frac{\partial \bar{v}_1}{\partial x_1} - \overline{v_2'^2}\frac{\partial \bar{v}_2}{\partial x_2} = -\frac{\partial \bar{v}_1}{\partial x_1}(\overline{v_1'^2} - \overline{v_2'^2})$$

$$k-\varepsilon \;:$$

¶See Section 12.3, Stagnation flow

▶Stagnation 2D flow



The flow pattern for stagnation flow.

▶Near the plate, strong deceleration, i.e. large $\frac{\partial \bar{v}_1}{\partial x_1}$. ▶Continuity equation ⇒ large $\frac{\partial \bar{v}_2}{\partial x_2}$

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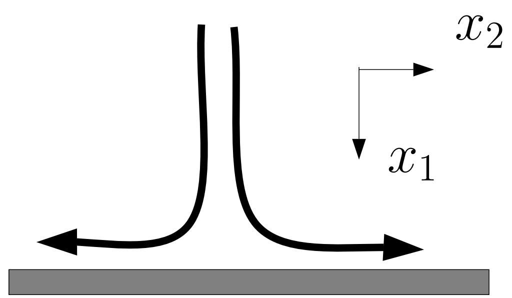
$$RSM/ASM \ : \ 0.5\,(P_{11}+P_{22}) = -\overline{v_1'^2}\frac{\partial \bar{v}_1}{\partial x_1} - \overline{v_2'^2}\frac{\partial \bar{v}_2}{\partial x_2} = -\frac{\partial \bar{v}_1}{\partial x_1}(\overline{v_1'^2} - \overline{v_2'^2})$$

$$k-\varepsilon \ : \ P^k = 2\nu_t\left\{\left(\frac{\partial \bar{v}_1}{\partial x_1}\right)^2 + \left(\frac{\partial \bar{v}_2}{\partial x_2}\right)^2\right\}$$

¶See Section 12.3, Stagnation flow

►Stagnation 2D flow



The flow pattern for stagnation flow.

►Near the plate, strong deceleration, i.e. large $\dfrac{\partial \bar{v}_1}{\partial x_1}$. ►Continuity equation ⇒ large $\dfrac{\partial \bar{v}_2}{\partial x_2}$

►The velocity gradient $\dfrac{\partial \bar{v}_1}{\partial x_2}$ and $\dfrac{\partial \bar{v}_2}{\partial x_1}$ are in this flow negligible.

$$RSM/ASM \;:\; 0.5\,(P_{11}+P_{22}) = -\overline{v_1'^2}\frac{\partial \bar{v}_1}{\partial x_1} - \overline{v_2'^2}\frac{\partial \bar{v}_2}{\partial x_2} = -\frac{\partial \bar{v}_1}{\partial x_1}(\overline{v_1'^2} - \overline{v_2'^2})$$

$$k-\varepsilon \;:\; P^k = 2\nu_t \left\{ \left(\frac{\partial \bar{v}_1}{\partial x_1}\right)^2 + \left(\frac{\partial \bar{v}_2}{\partial x_2}\right)^2 \right\}$$

For RSM/ASM, $\overline{v_1'^2} - \overline{v_2'^2}$ nearly cancels whereas for $k-\varepsilon$ they don't since the sum of squared velocity gradients is used.

¶See Section 13, Realizability

▶Realizability

¶See Section 13, Realizability

▶Realizability

¶See Section 13, Realizability
▶Realizability

$$\overline{v_i'^2}$$

¶See Section 13, Realizability

►Realizability

$$\overline{v_i'^2} \geq$$

¶See Section 13, Realizability

▶Realizability

$$\overline{v_i'^2} \geq 0 \text{ for all } i$$

¶See Section 13, Realizability

▶Realizability

$$\overline{v_i'^2} \geq 0 \text{ for all } i$$

$$\frac{\left|\overline{v_i' v_j'}\right|}{\left(\overline{v_i'^2}\,\overline{v_j'^2}\right)^{1/2}}$$

¶See Section 13, Realizability

▶Realizability

$$\overline{v_i'^2} \geq 0 \text{ for all } i$$

$$\frac{\left|\overline{v_i' v_j'}\right|}{\left(\overline{v_i'^2}\,\overline{v_j'^2}\right)^{1/2}} \leq$$

¶See Section 13, Realizability

▶Realizability

$$\overline{v_i'^2} \geq 0 \text{ for all } i$$

$$\frac{\left|\overline{v_i' v_j'}\right|}{\left(\overline{v_i'^2}\,\overline{v_j'^2}\right)^{1/2}} \leq 1 \text{ no summation over } i \text{ and } j,\ i \neq j$$

¶See Section 13, Realizability

▶Realizability

$$\overline{v_i'^2} \geq 0 \text{ for all } i$$

$$\frac{\left|\overline{v_i'v_j'}\right|}{\left(\overline{v_i'^2}\,\overline{v_j'^2}\right)^{1/2}} \leq 1 \text{ no summation over } i \text{ and } j,\ i \neq j$$

$$\overline{v_1'^2} = \frac{2}{3}k - 2\nu_t \frac{\partial \bar{v}_1}{\partial x_1} = \frac{2}{3}k - 2\nu_t \bar{s}_{11}$$

¶See Section 13, Realizability

▶Realizability

$$\overline{v_i'^2} \geq 0 \text{ for all } i$$

$$\frac{\left|\overline{v_i' v_j'}\right|}{\left(\overline{v_i'^2}\;\overline{v_j'^2}\right)^{1/2}} \leq 1 \text{ no summation over } i \text{ and } j,\; i \neq j$$

$$\overline{v_1'^2} = \frac{2}{3}k - 2\nu_t \frac{\partial \bar{v}_1}{\partial x_1} = \frac{2}{3}k - 2\nu_t \bar{s}_{11}$$

$\bar{s}_{11}$ largest in the principal coordinate directions. Hence, let's find the eigenvalues of $\bar{s}_{ij}$

¶See Section 13, Realizability

▶Realizability

$$\overline{v_i'^2} \geq 0 \text{ for all } i$$

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$\bar{s}_{11}$ largest in the principal coordinate directions. Hence, let's find the eigenvalues of $\bar{s}_{ij}$

$$|\bar{s}_{ij} - \delta_{ij}\lambda| = 0$$

¶See Section 13, Realizability

▶Realizability

$$\overline{v_i'^2} \geq 0 \text{ for all } i$$

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$$\overline{v_1'^2} = \frac{2}{3}k - 2\nu_t\frac{\partial \bar{v}_1}{\partial x_1} = \frac{2}{3}k - 2\nu_t\bar{s}_{11}$$

$\bar{s}_{11}$ largest in the principal coordinate directions. Hence, let's find the eigenvalues of $\bar{s}_{ij}$

$$|\bar{s}_{ij} - \delta_{ij}\lambda| = 0$$

which gives in 2D

$$\begin{vmatrix} \bar{s}_{11} - \lambda & \bar{s}_{12} \\ \bar{s}_{21} & \bar{s}_{22} - \lambda \end{vmatrix} = 0$$

¶See Section 13, Realizability

▶Realizability

$$\overline{v_i'^2} \geq 0 \text{ for all } i$$

$$\frac{\left|\overline{v_i' v_j'}\right|}{\left(\overline{v_i'^2}\,\overline{v_j'^2}\right)^{1/2}} \leq 1 \text{ no summation over } i \text{ and } j,\ i \neq j$$

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The resulting equation characteristic equation is

¶See Section 13, Realizability

▶Realizability

$$
\begin{aligned}
\overline{v_i'^2} &\geq 0 \text{ for all } i \\
\frac{\left|\overline{v_i'v_j'}\right|}{\left(\overline{v_i'^2}\,\overline{v_j'^2}\right)^{1/2}} &\leq 1 \text{ no summation over } i \text{ and } j,\ i \neq j
\end{aligned}
$$

$$
\overline{v_1'^2} = \frac{2}{3}k - 2\nu_t \frac{\partial \bar{v}_1}{\partial x_1} = \frac{2}{3}k - 2\nu_t \bar{s}_{11}
$$

$\bar{s}_{11}$ largest in the principal coordinate directions. Hence, let's find the eigenvalues of $\bar{s}_{ij}$

$$
|\bar{s}_{ij} - \delta_{ij}\lambda| = 0
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which gives in 2D

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\begin{vmatrix} \bar{s}_{11} - \lambda & \bar{s}_{12} \\ \bar{s}_{21} & \bar{s}_{22} - \lambda \end{vmatrix} = 0
$$

The resulting equation characteristic equation is

$$
\lambda^2 - I_1^{2D}\lambda + I_2^{2D} = 0
$$

¶See Section 13, Realizability

▶Realizability

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$$\frac{\left|\overline{v_i'v_j'}\right|}{\left(\overline{v_i'^2}\,\overline{v_j'^2}\right)^{1/2}} \leq 1 \text{ no summation over } i \text{ and } j,\ i \neq j$$

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The resulting equation characteristic equation is

$$\lambda^2 - I_1^{2D}\lambda + I_2^{2D} = 0$$

$$I_1^{2D} = \bar{s}_{ii} = 0 \quad \textbf{continuity equation}$$

¶See Section 13, Realizability

▶Realizability

$$\overline{v_i'^2} \geq 0 \text{ for all } i$$

$$\frac{\left|\overline{v_i' v_j'}\right|}{\left(\overline{v_i'^2}\,\overline{v_j'^2}\right)^{1/2}} \leq 1 \text{ no summation over } i \text{ and } j,\ i \neq j$$

$$\overline{v_1'^2} = \frac{2}{3}k - 2\nu_t \frac{\partial \bar{v}_1}{\partial x_1} = \frac{2}{3}k - 2\nu_t \bar{s}_{11}$$

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which gives in 2D

$$\begin{vmatrix} \bar{s}_{11} - \lambda & \bar{s}_{12} \\ \bar{s}_{21} & \bar{s}_{22} - \lambda \end{vmatrix} = 0$$

The resulting equation characteristic equation is

$$\begin{aligned} \lambda^2 &- I_1^{2D}\lambda + I_2^{2D} = 0 \\ I_1^{2D} &= \bar{s}_{ii} = 0 \quad \textbf{continuity equation} \\ I_2^{2D} &= \frac{1}{2}(\bar{s}_{ii}\bar{s}_{jj} - \bar{s}_{ij}\bar{s}_{ij}) = \det(C_{ij}) = -\bar{s}_{ij}\bar{s}_{ij}/2 \end{aligned}$$

¶See Section 13, Realizability

▶Realizability

$$\overline{v_i'^2} \geq 0 \text{ for all } i$$
$$\frac{\left|\overline{v_i' v_j'}\right|}{\left(\overline{v_i'^2}\,\overline{v_j'^2}\right)^{1/2}} \leq 1 \text{ no summation over } i \text{ and } j,\ i \neq j$$

$$\overline{v_1'^2} = \frac{2}{3}k - 2\nu_t \frac{\partial \bar{v}_1}{\partial x_1} = \frac{2}{3}k - 2\nu_t \bar{s}_{11}$$

$\bar{s}_{11}$ largest in the principal coordinate directions. Hence, let's find the eigenvalues of $\bar{s}_{ij}$

$$|\bar{s}_{ij} - \delta_{ij}\lambda| = 0$$

which gives in 2D

$$\left| \begin{array}{cc} \bar{s}_{11} - \lambda & \bar{s}_{12} \\ \bar{s}_{21} & \bar{s}_{22} - \lambda \end{array} \right| = 0$$

The resulting equation characteristic equation is

$$\lambda^2 - I_1^{2D}\lambda + I_2^{2D} = 0$$
$$I_1^{2D} = \bar{s}_{ii} = 0 \quad \textbf{continuity equation}$$
$$I_2^{2D} = \frac{1}{2}(\bar{s}_{ii}\bar{s}_{jj} - \bar{s}_{ij}\bar{s}_{ij}) = \det(C_{ij}) = -\bar{s}_{ij}\bar{s}_{ij}/2$$

$$\lambda^2 + I_2^{2D} = 0$$

$$\lambda^2 + I_2^{2D} = 0$$

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$$\lambda_{1,2} = \pm \left(-I_2^{2D}\right)^{1/2}$$

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$$\lambda^2 + I_2^{2D} = 0$$

$$\lambda_{1,2} = \pm \left(-I_2^{2D}\right)^{1/2} = \pm \left(\frac{\bar{s}_{ij}\bar{s}_{ij}}{2}\right)^{1/2}$$

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$$\lambda_{1,2} = \pm \left(-I_2^{2D}\right)^{1/2} = \pm \left(\frac{\bar{s}_{ij}\bar{s}_{ij}}{2}\right)^{1/2}$$

$$\lambda^2 + I_2^{2D} = 0$$

$$\lambda_{1,2} = \pm \left(-I_2^{2D}\right)^{1/2} = \pm \left(\frac{\bar{s}_{ij}\bar{s}_{ij}}{2}\right)^{1/2}$$

$$\overline{v_1'^2}$$

$$\lambda^2 + I_2^{2D} = 0$$

$$\lambda_{1,2} = \pm \left(-I_2^{2D}\right)^{1/2} \; = \; \pm \left(\frac{\bar{s}_{ij}\bar{s}_{ij}}{2}\right)^{1/2}$$

$$\overline{v_1'^2} \; = \; \frac{2}{3}k - 2\nu_t \bar{s}_{11}$$

$$\lambda^2 + I_2^{2D} = 0$$

$$\lambda_{1,2} = \pm \left(-I_2^{2D}\right)^{1/2} \;=\; \pm \left(\frac{\bar{s}_{ij}\bar{s}_{ij}}{2}\right)^{1/2}$$

$$\overline{v_1'^2} \;=\; \frac{2}{3}k - 2\nu_t \bar{s}_{11}$$

$$\left(\overline{v_1'^2}\right)_{\lambda_1}$$

$$\lambda^2 + I_2^{2D} = 0$$

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$$\overline{v_1'^2} = \frac{2}{3}k - 2\nu_t \bar{s}_{11}$$

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$$\left(\overline{v_1'^2}\right)_{\lambda_1} = \frac{2}{3}k - 2\nu_t \lambda_1$$

$$\lambda^2 + I_2^{2D} = 0$$

$$\lambda_{1,2} = \pm \left(-I_2^{2D}\right)^{1/2} \; = \; \pm \left(\frac{\bar{s}_{ij}\bar{s}_{ij}}{2}\right)^{1/2}$$

$$\overline{v_1'^2} \; = \; \frac{2}{3}k - 2\nu_t \bar{s}_{11}$$

$$\left(\overline{v_1'^2}\right)_{\lambda_1} \; = \; \frac{2}{3}k - 2\nu_t \lambda_1 = \frac{2}{3}k - 2\nu_t \left(\frac{\bar{s}_{ij}\bar{s}_{ij}}{2}\right)^{1/2}$$

$$\lambda^2 + I_2^{2D} = 0$$

$$\lambda_{1,2} = \pm \left(-I_2^{2D}\right)^{1/2} \; = \; \pm \left(\frac{\bar{s}_{ij}\bar{s}_{ij}}{2}\right)^{1/2}$$

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$$\lambda^2 + I_2^{2D} = 0$$

$$\lambda_{1,2} = \pm \left(-I_2^{2D}\right)^{1/2} \ = \ \pm \left(\frac{\bar{s}_{ij}\bar{s}_{ij}}{2}\right)^{1/2}$$

$$\overline{v_1'^2} \ = \ \frac{2}{3}k - 2\nu_t \bar{s}_{11}$$

$$\left(\overline{v_1'^2}\right)_{\lambda_1} \ = \ \frac{2}{3}k - 2\nu_t\lambda_1 = \frac{2}{3}k - 2\nu_t \left(\frac{\bar{s}_{ij}\bar{s}_{ij}}{2}\right)^{1/2} > 0$$

$$\Rightarrow \nu_t \leq \frac{k}{3|\lambda_1|} = \frac{k}{3}\left(\frac{2}{\bar{s}_{ij}\bar{s}_{ij}}\right)^{1/2}$$

$$\lambda^2 + I_2^{2D} = 0$$

$$\lambda_{1,2} = \pm\left(-I_2^{2D}\right)^{1/2} \; = \; \pm\left(\frac{\bar{s}_{ij}\bar{s}_{ij}}{2}\right)^{1/2}$$

$$\overline{v_1'^2} \; = \; \frac{2}{3}k - 2\nu_t\bar{s}_{11}$$

$$\left(\overline{v_1'^2}\right)_{\lambda_1} \; = \; \frac{2}{3}k - 2\nu_t\lambda_1 = \frac{2}{3}k - 2\nu_t\left(\frac{\bar{s}_{ij}\bar{s}_{ij}}{2}\right)^{1/2} > 0$$

$$\Rightarrow \nu_t \leq \frac{k}{3|\lambda_1|} = \frac{k}{3}\left(\frac{2}{\bar{s}_{ij}\bar{s}_{ij}}\right)^{1/2}$$

In 3D

$$\lambda^2 + I_2^{2D} = 0$$

$$\lambda_{1,2} = \pm \left(-I_2^{2D}\right)^{1/2} = \pm \left(\frac{\bar{s}_{ij}\bar{s}_{ij}}{2}\right)^{1/2}$$

$$\overline{v_1'^2} = \frac{2}{3}k - 2\nu_t \bar{s}_{11}$$

$$\left(\overline{v_1'^2}\right)_{\lambda_1} = \frac{2}{3}k - 2\nu_t\lambda_1 = \frac{2}{3}k - 2\nu_t \left(\frac{\bar{s}_{ij}\bar{s}_{ij}}{2}\right)^{1/2} > 0$$

$$\Rightarrow \nu_t \leq \frac{k}{3|\lambda_1|} = \frac{k}{3}\left(\frac{2}{\bar{s}_{ij}\bar{s}_{ij}}\right)^{1/2}$$

In 3D

$$|\lambda_k| = k\left(\frac{2\bar{s}_{ij}\bar{s}_{ij}}{3}\right)^{1/2}$$

$$\lambda^2 + I_2^{2D} = 0$$

$$\lambda_{1,2} = \pm\left(-I_2^{2D}\right)^{1/2} = \pm\left(\frac{\bar{s}_{ij}\bar{s}_{ij}}{2}\right)^{1/2}$$

$$\overline{v_1'^2} = \frac{2}{3}k - 2\nu_t \bar{s}_{11}$$

$$\left(\overline{v_1'^2}\right)_{\lambda_1} = \frac{2}{3}k - 2\nu_t \lambda_1 = \frac{2}{3}k - 2\nu_t \left(\frac{\bar{s}_{ij}\bar{s}_{ij}}{2}\right)^{1/2} > 0$$

$$\Rightarrow \nu_t \leq \frac{k}{3|\lambda_1|} = \frac{k}{3}\left(\frac{2}{\bar{s}_{ij}\bar{s}_{ij}}\right)^{1/2}$$

In 3D

$$|\lambda_k| = k\left(\frac{2\bar{s}_{ij}\bar{s}_{ij}}{3}\right)^{1/2}$$

¶See Section 14, Non-linear Eddy-viscosity Models

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¶See Section 14, Non-linear Eddy-viscosity Models

►It is non-linear in velocity gradients.

¶See Section 14, Non-linear Eddy-viscosity Models

- It is non-linear in velocity gradients.
- The advantage is better normal stresses and a certain ability to handle streamline curvature.

¶See Section 14, Non-linear Eddy-viscosity Models

▶It is non-linear in velocity gradients.

▶The advantage is better normal stresses and a certain ability to handle streamline curvature.

$$a_{ij} \equiv \frac{\overline{v_i' v_j'}}{k} - \frac{2}{3}\delta_{ij}$$

$$\begin{aligned}
a_{ij} &= \boxed{-2c_\mu \tau \bar{s}_{ij}} + c_1\tau^2\left(\bar{s}_{ik}\bar{s}_{kj} - \frac{1}{3}\bar{s}_{mk}\bar{s}_{mk}\delta_{ij}\right) + c_2\tau^2\left(\bar{\Omega}_{ik}\bar{s}_{kj} - \bar{s}_{ik}\bar{\Omega}_{kj}\right) \\
&+ c_3\tau^2\left(\bar{\Omega}_{ik}\bar{\Omega}_{jk} - \frac{1}{3}\bar{\Omega}_{mk}\bar{\Omega}_{mk}\delta_{ij}\right) + c_4\tau^3\left(\bar{s}_{ik}\bar{s}_{km}\bar{\Omega}_{mj} - \bar{\Omega}_{im}\bar{s}_{mk}\bar{s}_{kj}\right) \\
&+ c_5\tau^3\left(\bar{\Omega}_{im}\bar{\Omega}_{mm}\bar{s}_{mj} + \bar{s}_{im}\bar{\Omega}_{mm}\bar{\Omega}_{mj} - \frac{2}{3}\bar{\Omega}_{mn}\bar{\Omega}_{nm}\bar{s}_{mm}\delta_{ij}\right) + c_6\tau^3\bar{s}_{km}\bar{s}_{km}\bar{s}_{ij} + c_7\tau^3\bar{\Omega}_{km}\bar{\Omega}_{km}\bar{s}_{ij} \\
\bar{s}_{ij} &= \frac{1}{2}\left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i}\right), \quad \bar{\Omega}_{ij} = \frac{1}{2}\left(\frac{\partial \bar{v}_i}{\partial x_j} - \frac{\partial \bar{v}_j}{\partial x_i}\right), \quad \tau = \frac{k}{\varepsilon} \text{ or } \tau = \frac{1}{c_\mu \omega}
\end{aligned} \tag{33.2}$$

¶See Section 14, Non-linear Eddy-viscosity Models

▶It is non-linear in velocity gradients.

▶The advantage is better normal stresses and a certain ability to handle streamline curvature.

$$
\begin{aligned}
a_{ij} &\equiv \frac{\overline{v'_i v'_j}}{k} - \frac{2}{3}\delta_{ij}\\
a_{ij} &= \boxed{-2c_\mu \tau \bar{s}_{ij}} + c_1\tau^2\left(\bar{s}_{ik}\bar{s}_{kj} - \frac{1}{3}\bar{s}_{mk}\bar{s}_{mk}\delta_{ij}\right) + c_2\tau^2\left(\bar{\Omega}_{ik}\bar{s}_{kj} - \bar{s}_{ik}\bar{\Omega}_{kj}\right)\\
&+ c_3\tau^2\left(\bar{\Omega}_{ik}\bar{\Omega}_{jk} - \frac{1}{3}\bar{\Omega}_{mk}\bar{\Omega}_{mk}\delta_{ij}\right) + c_4\tau^3\left(\bar{s}_{ik}\bar{s}_{km}\bar{\Omega}_{mj} - \bar{\Omega}_{im}\bar{s}_{mk}\bar{s}_{kj}\right)\\
&+ c_5\tau^3\left(\bar{\Omega}_{im}\bar{\Omega}_{mm}\bar{s}_{mj} + \bar{s}_{im}\bar{\Omega}_{mm}\bar{\Omega}_{mj} - \frac{2}{3}\bar{\Omega}_{mn}\bar{\Omega}_{nm}\bar{s}_{mm}\delta_{ij}\right) + c_6\tau^3\bar{s}_{km}\bar{s}_{km}\bar{s}_{ij} + c_7\tau^3\bar{\Omega}_{km}\bar{\Omega}_{km}\bar{s}_{ij}\\
\bar{s}_{ij} &= \frac{1}{2}\left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i}\right), \quad \bar{\Omega}_{ij} = \frac{1}{2}\left(\frac{\partial \bar{v}_i}{\partial x_j} - \frac{\partial \bar{v}_j}{\partial x_i}\right), \quad \tau = \frac{k}{\varepsilon} \text{ or } \tau = \frac{1}{c_\mu \omega}
\end{aligned}
\tag{33.2}
$$

▶symmetric

¶See Section 14, Non-linear Eddy-viscosity Models

- It is non-linear in velocity gradients.
- The advantage is better normal stresses and a certain ability to handle streamline curvature.

$$
\begin{aligned}
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¶See Section 14, Non-linear Eddy-viscosity Models

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▶ Let's verify that the three first terms are indeed symmetric and traceless

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▶We find that the nornal stresses are indeed not equal (coontrary to the standard linear $k-\varepsilon$ model)

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¶See Section 15, The V2F Model

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▶Next, let's see how the f equation behaves near the wall.

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▶We see that the $\overline{v_2'^2}$ eq. in the V2F model reverts back to the origonal $\overline{v_2'^2}$ equation. ▶Good.

▶Next, let's see how the f equation behaves near the wall.

▶We formulate a simplified f in one dimension

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▶Now solve it (with a 1D finite volume code) for different L and S

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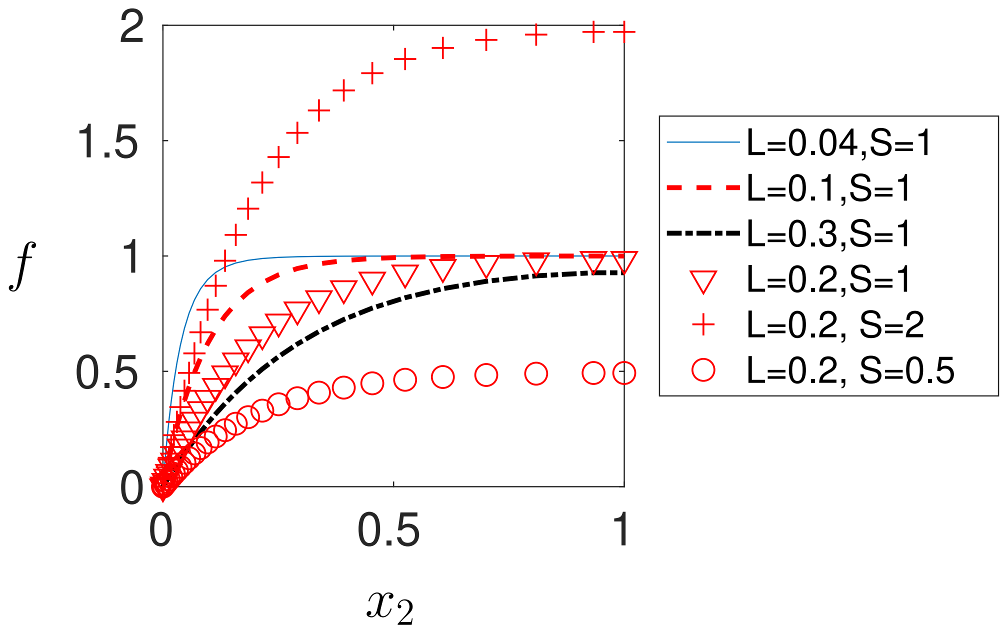

Solution of Eq. 34.1 for different L and S

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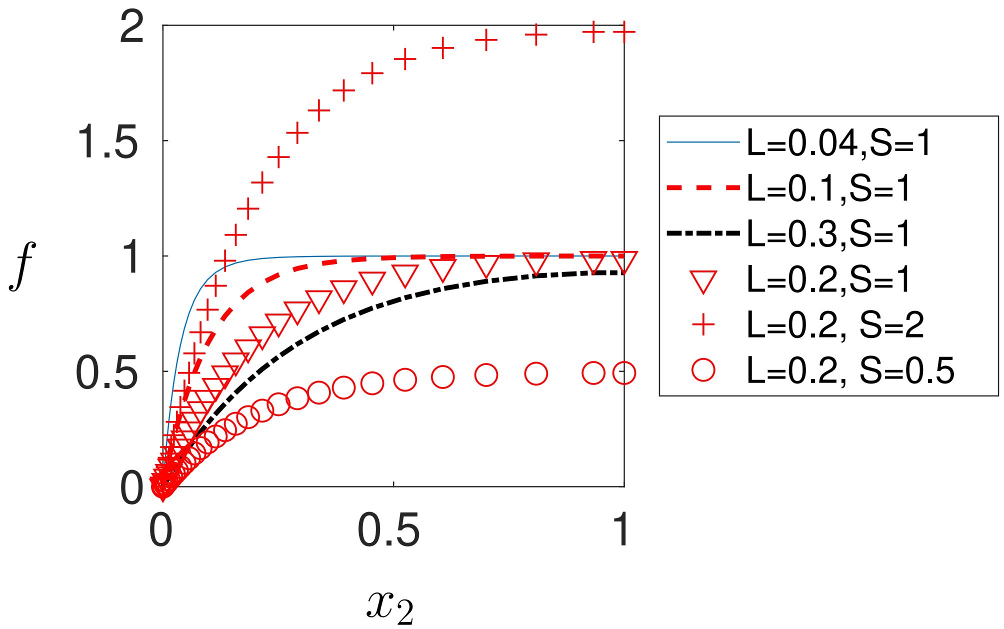

Solution of Eq. 34.1 for different L and S

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$$\overline{v_2'^2} = A x_2^2 + \frac{B}{x_2} - \varepsilon f \frac{x_2^4}{20\nu^2}$$

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$$L^2\frac{\partial^2 f}{\partial x_j \partial x_j} - f = -\frac{\Phi_{22}}{\rho k} - \frac{1}{T}\left(\frac{v^2}{k} - \frac{2}{3}\right)$$

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► How does the V2F model behave near a vertical wall?

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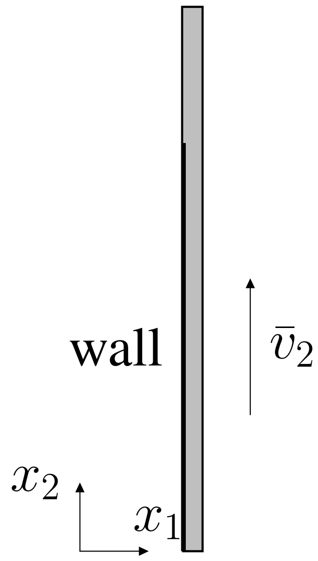

Boundary later along a vertical wall

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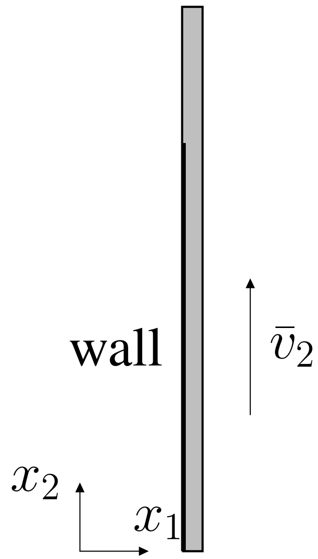

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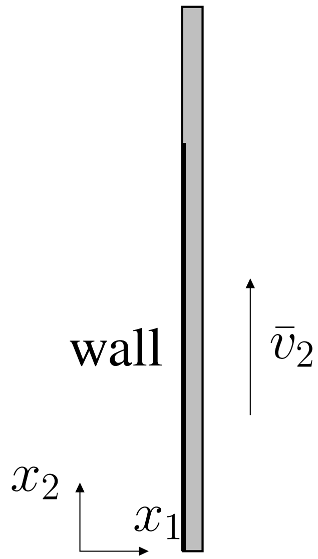

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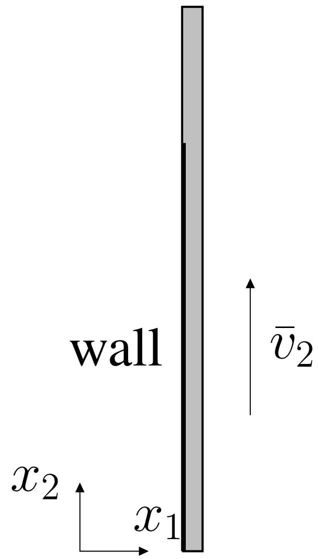

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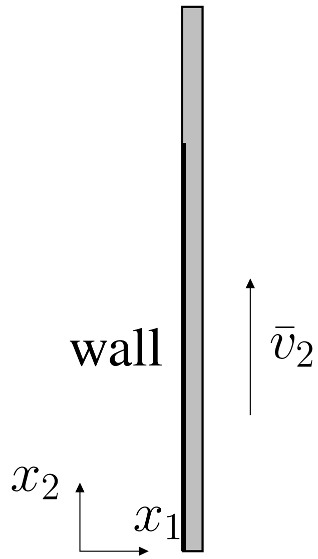

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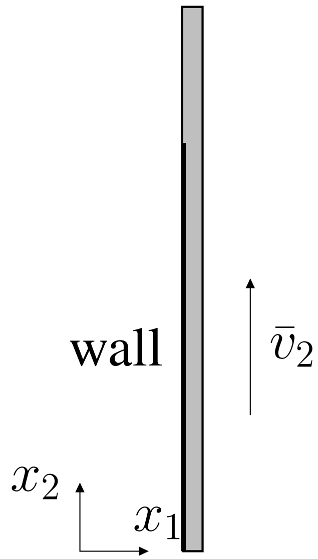

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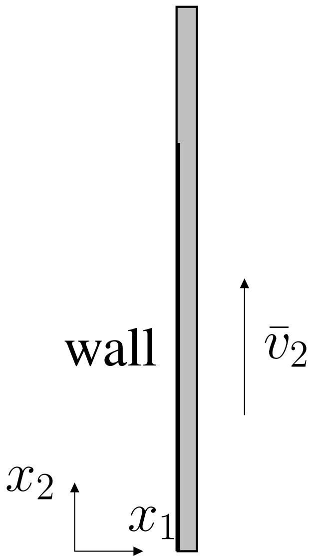

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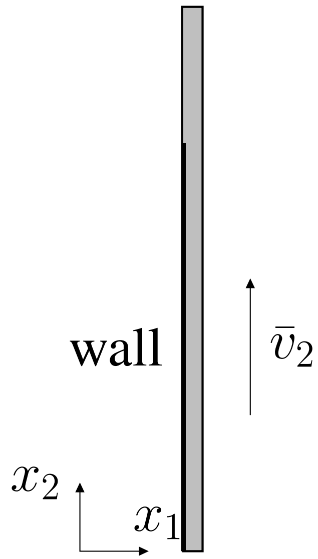

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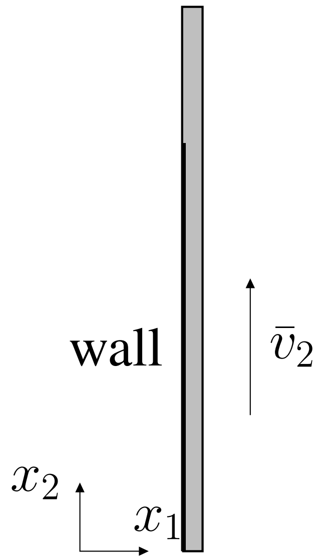

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- Hence, in this case (the vertical plate), v^2 corresponds to $\overline{v_1'^2}$
- P^k in the expression of Φ_{22} explains why v^2 is equal to $\overline{v_2'^2}$, $\overline{v_1'^2}$ or $\overline{v_3'^2}$ depending on orientation of the nearest wall (the largest velocity gradient).

¶See Section 16, The SST Model

►The SST (Shear Stress Transport) model

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1.(a)

¶See Section 16, The SST Model

►The SST (Shear Stress Transport) model

1. Combination of a $k-\omega$ model (in the inner boundary layer) and $k-\varepsilon$ model (in the outer region of the boundary layer as well as outside of it)

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¶See Section 16, The SST Model

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¶See Section 16, The SST Model

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¶See Section 16, The SST Model

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$$\frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{\varepsilon}{\beta^* k}\right) = \frac{1}{\beta^* k}\frac{d\varepsilon}{dt} - \frac{\varepsilon}{\beta^* k^2}\frac{dk}{dt} =$$

¶See Section 16, The SST Model

▶The SST (Shear Stress Transport) model

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2. A limitation of the shear stress in adverse pressure gradient regions

▶$\omega = \varepsilon/(\beta^* k) = \varepsilon/(c_\mu k)$. Use this to obtain an eq. for ω

$$\frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{\varepsilon}{\beta^* k}\right) = \frac{1}{\beta^* k}\frac{d\varepsilon}{dt} - \frac{\varepsilon}{\beta^* k^2}\frac{dk}{dt} = \frac{1}{\beta^* k}\frac{d\varepsilon}{dt} - \frac{\omega}{k}\frac{dk}{dt}$$

¶See Section 16, The SST Model

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¶See Section 16, The SST Model

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$$P_\omega=\frac{1}{\beta^* k}P_\varepsilon-\frac{\omega}{k}P^k=\frac{1}{\beta^* k}C_{\varepsilon 1}\frac{\varepsilon}{k}P^k-\frac{\omega}{k}P^k=$$

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▶Destruction term

¶See Section 16, The SST Model

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$$\frac{d\omega}{dt} = \frac{1}{\beta^* k}\frac{d\varepsilon}{dt} - \frac{\omega}{k}\frac{dk}{dt}$$

▶ Viscous diffusion term

$$\frac{d\omega}{dt} = \frac{1}{\beta^* k}\frac{d\varepsilon}{dt} - \frac{\omega}{k}\frac{dk}{dt}$$

- Viscous diffusion term

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▶Viscous diffusion term

$$D_\omega^\nu =$$

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▶Viscous diffusion term

$$D_\omega^\nu = \frac{\nu}{\beta^* k}\frac{\partial^2 \varepsilon}{\partial x_j^2} - \frac{\nu\omega}{k}\frac{\partial^2 k}{\partial x_j^2} =$$

$$\frac{d\omega}{dt} = \frac{1}{\beta^* k}\frac{d\varepsilon}{dt} - \frac{\omega}{k}\frac{dk}{dt}$$

▶Viscous diffusion term

$$D_\omega^\nu = \frac{\nu}{\beta^* k}\frac{\partial^2 \varepsilon}{\partial x_j^2} - \frac{\nu\omega}{k}\frac{\partial^2 k}{\partial x_j^2} = \frac{\nu}{k}\frac{\partial^2 \omega k}{\partial x_j^2} - \frac{\nu\omega}{k}\frac{\partial^2 k}{\partial x_j^2}$$

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$$=$$

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$$= \frac{\nu}{k}\left[\frac{\partial}{\partial x_j}\left(\omega\frac{\partial k}{\partial x_j} + k\frac{\partial \omega}{\partial x_j}\right)\right] - \nu\frac{\omega}{k}\frac{\partial^2 k}{\partial x_j^2}$$

$$\frac{d\omega}{dt} = \frac{1}{\beta^* k}\frac{d\varepsilon}{dt} - \frac{\omega}{k}\frac{dk}{dt}$$

▶Viscous diffusion term

$$\begin{aligned} D_\omega^\nu &= \frac{\nu}{\beta^* k}\frac{\partial^2 \varepsilon}{\partial x_j^2} - \frac{\nu\omega}{k}\frac{\partial^2 k}{\partial x_j^2} = \frac{\nu}{k}\frac{\partial^2 \omega k}{\partial x_j^2} - \frac{\nu\omega}{k}\frac{\partial^2 k}{\partial x_j^2} \\ &= \frac{\nu}{k}\left[\frac{\partial}{\partial x_j}\left(\omega\frac{\partial k}{\partial x_j} + k\frac{\partial \omega}{\partial x_j}\right)\right] - \nu\frac{\omega}{k}\frac{\partial^2 k}{\partial x_j^2} = \frac{2\nu}{k}\frac{\partial \omega}{\partial x_j}\frac{\partial k}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\nu\frac{\partial \omega}{\partial x_j}\right) \end{aligned}$$

$$\frac{d\omega}{dt}=\frac{1}{\beta^* k}\frac{d\varepsilon}{dt}-\frac{\omega}{k}\frac{dk}{dt}$$

▶Viscous diffusion term

$$\begin{aligned}
D^\nu_\omega &= \frac{\nu}{\beta^* k}\frac{\partial^2\varepsilon}{\partial x_j^2}-\frac{\nu\omega}{k}\frac{\partial^2 k}{\partial x_j^2}=\frac{\nu}{k}\frac{\partial^2\omega k}{\partial x_j^2}-\frac{\nu\omega}{k}\frac{\partial^2 k}{\partial x_j^2}\\
&= \frac{\nu}{k}\left[\frac{\partial}{\partial x_j}\left(\omega\frac{\partial k}{\partial x_j}+k\frac{\partial\omega}{\partial x_j}\right)\right]-\nu\frac{\omega}{k}\frac{\partial^2 k}{\partial x_j^2}=\frac{2\nu}{k}\frac{\partial\omega}{\partial x_j}\frac{\partial k}{\partial x_j}+\frac{\partial}{\partial x_j}\left(\nu\frac{\partial\omega}{\partial x_j}\right)
\end{aligned}$$

▶The ω eq. (which really is an ε eq. when the $k-\varepsilon$ constants are used) reads

$$\frac{d\omega}{dt} = \frac{1}{\beta^* k}\frac{d\varepsilon}{dt} - \frac{\omega}{k}\frac{dk}{dt}$$

►Viscous diffusion term

$$\begin{aligned} D^\nu_\omega &= \frac{\nu}{\beta^* k}\frac{\partial^2 \varepsilon}{\partial x_j^2} - \frac{\nu\omega}{k}\frac{\partial^2 k}{\partial x_j^2} = \frac{\nu}{k}\frac{\partial^2 \omega k}{\partial x_j^2} - \frac{\nu\omega}{k}\frac{\partial^2 k}{\partial x_j^2} \\ &= \frac{\nu}{k}\left[\frac{\partial}{\partial x_j}\left(\omega\frac{\partial k}{\partial x_j} + k\frac{\partial \omega}{\partial x_j}\right)\right] - \nu\frac{\omega}{k}\frac{\partial^2 k}{\partial x_j^2} = \frac{2\nu}{k}\frac{\partial \omega}{\partial x_j}\frac{\partial k}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\nu\frac{\partial \omega}{\partial x_j}\right) \end{aligned}$$

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►Viscous diffusion term

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►The ω eq. (which really is an ε eq. when the $k-\varepsilon$ constants are used) reads

$$\frac{\partial}{\partial x_j}(\bar{v}_j\omega) =$$

$$\frac{d\omega}{dt} = \frac{1}{\beta^* k}\frac{d\varepsilon}{dt} - \frac{\omega}{k}\frac{dk}{dt}$$

▶Viscous diffusion term

$$\begin{aligned}
D^\nu_\omega &= \frac{\nu}{\beta^* k}\frac{\partial^2 \varepsilon}{\partial x_j^2} - \frac{\nu\omega}{k}\frac{\partial^2 k}{\partial x_j^2} = \frac{\nu}{k}\frac{\partial^2 \omega k}{\partial x_j^2} - \frac{\nu\omega}{k}\frac{\partial^2 k}{\partial x_j^2} \\
&= \frac{\nu}{k}\left[\frac{\partial}{\partial x_j}\left(\omega\frac{\partial k}{\partial x_j} + k\frac{\partial \omega}{\partial x_j}\right)\right] - \nu\frac{\omega}{k}\frac{\partial^2 k}{\partial x_j^2} = \frac{2\nu}{k}\frac{\partial \omega}{\partial x_j}\frac{\partial k}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\nu\frac{\partial \omega}{\partial x_j}\right)
\end{aligned}$$

▶The ω eq. (which really is an ε eq. when the $k-\varepsilon$ constants are used) reads

$$\frac{\partial}{\partial x_j}(\bar{v}_j\omega) = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}\right] + \alpha\frac{\omega}{k}P^k - \beta\omega^2 + \frac{2}{k}\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial k}{\partial x_i}\frac{\partial \omega}{\partial x_i}$$

$$\frac{d\omega}{dt} = \frac{1}{\beta^* k}\frac{d\varepsilon}{dt} - \frac{\omega}{k}\frac{dk}{dt}$$

▶Viscous diffusion term

$$\begin{aligned} D_\omega^\nu &= \frac{\nu}{\beta^* k}\frac{\partial^2 \varepsilon}{\partial x_j^2} - \frac{\nu\omega}{k}\frac{\partial^2 k}{\partial x_j^2} = \frac{\nu}{k}\frac{\partial^2 \omega k}{\partial x_j^2} - \frac{\nu\omega}{k}\frac{\partial^2 k}{\partial x_j^2} \\ &= \frac{\nu}{k}\left[\frac{\partial}{\partial x_j}\left(\omega\frac{\partial k}{\partial x_j} + k\frac{\partial \omega}{\partial x_j}\right)\right] - \nu\frac{\omega}{k}\frac{\partial^2 k}{\partial x_j^2} = \frac{2\nu}{k}\frac{\partial \omega}{\partial x_j}\frac{\partial k}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\nu\frac{\partial \omega}{\partial x_j}\right) \end{aligned}$$

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$$\frac{d\omega}{dt} = \frac{1}{\beta^* k}\frac{d\varepsilon}{dt} - \frac{\omega}{k}\frac{dk}{dt}$$

►Viscous diffusion term

$$\begin{aligned}
D^\nu_\omega &= \frac{\nu}{\beta^* k}\frac{\partial^2 \varepsilon}{\partial x_j^2} - \frac{\nu\omega}{k}\frac{\partial^2 k}{\partial x_j^2} = \frac{\nu}{k}\frac{\partial^2 \omega k}{\partial x_j^2} - \frac{\nu\omega}{k}\frac{\partial^2 k}{\partial x_j^2} \\
&= \frac{\nu}{k}\left[\frac{\partial}{\partial x_j}\left(\omega\frac{\partial k}{\partial x_j} + k\frac{\partial \omega}{\partial x_j}\right)\right] - \nu\frac{\omega}{k}\frac{\partial^2 k}{\partial x_j^2} = \frac{2\nu}{k}\frac{\partial \omega}{\partial x_j}\frac{\partial k}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\nu\frac{\partial \omega}{\partial x_j}\right)
\end{aligned}$$

►The ω eq. (which really is an ε eq. when the $k-\varepsilon$ constants are used) reads

$$\begin{aligned}
\frac{\partial}{\partial x_j}(\bar{v}_j\omega) &= \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_j}\right] + \alpha\frac{\omega}{k}P^k - \beta\omega^2 + \frac{2}{k}\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial k}{\partial x_i}\frac{\partial \omega}{\partial x_i} \\
\alpha &= C_{\varepsilon 1} - 1 = 0.44, \beta = (C_{\varepsilon 2} - 1)\beta^* = 0.0828
\end{aligned}$$

$$\frac{d\omega}{dt} = \frac{1}{\beta^* k}\frac{d\varepsilon}{dt} - \frac{\omega}{k}\frac{dk}{dt}$$

▶Viscous diffusion term

$$\begin{aligned} D_\omega^\nu &= \frac{\nu}{\beta^* k}\frac{\partial^2 \varepsilon}{\partial x_j^2} - \frac{\nu\omega}{k}\frac{\partial^2 k}{\partial x_j^2} = \frac{\nu}{k}\frac{\partial^2 \omega k}{\partial x_j^2} - \frac{\nu\omega}{k}\frac{\partial^2 k}{\partial x_j^2} \\ &= \frac{\nu}{k}\left[\frac{\partial}{\partial x_j}\left(\omega\frac{\partial k}{\partial x_j} + k\frac{\partial \omega}{\partial x_j}\right)\right] - \nu\frac{\omega}{k}\frac{\partial^2 k}{\partial x_j^2} = \frac{2\nu}{k}\frac{\partial \omega}{\partial x_j}\frac{\partial k}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\nu\frac{\partial \omega}{\partial x_j}\right) \end{aligned}$$

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▶Inner region: $k - \omega$ coefficients; outer region: $k - \varepsilon$ coefficients. Blending function reads

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▶Two formulas for ν_t. ▶$\Omega = \partial \bar{v}_1/\partial x_2$. ▶Formulate JK model with Boussinesq.

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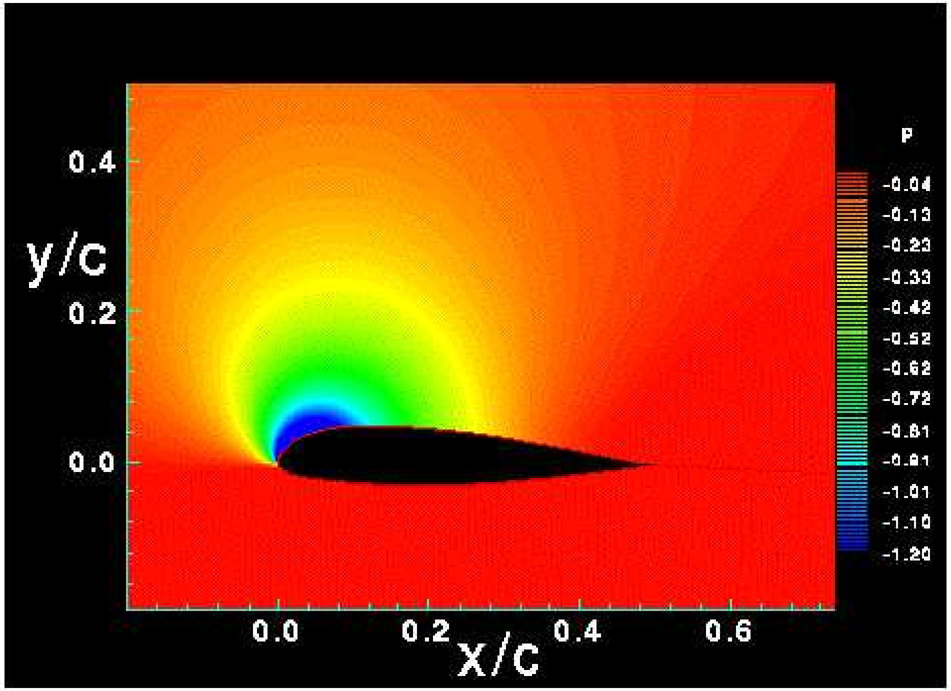
►Adverse pressure gradient flow (APG).



Flow around an airfoil. Angle of attack, $\alpha = 13^o$. Pressure contours.

On-line Lecture 6

¶See Section 5, Turbulence

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▶$v_i = \bar{v}_i + v'_i$, is irregular and consists of eddies of different size

On-line Lecture 6

¶See Section 5, Turbulence

- $v_i = \bar{v}_i + v_i'$, is irregular and consists of eddies of different size
- increases diffusivity

On-line Lecture 6

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▶ $v_i = \bar{v}_i + v_i'$, is irregular and consists of eddies of different size

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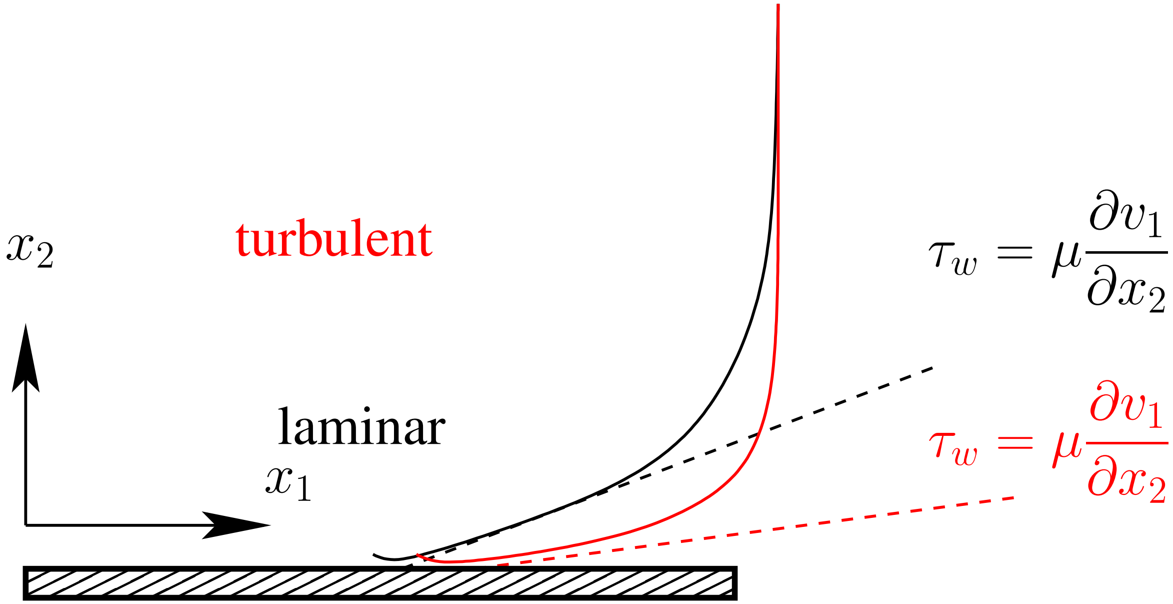

Difference between a laminar and turbulent boundary later

On-line Lecture 6

¶See Section 5, Turbulence

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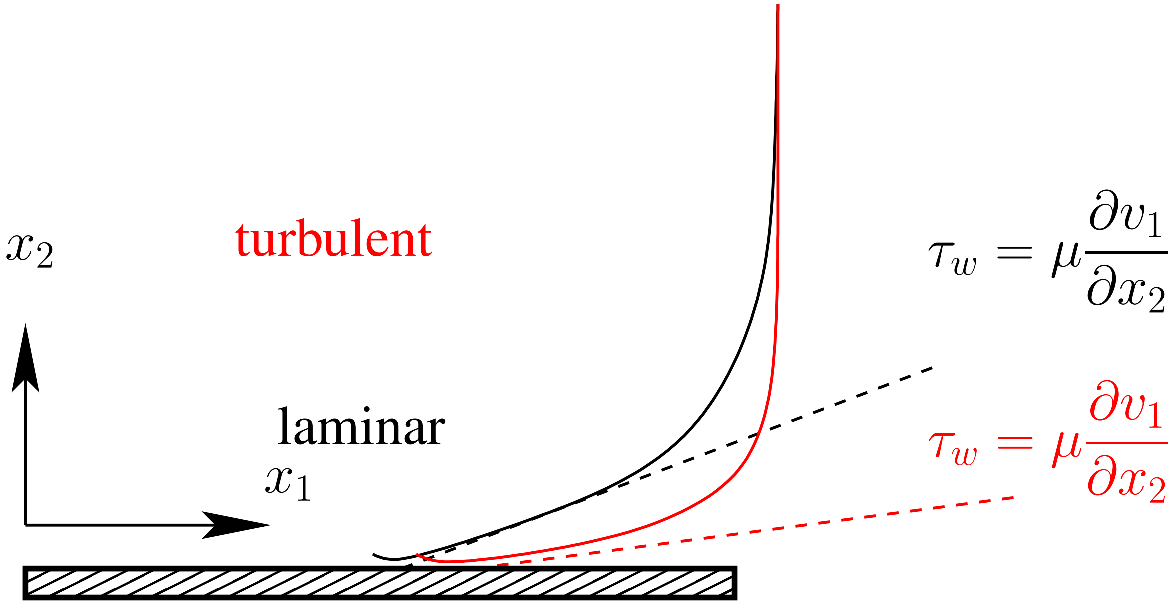

Difference between a laminar and turbulent boundary later

▶occurs at large Reynolds numbers. Pipes: $Re_D = \dfrac{VD}{\nu} \simeq 2300$; boundary layers: $Re_x = \dfrac{Vx}{\nu} \simeq 500\,000$.

On-line Lecture 6

¶See Section 5, Turbulence

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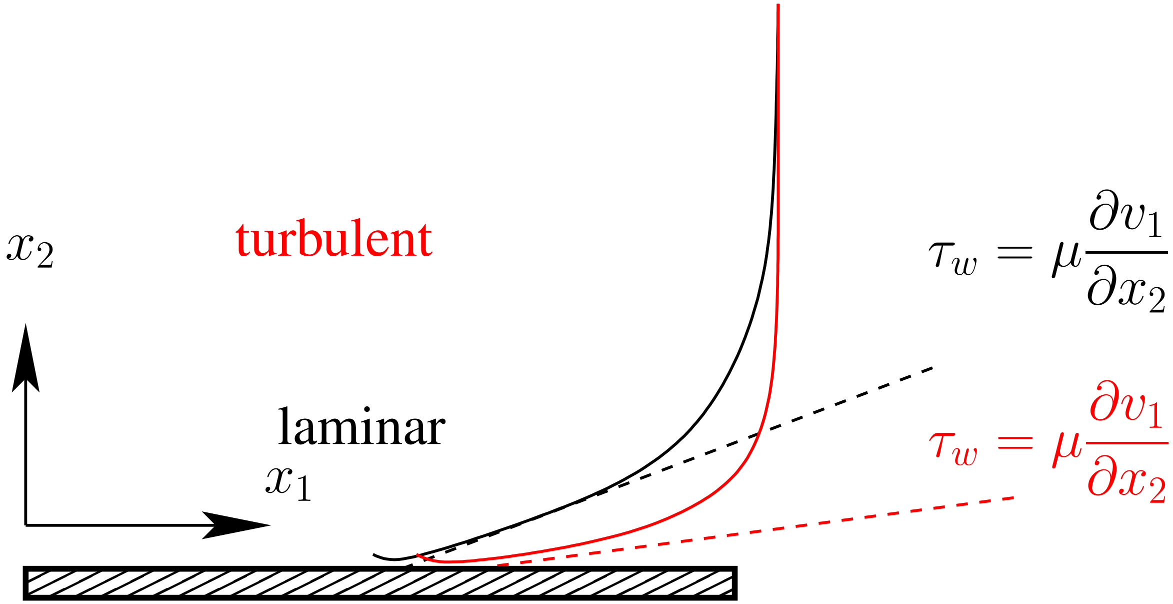

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On-line Lecture 6

¶See Section 5, Turbulence

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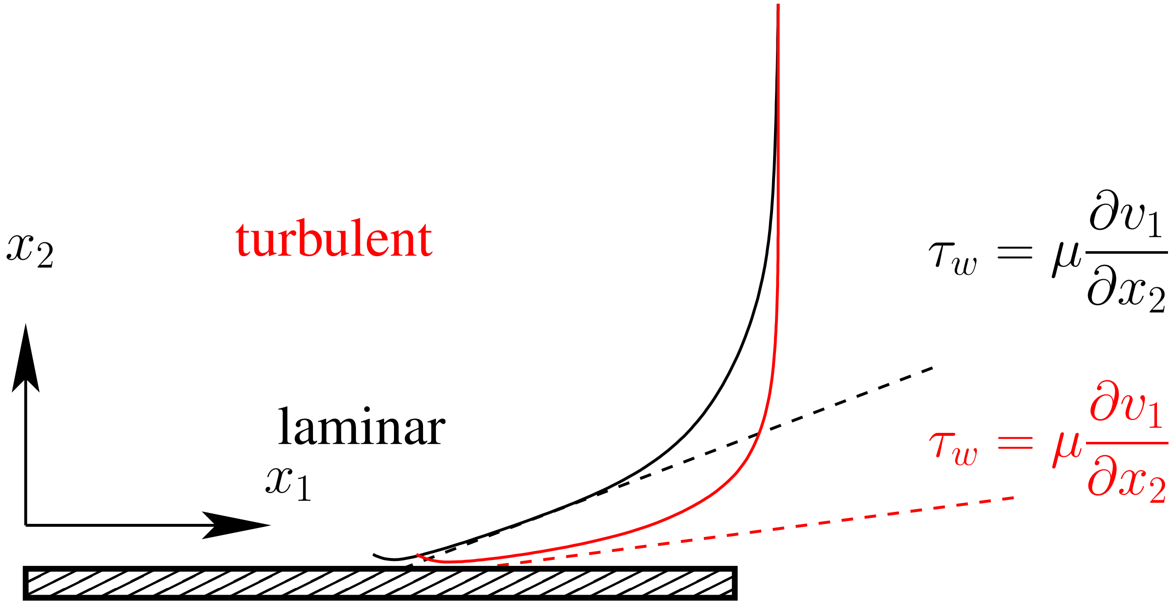
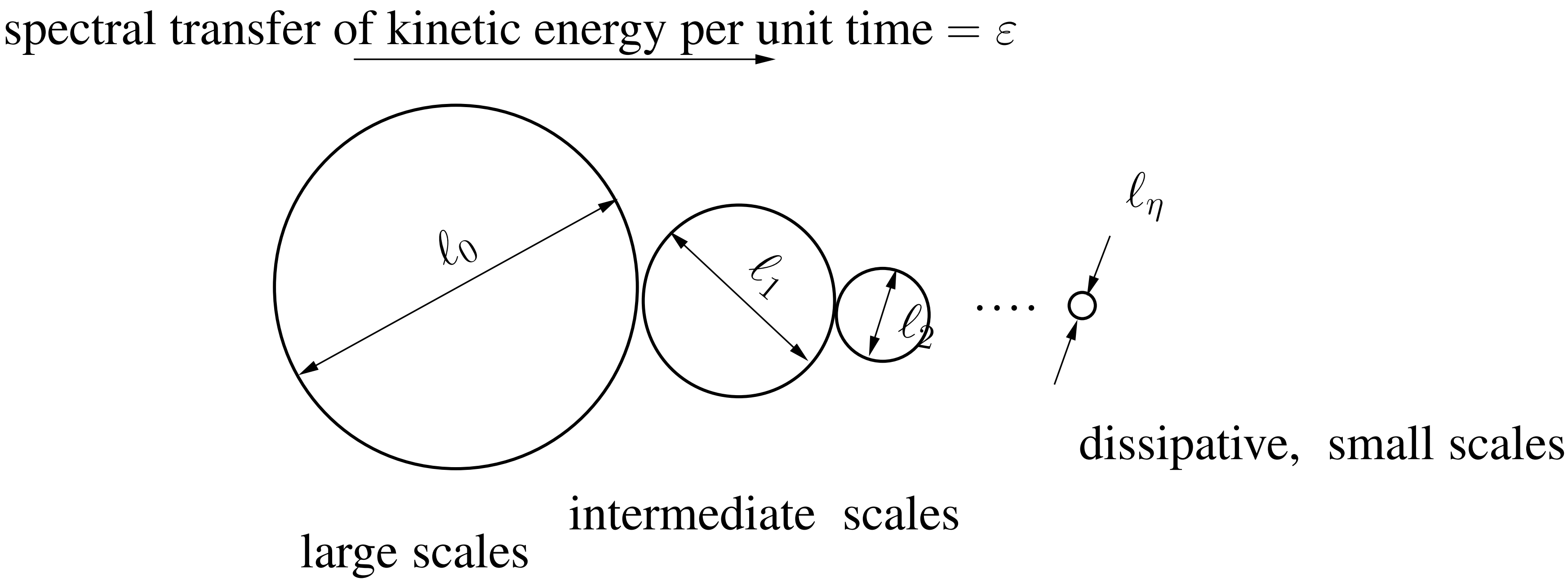
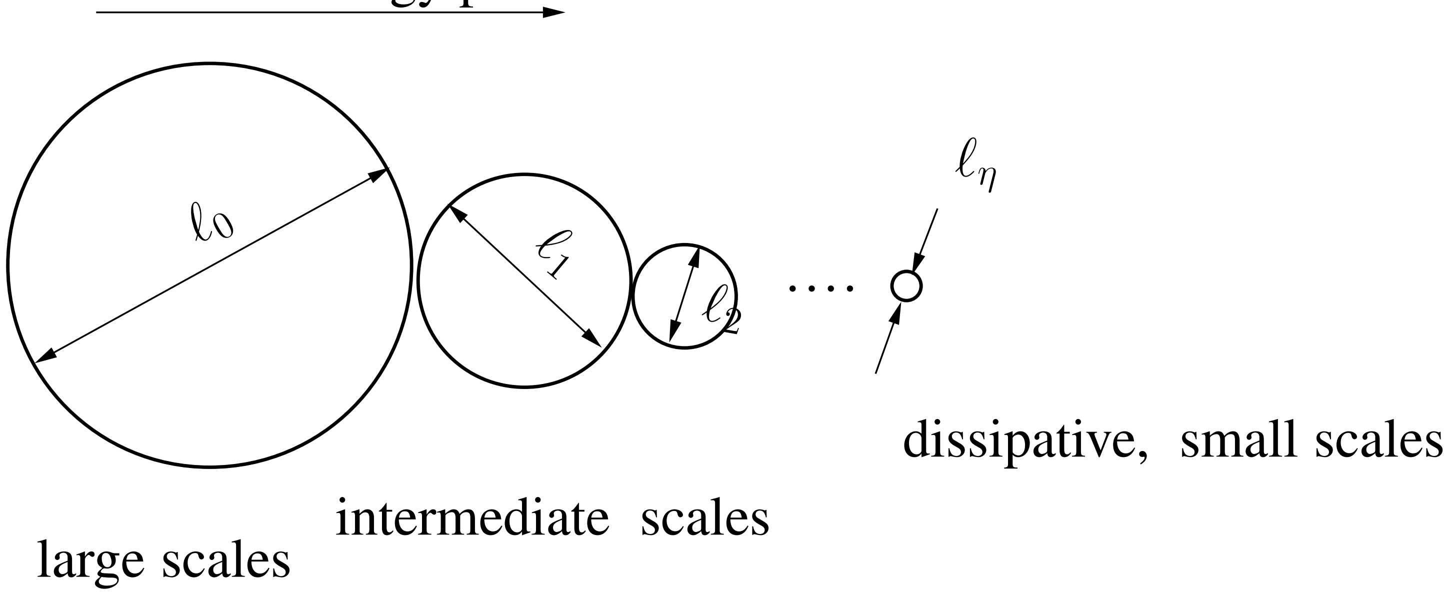


Difference between a laminar and turbulent boundary later

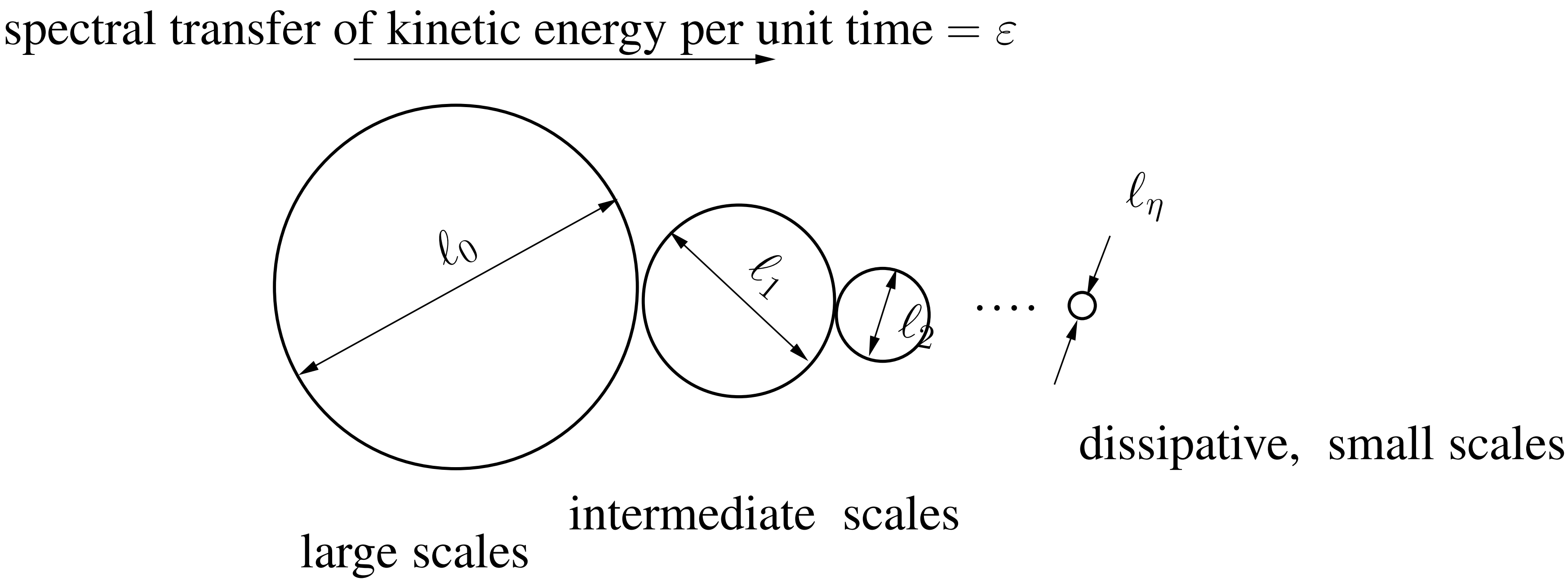
- occurs at large Reynolds numbers. Pipes: $Re_D = \frac{VD}{\nu} \simeq 2300$; boundary layers: $Re_x = \frac{Vx}{\nu} \simeq 500\,000$.
- is three-dimensional
- is dissipative. Kinetic energy, $v_i'v_i'/2$, in the small (dissipative) eddies are transformed into thermal energy (increases temperature).


spectral transfer of kinetic energy per unit time = ε
ℓ_0
ℓ_1
ℓ_2
ℓ_η
dissipative, small scales
intermediate scales
large scales

spectral transfer of kinetic energy per unit time $= \varepsilon$

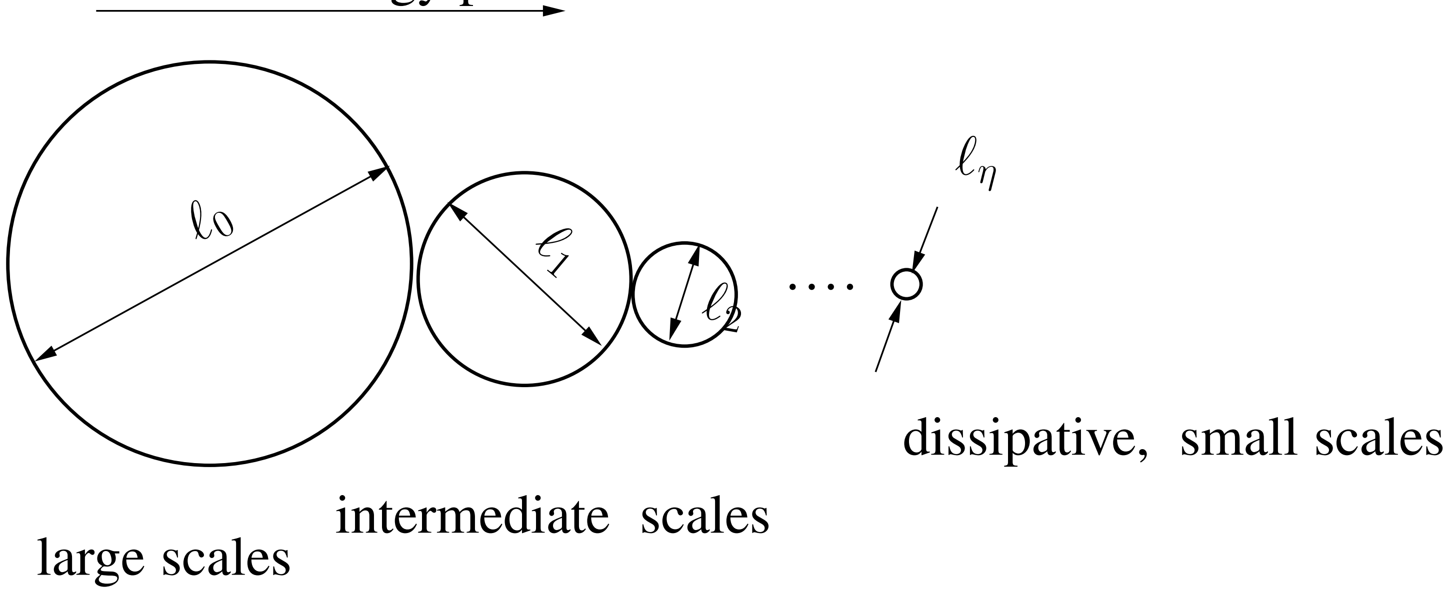


▶ Dissipation $\varepsilon = \nu \overline{\frac{\partial v_i'}{\partial x_j}\frac{\partial v_i'}{\partial x_j}}$



- Dissipation $\varepsilon = \nu \overline{\frac{\partial v_i'}{\partial x_j}\frac{\partial v_i'}{\partial x_j}}$
- All dissipation energy (say 90%) takes place at the small scales.

spectral transfer of kinetic energy per unit time $= \varepsilon$



- Dissipation $\varepsilon = \nu \overline{\frac{\partial v_i'}{\partial x_j}\frac{\partial v_i'}{\partial x_j}}$
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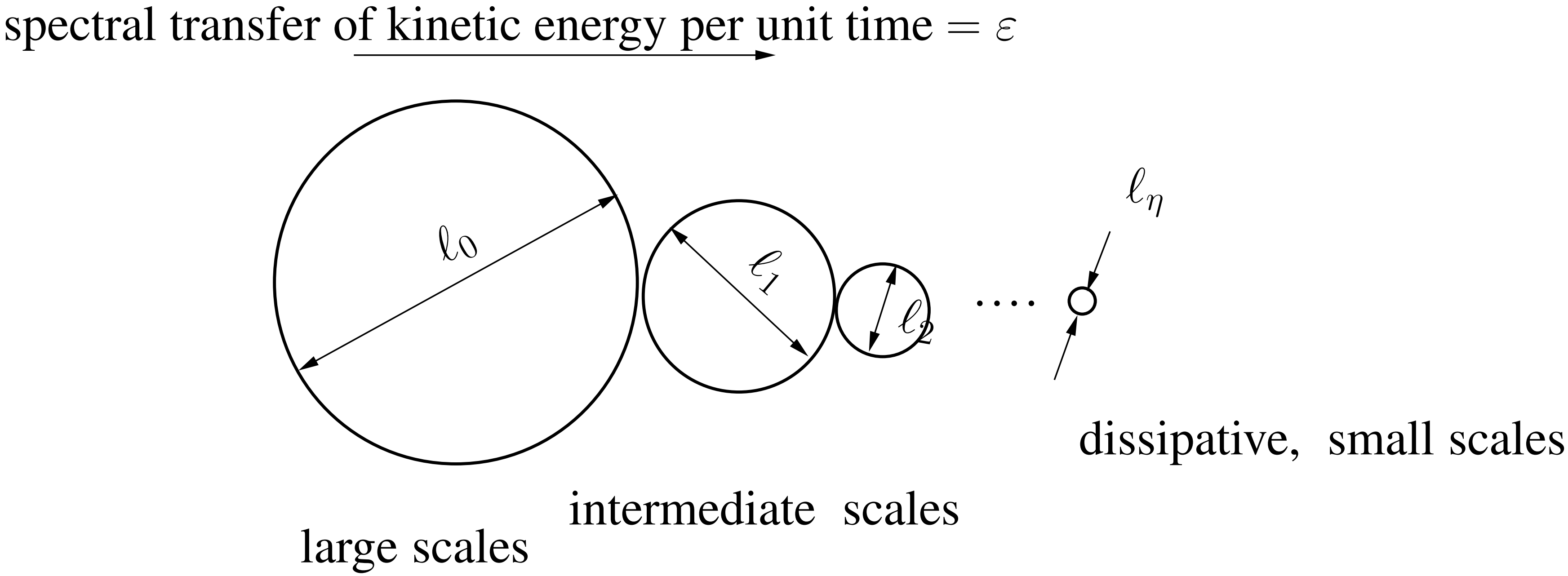
- We want to characterize the dissipation of kinetic energy at small scales in two relevant quantities:



▶Dissipation $\varepsilon = \nu \overline{\frac{\partial v_i'}{\partial x_j}\frac{\partial v_i'}{\partial x_j}}$ ▶All dissipation energy (say 90%) takes place at the small scales.

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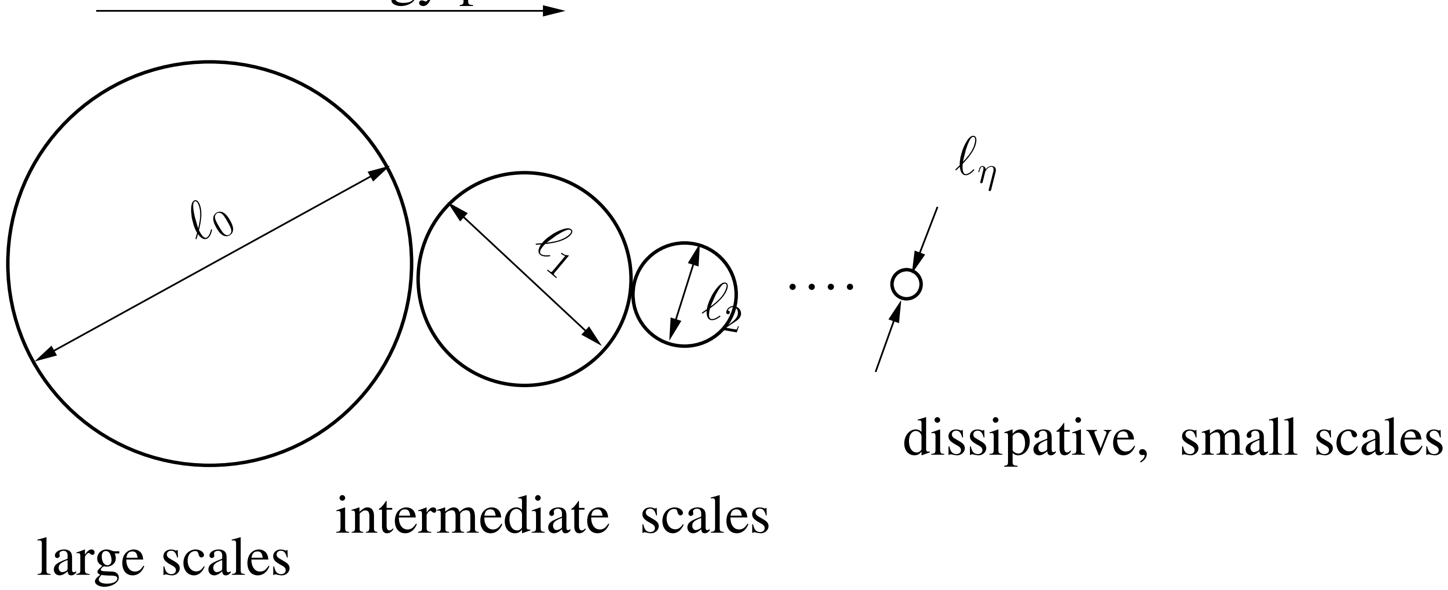
spectral transfer of kinetic energy per unit time $= \varepsilon$



▶Dissipation $\varepsilon = \nu \overline{\frac{\partial v_i'}{\partial x_j}\frac{\partial v_i'}{\partial x_j}}$ ▶All dissipation energy (say 90%) takes place at the small scales.

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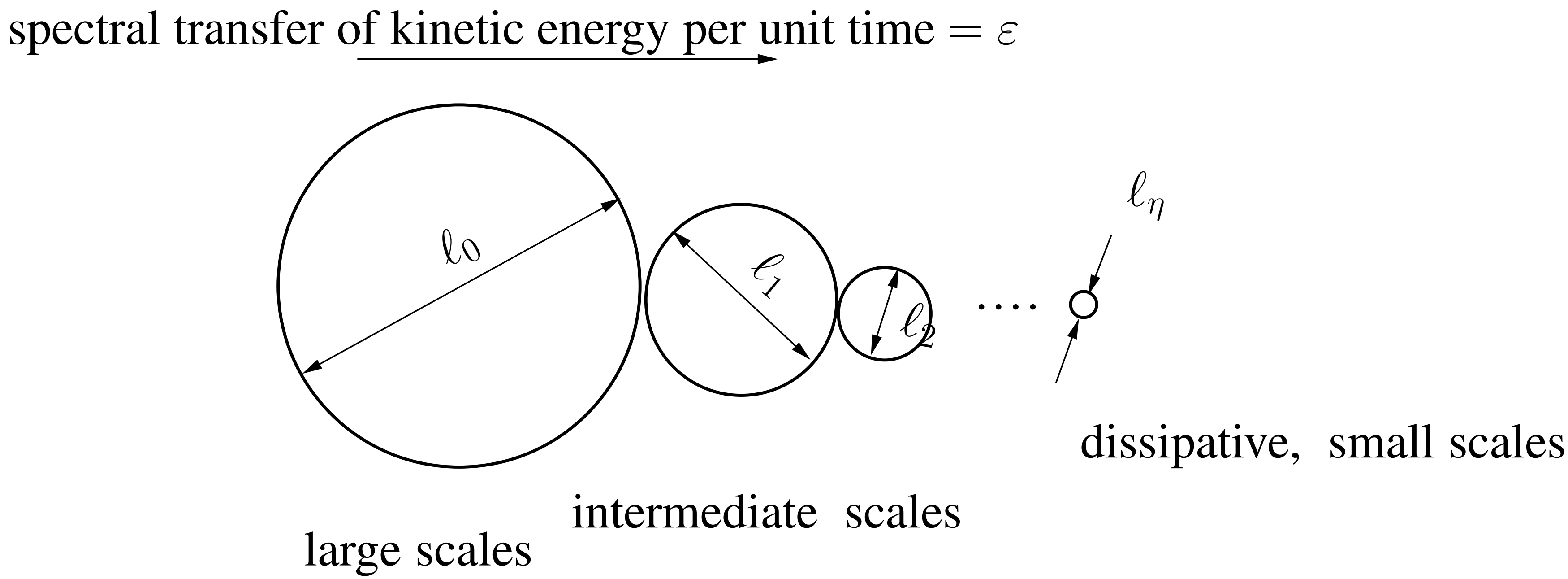

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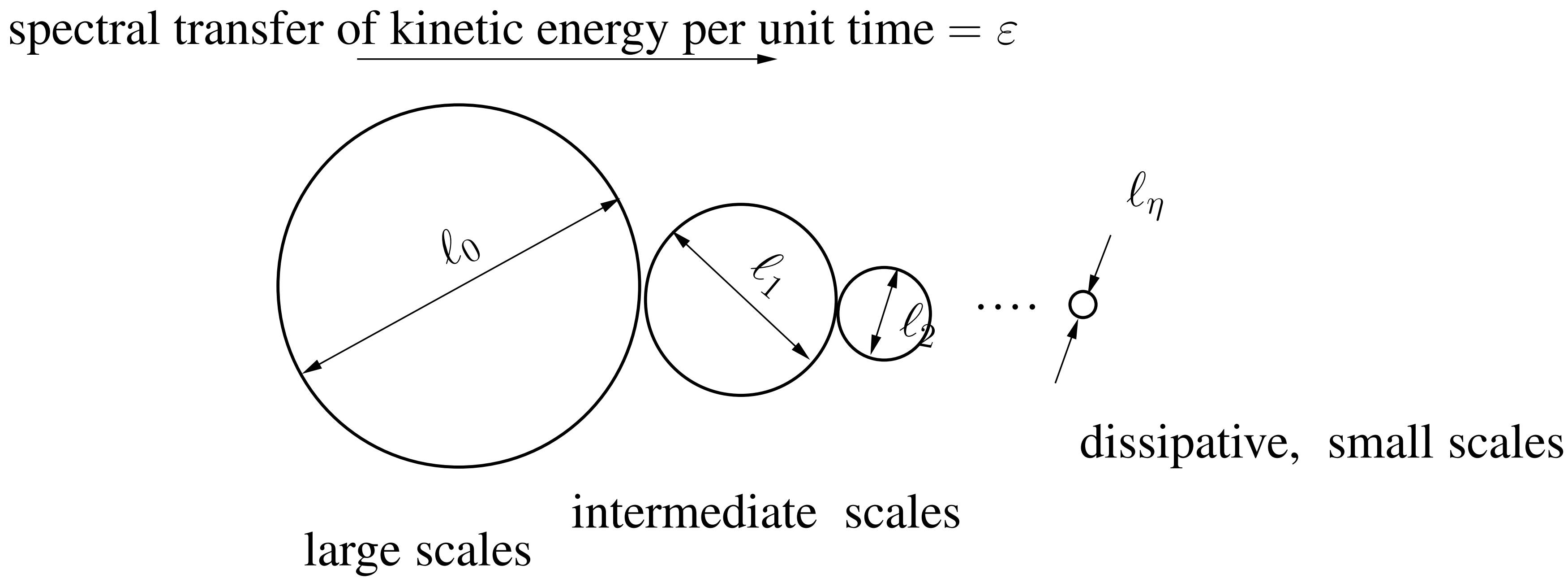
$$v_\eta \quad = \quad \nu^a \qquad \varepsilon^b$$



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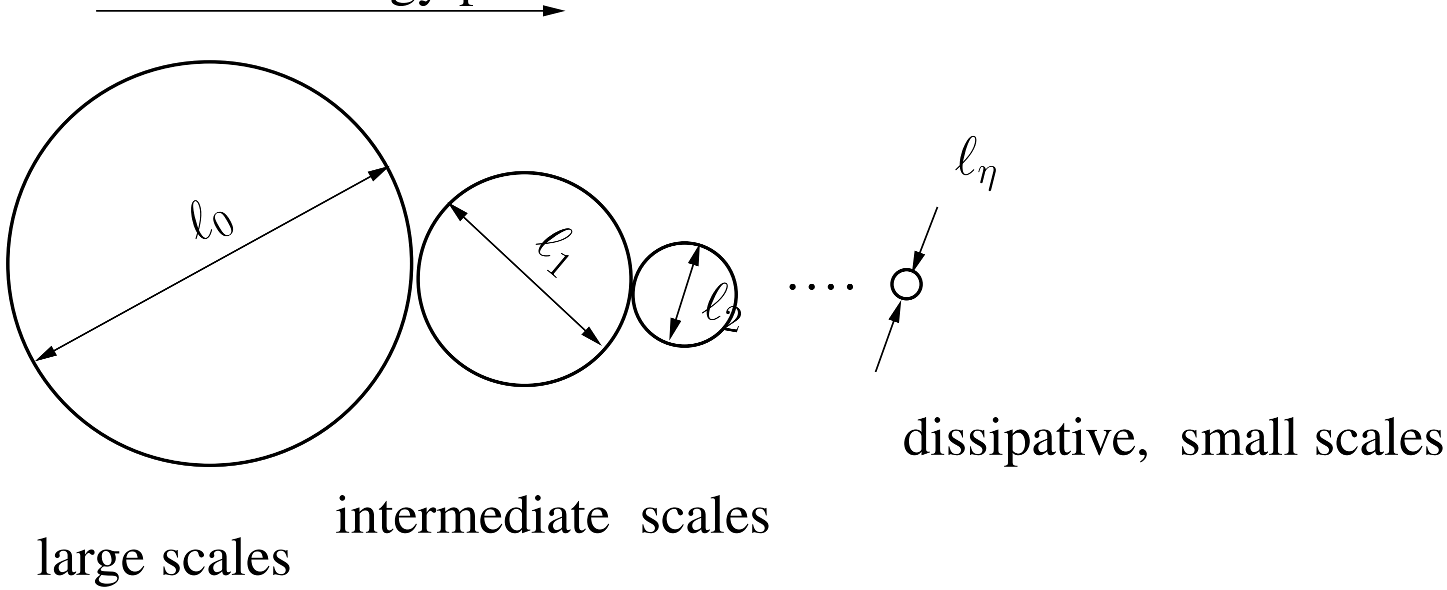
spectral transfer of kinetic energy per unit time $= \varepsilon$



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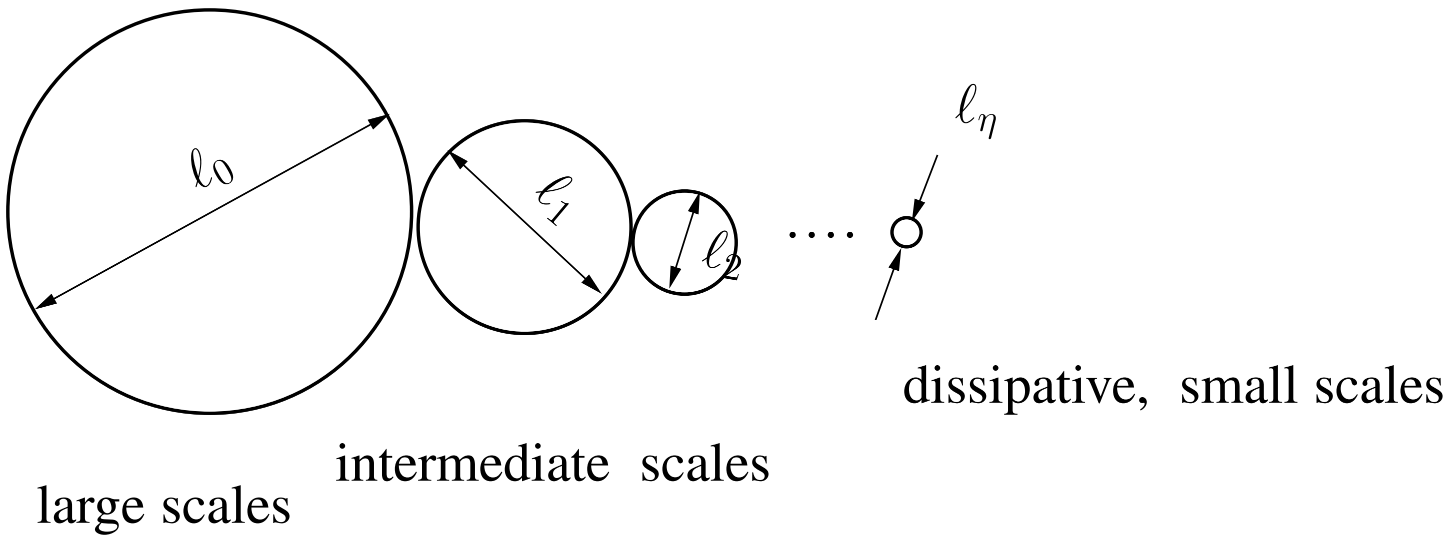

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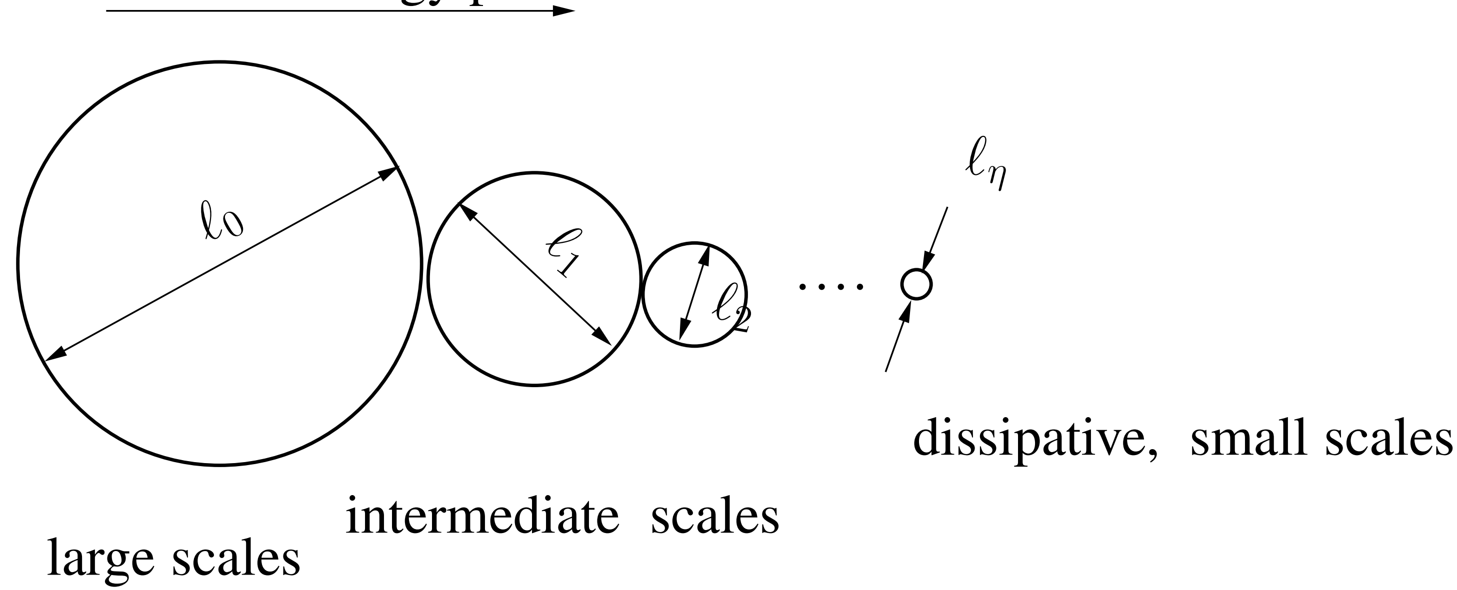

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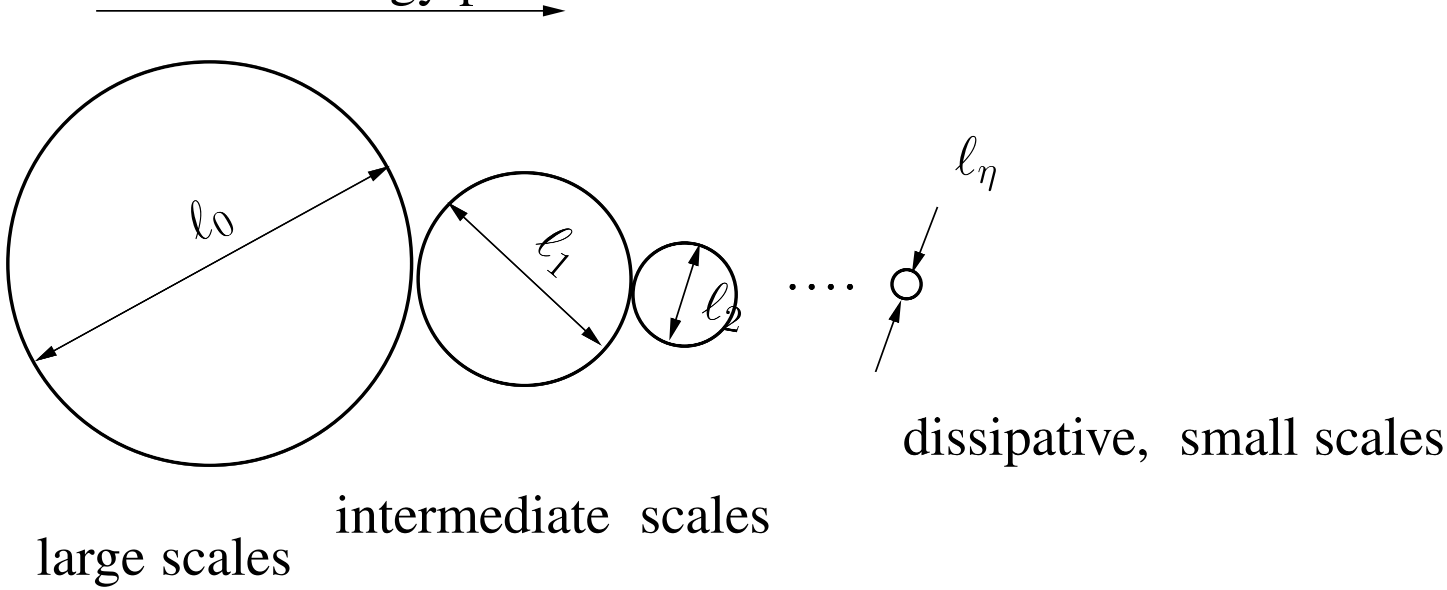

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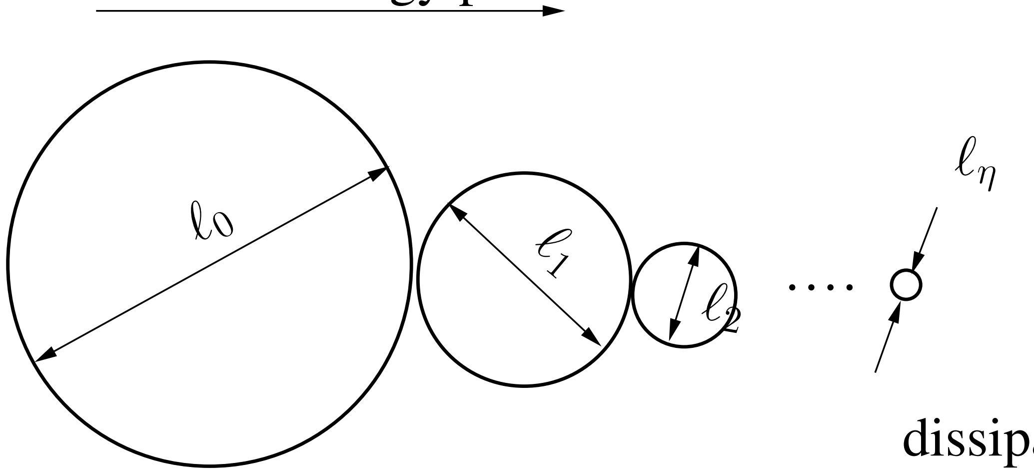

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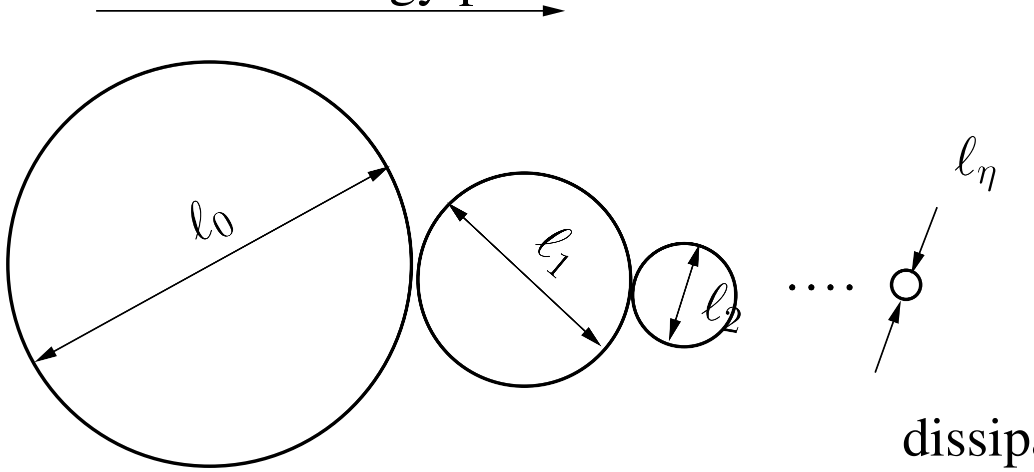
$$[m]$$

spectral transfer of kinetic energy per unit time = ε



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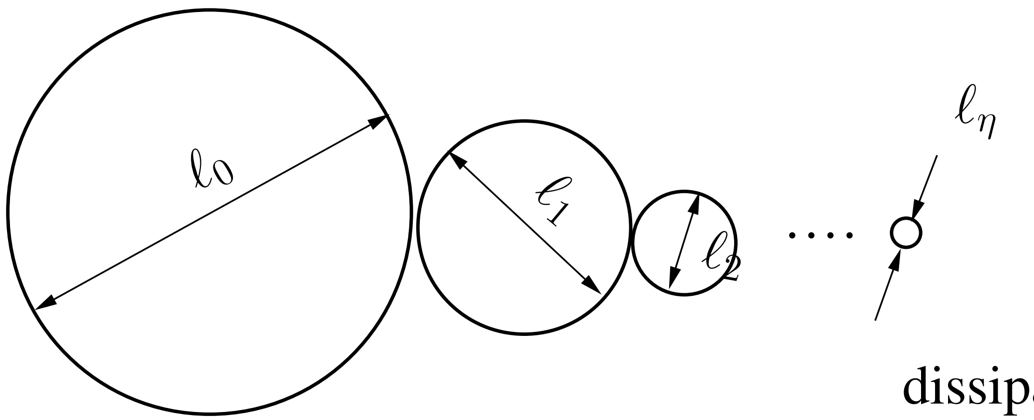

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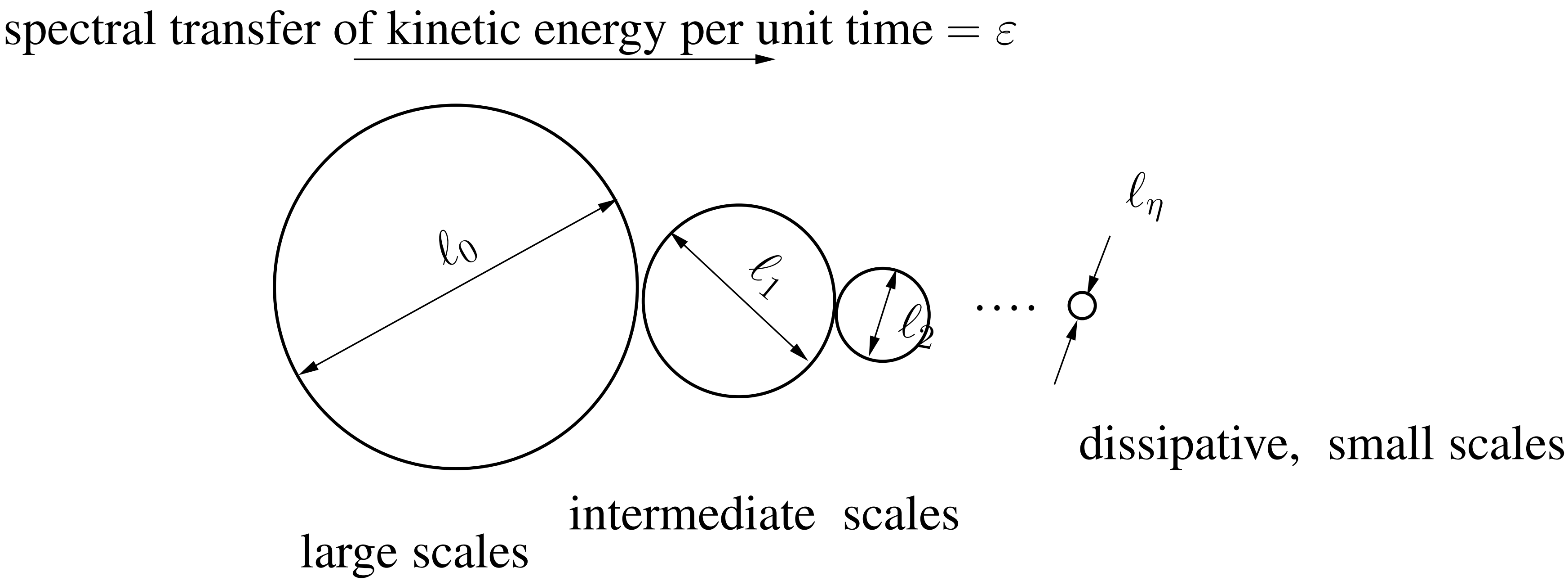

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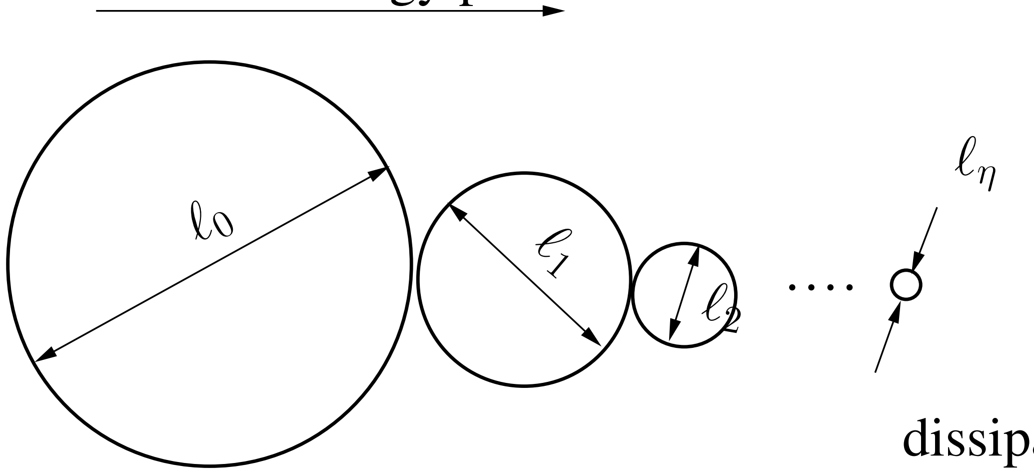

dissipative, small scales

intermediate scales

large scales

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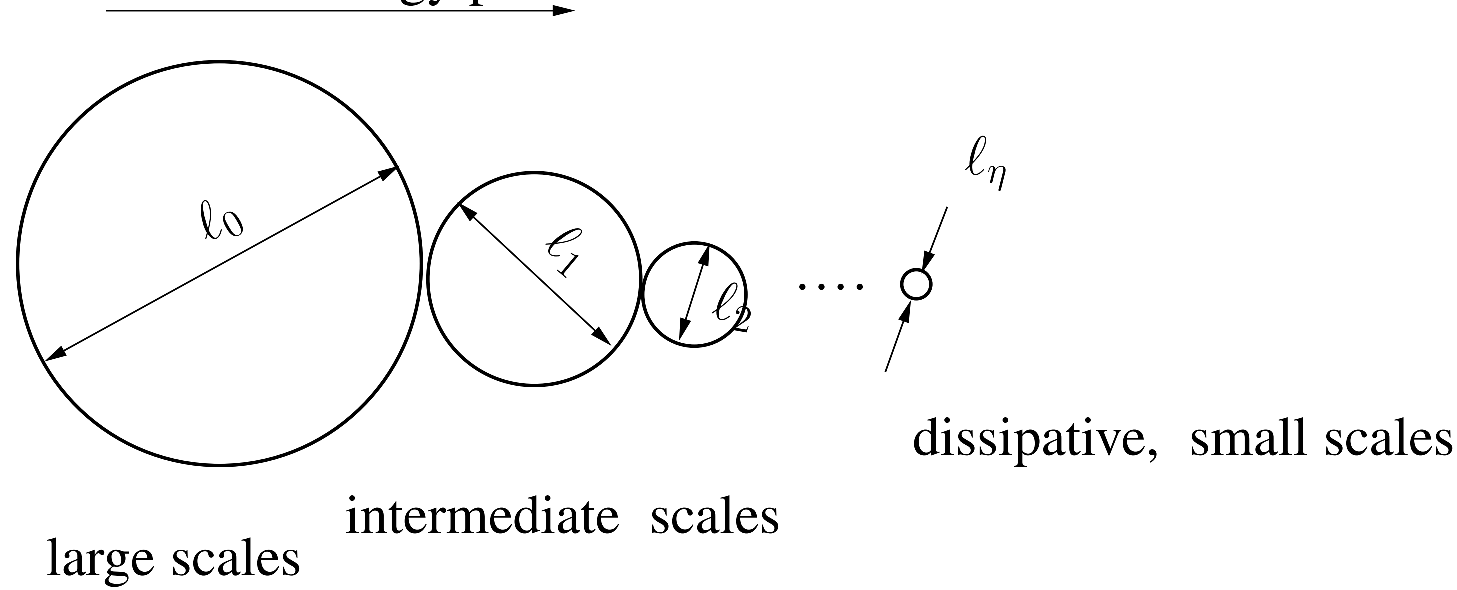

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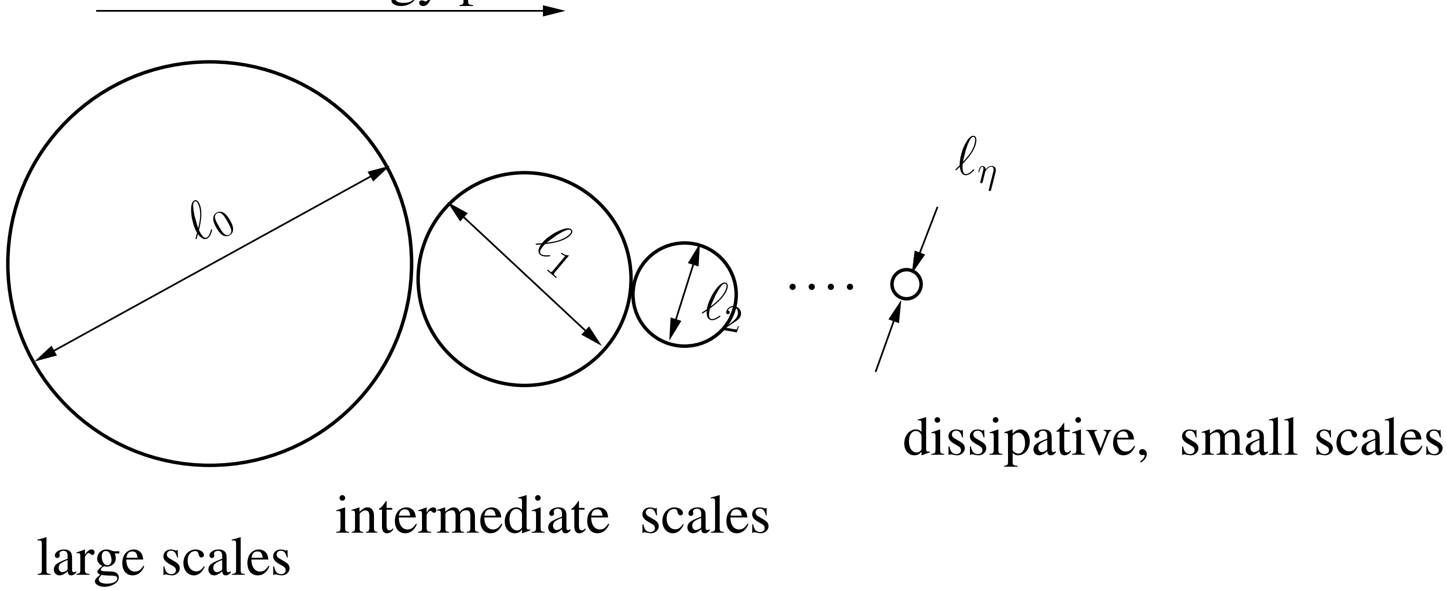

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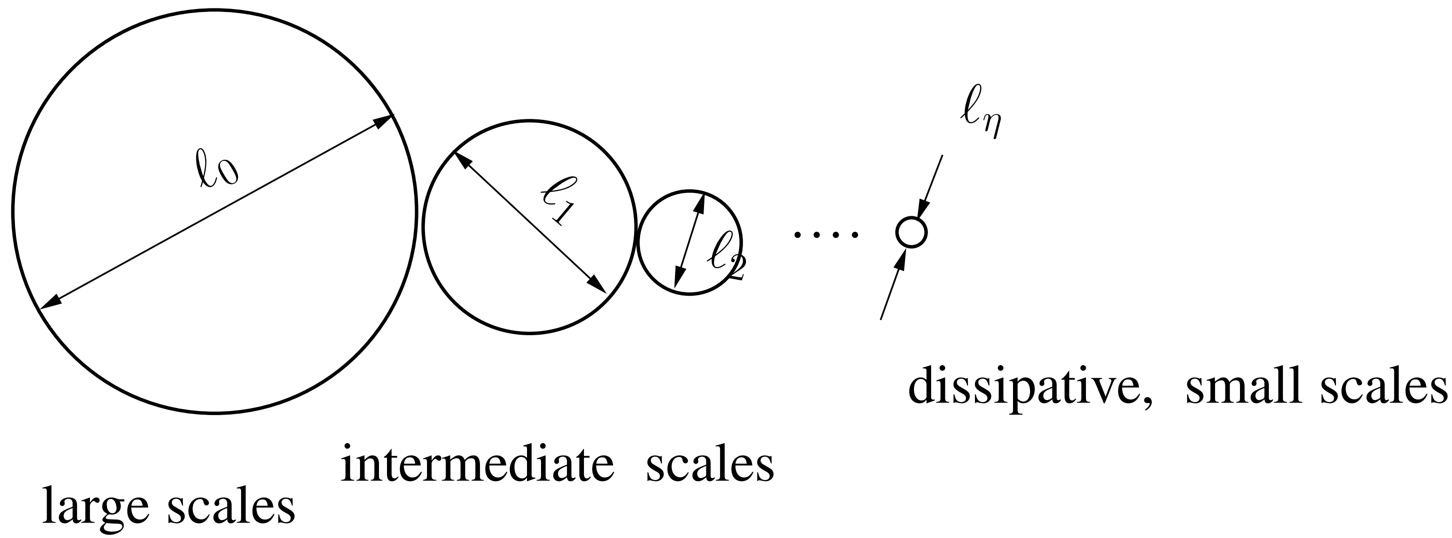

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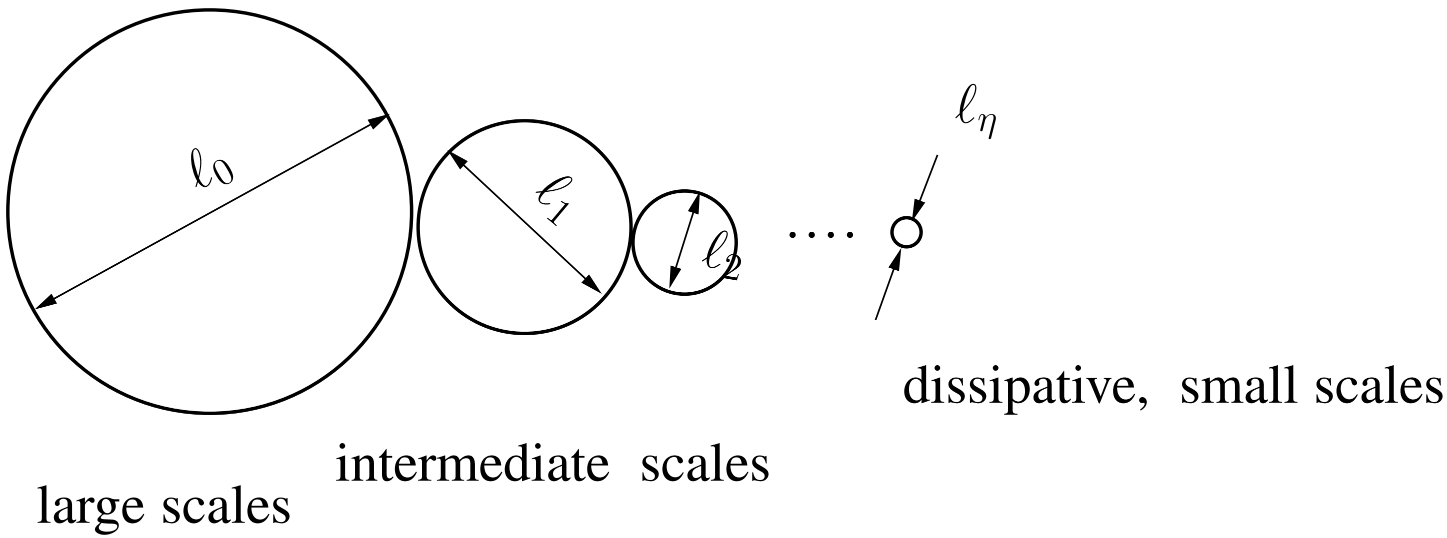

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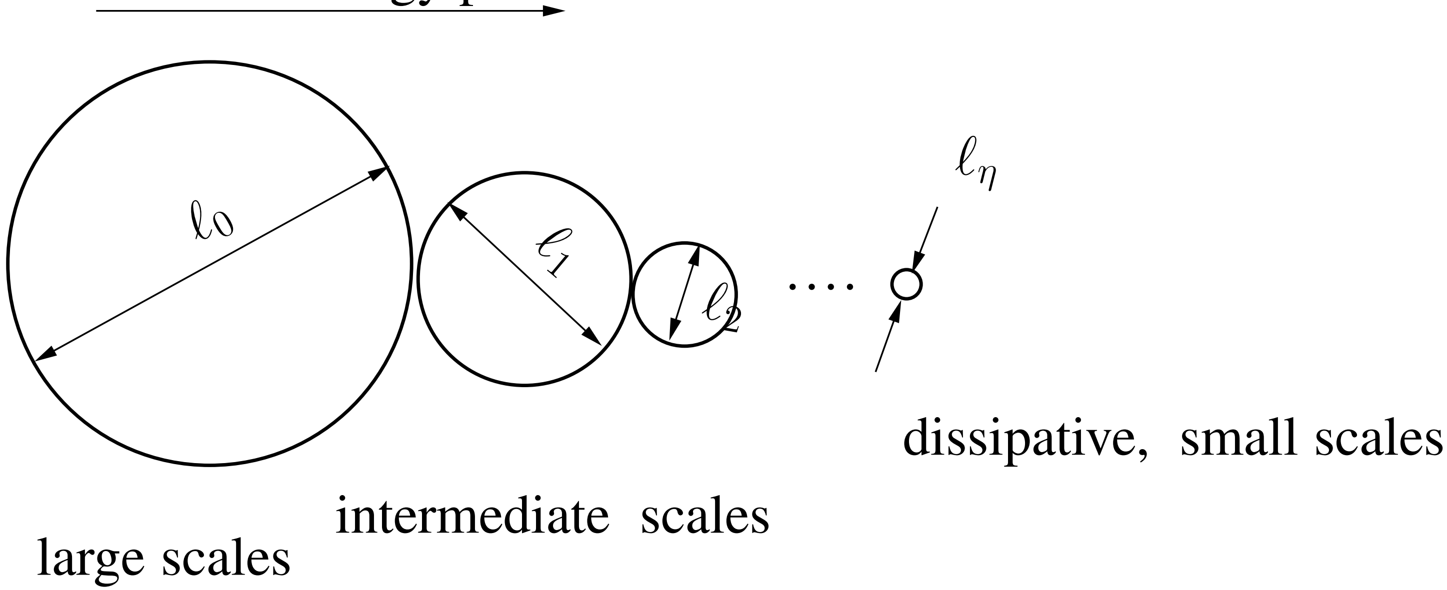

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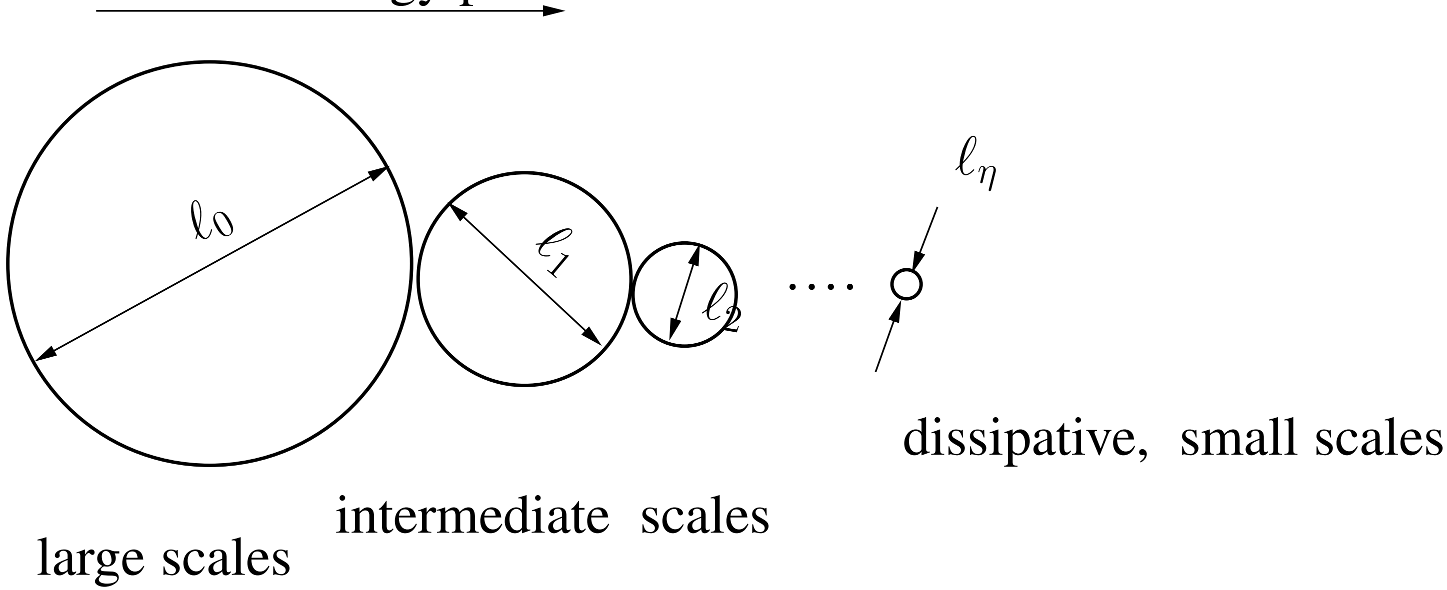
▶This gives the Kolmogorov scales, $a = b = 1/4$

spectral transfer of kinetic energy per unit time = ε



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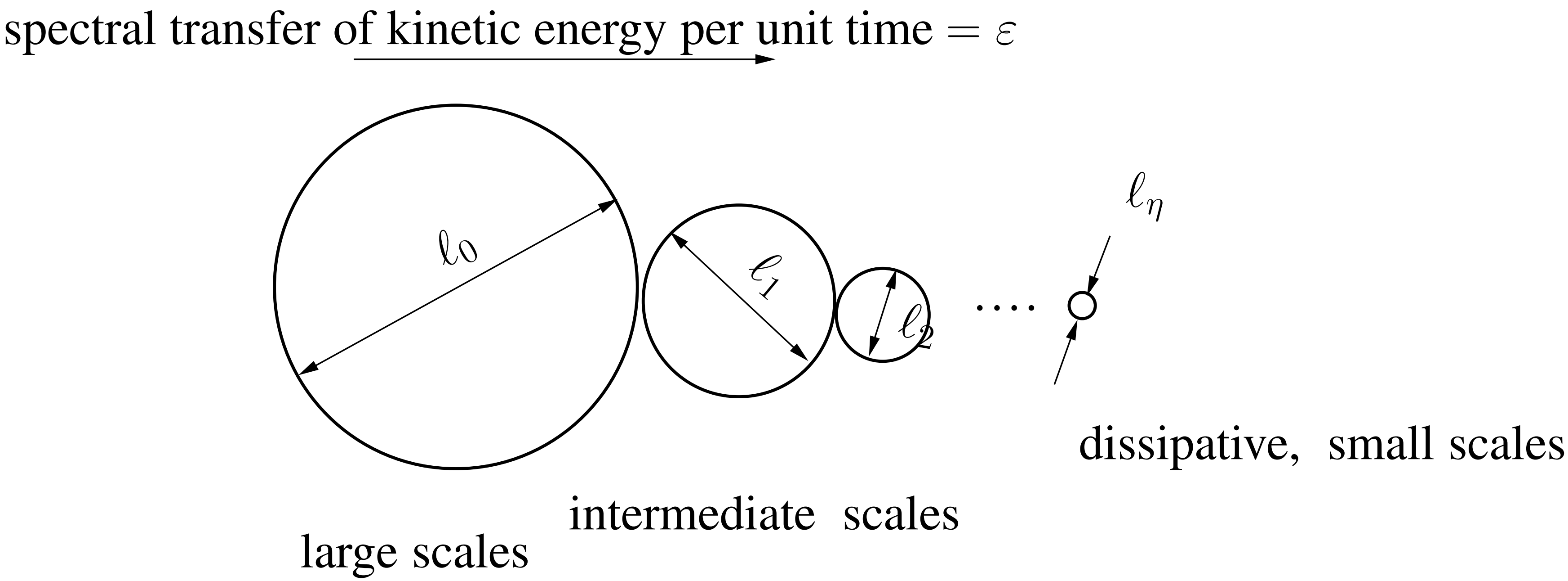

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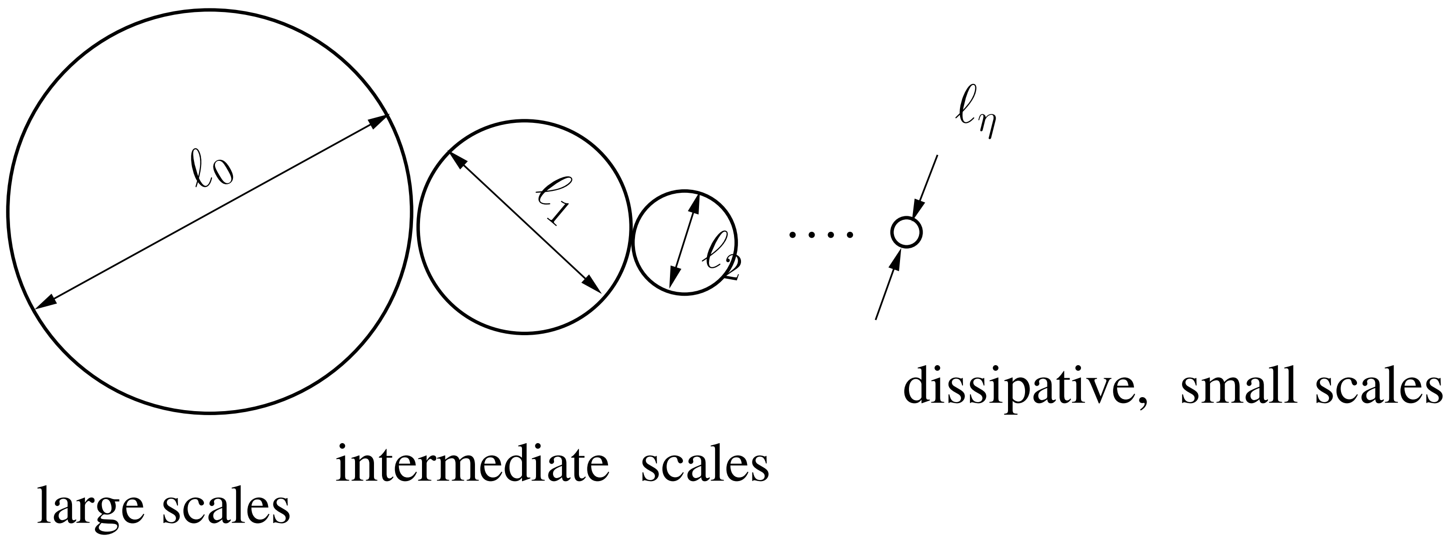

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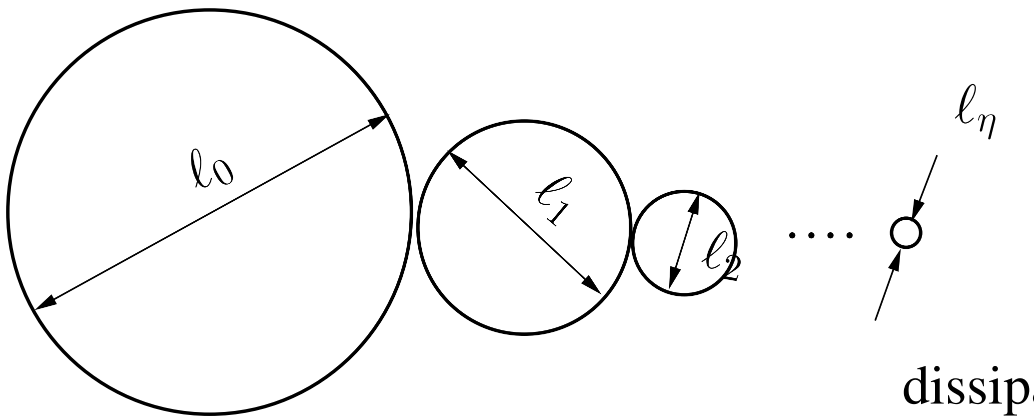

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▶ Hence, you can compute the kinetic energy by:

- integrating in Fourier (wavenumber) space (right-hand side)
- or integrating in physical space over all fluctuations (left-hand side)

▶Spectrum for turbulent kinetic energy, k

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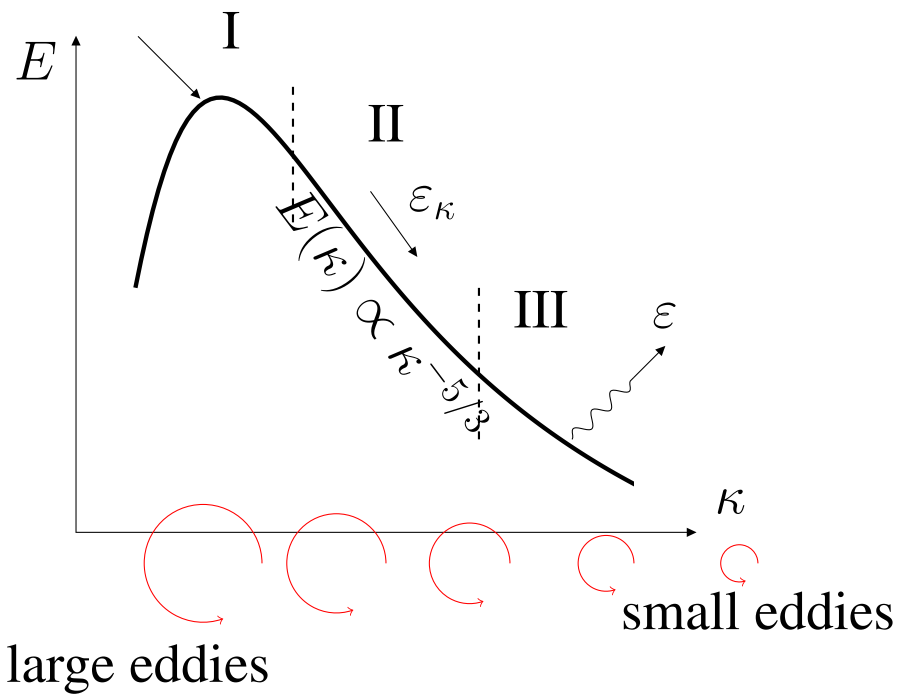
$$-\langle \bar{v}_i' \bar{v}_j' \rangle \frac{\partial \langle \bar{v}_i \rangle}{\partial x_j}$$



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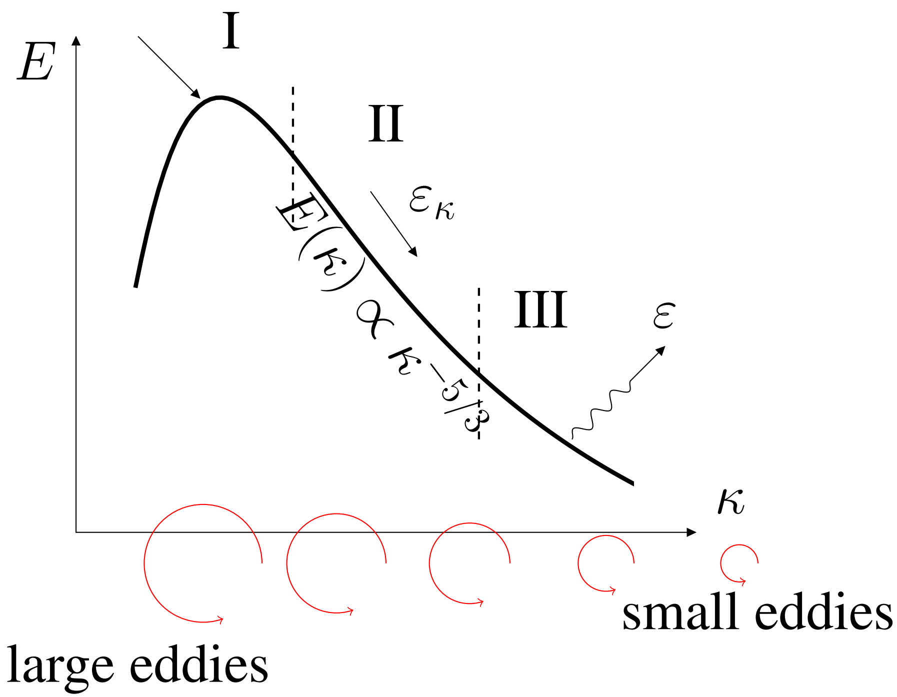
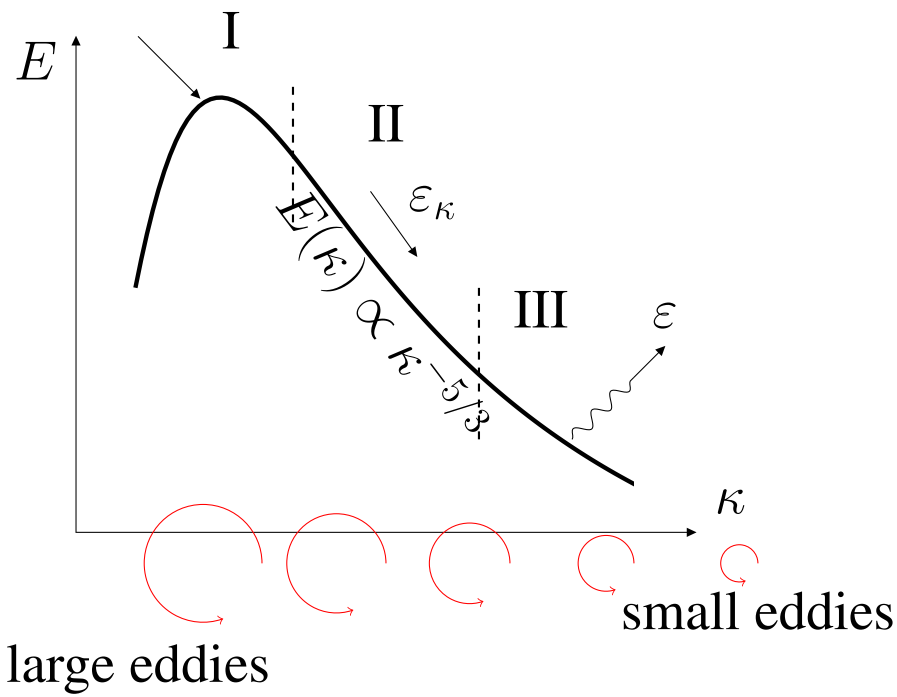


- $E(\kappa_n) \propto a_n^2 + b_n^2$, see the Fourier series on the previous slide

▶ Spectrum for turbulent kinetic energy, k

$$-\langle \bar{v}_i' \bar{v}_j' \rangle \frac{\partial \langle \bar{v}_i \rangle}{\partial x_j}$$



▶ $E(\kappa_n) \propto a_n^2 + b_n^2$, see the Fourier series on the previous slide ▶

$$k = \int_0^\infty E(\kappa) d\kappa = \sum_0^\infty E(\kappa_n) \Delta \kappa_n \tag{35.2}$$

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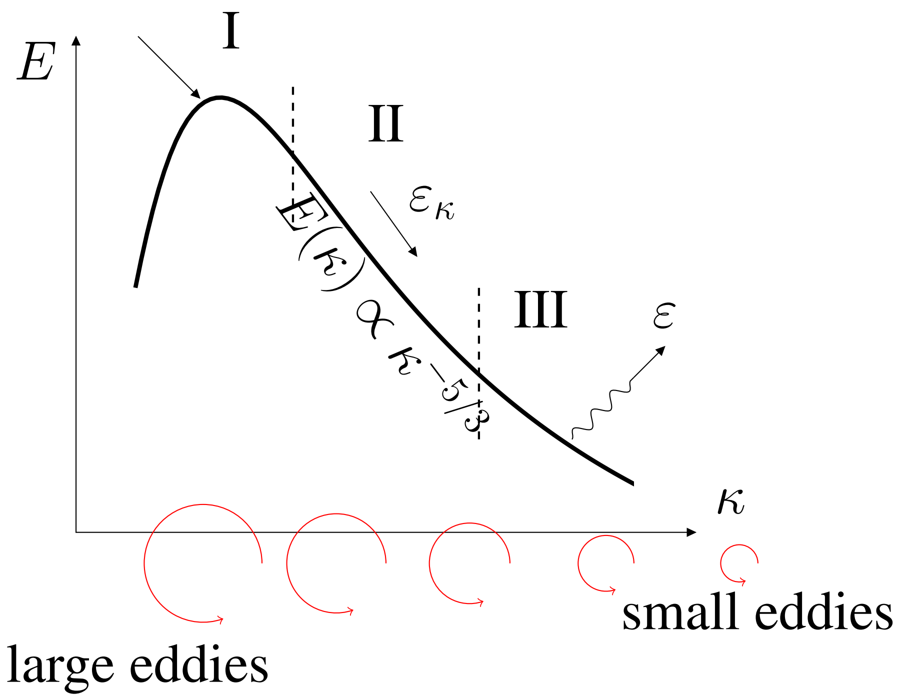

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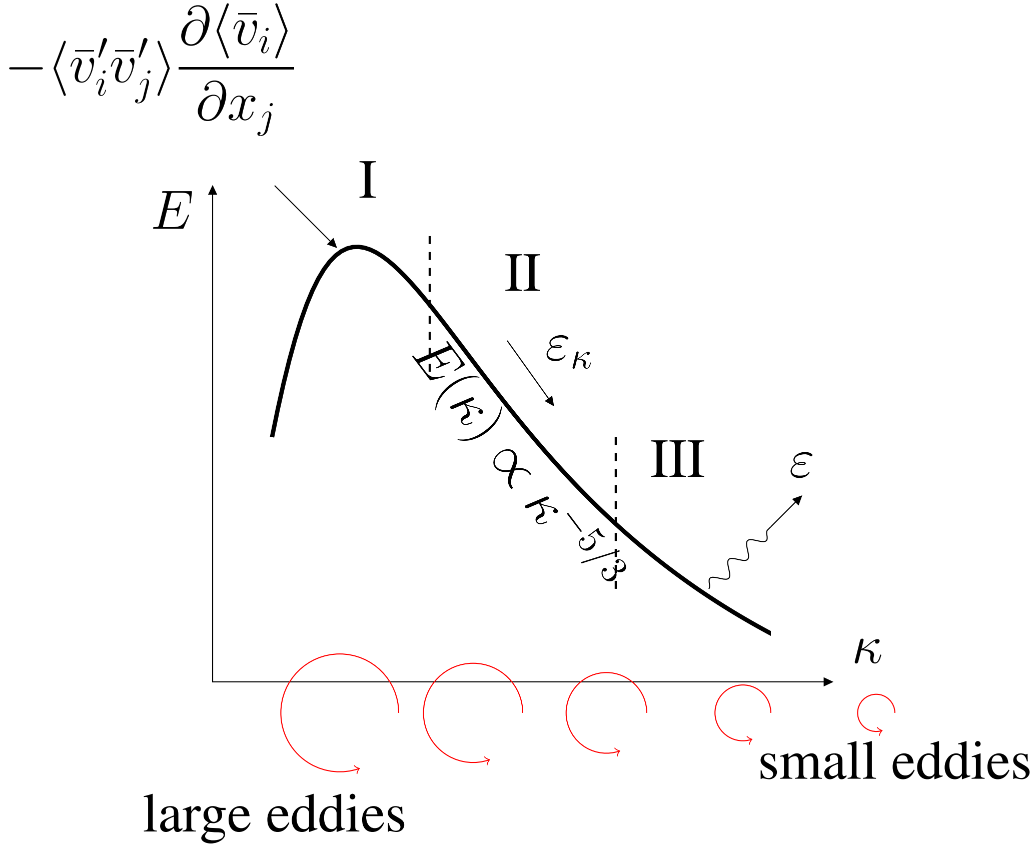
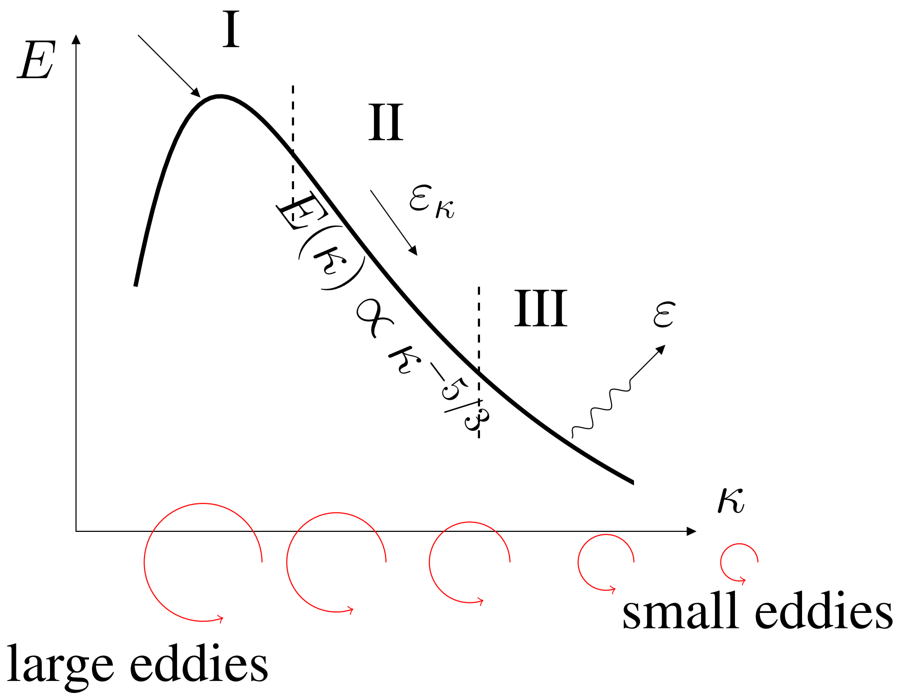
▶which corresponds to Parseval's formula


$-\langle \bar{v}_i' \bar{v}_j' \rangle \frac{\partial \langle \bar{v}_i \rangle}{\partial x_j}$
I
II
III
E
ε_κ
ε
$E(\kappa) \propto \kappa^{-5/3}$
κ
large eddies
small eddies

$$-\langle \bar{v}_i' \bar{v}_j' \rangle \frac{\partial \langle \bar{v}_i \rangle}{\partial x_j}$$



▶ The turbulence spectrum is divided into three regions:

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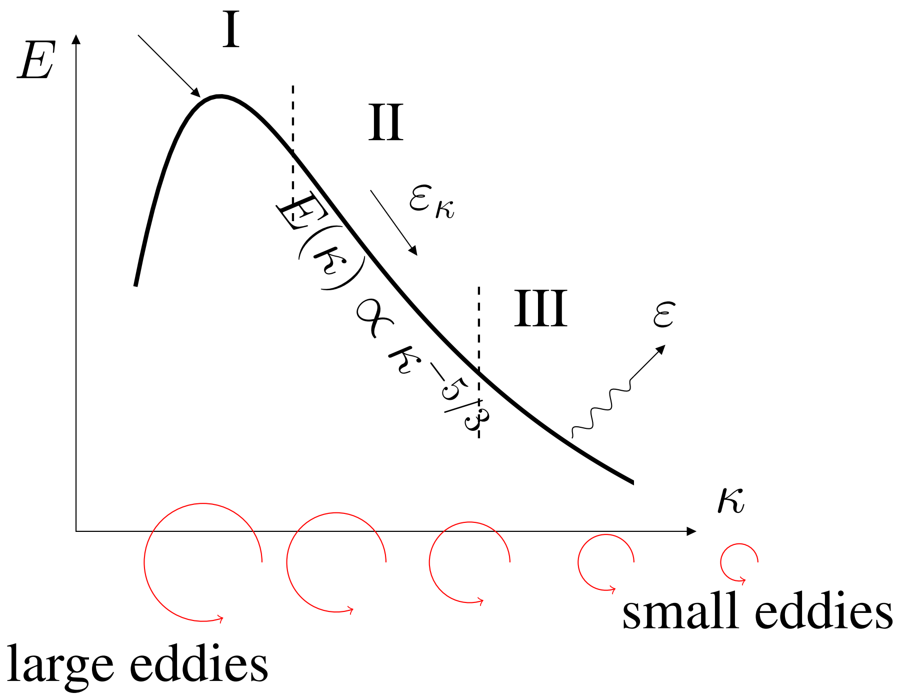

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I. Large eddies carry most of the turb. kinetic energy. They extract energy from the mean flow, P^k.

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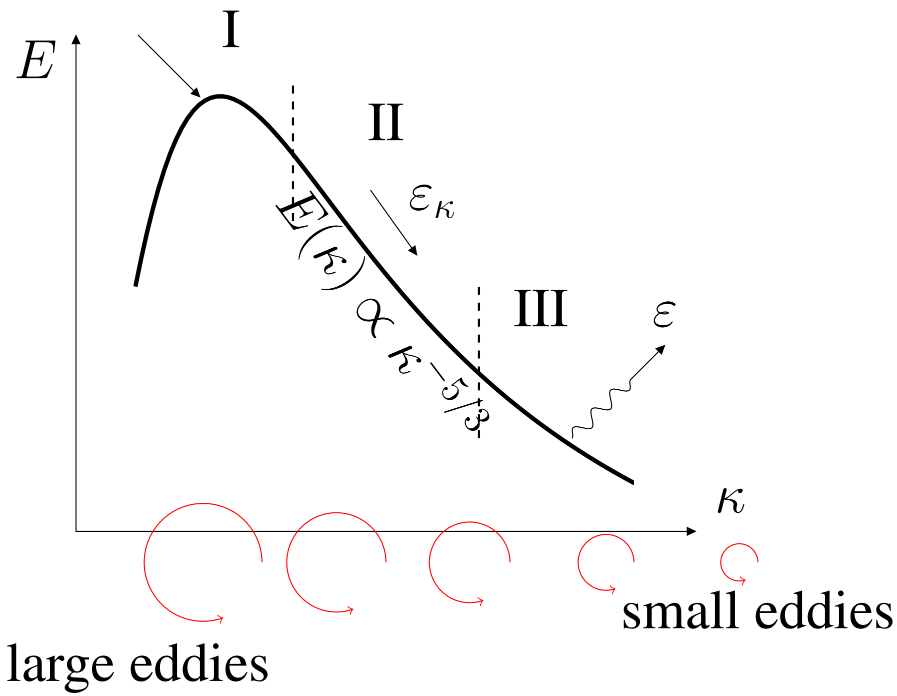

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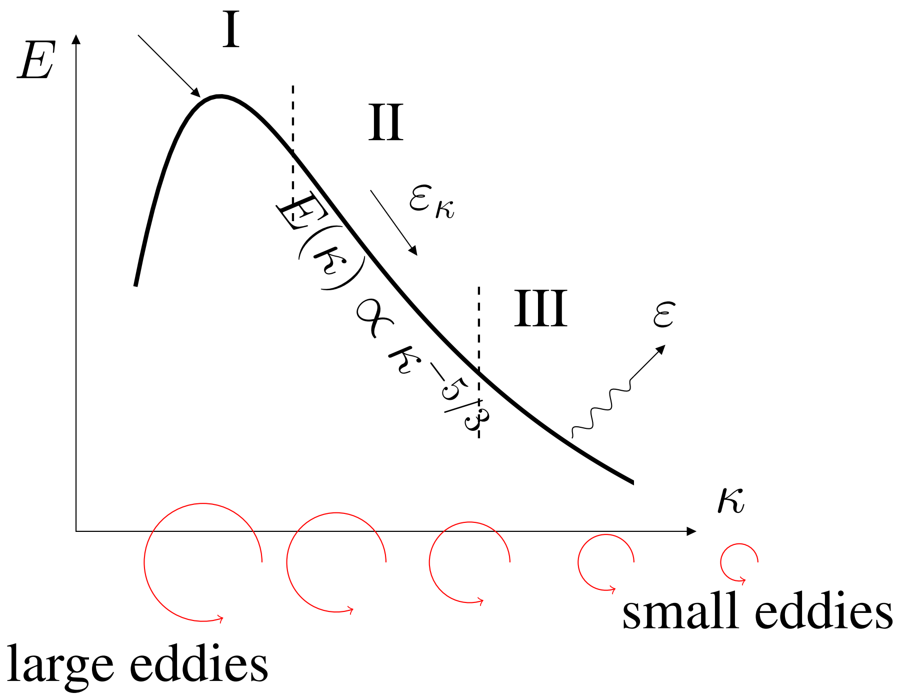

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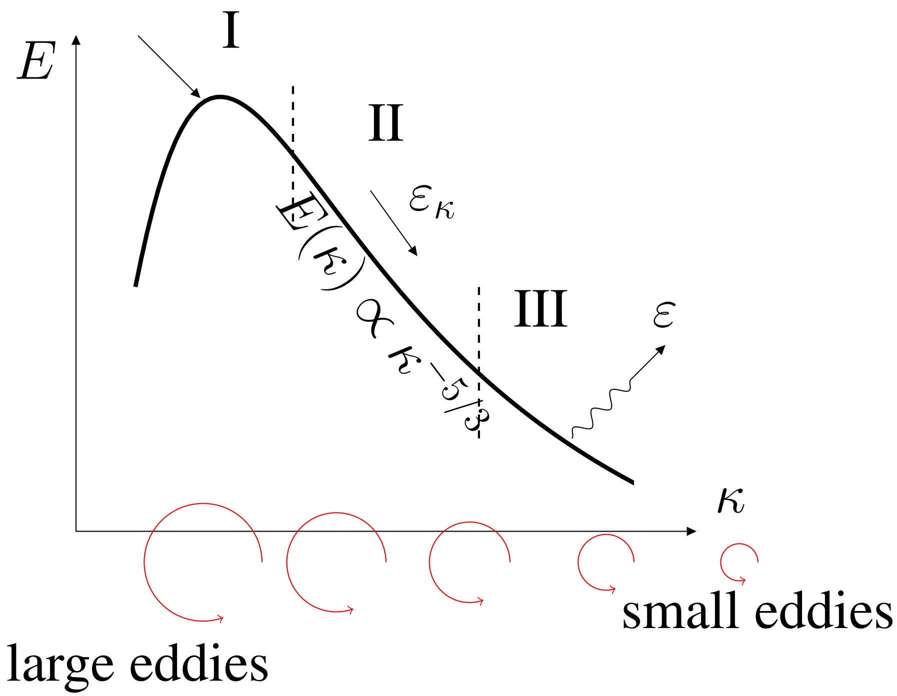
II. Inertial subrange. Independent of both large eddies (mean flow) and viscosity. Isotropic eddies.

III. Dissipation range. Isotropic eddies ($\overline{v_i' v_j'} = c_1 \delta_{ij}$) described by the Kolmogorov scales.

▶Turb. kinetic energy in Region II

$$-\langle \bar{v}_i' \bar{v}_j' \rangle \frac{\partial \langle \bar{v}_i \rangle}{\partial x_j}$$



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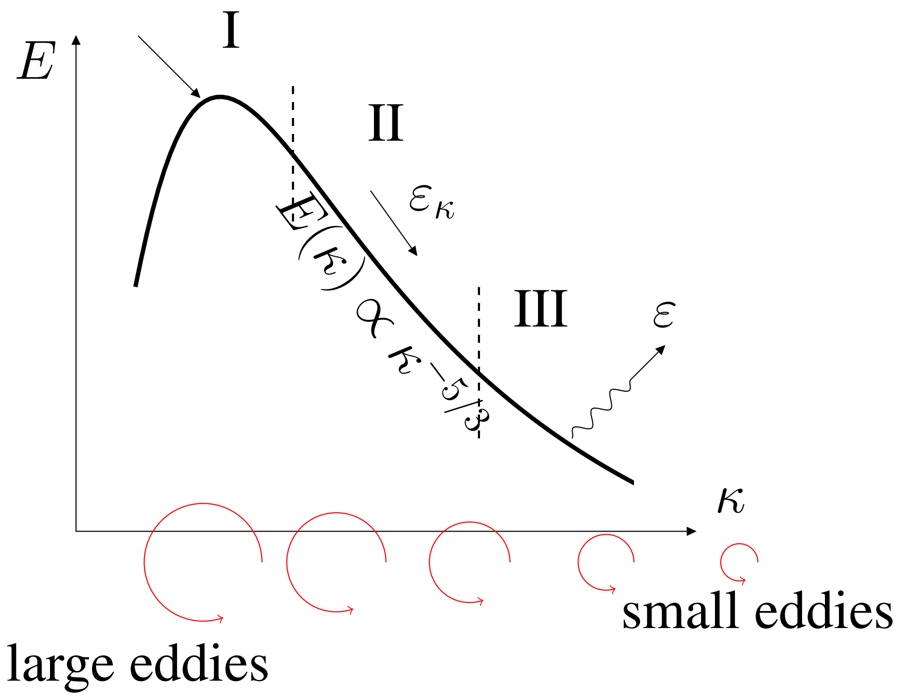

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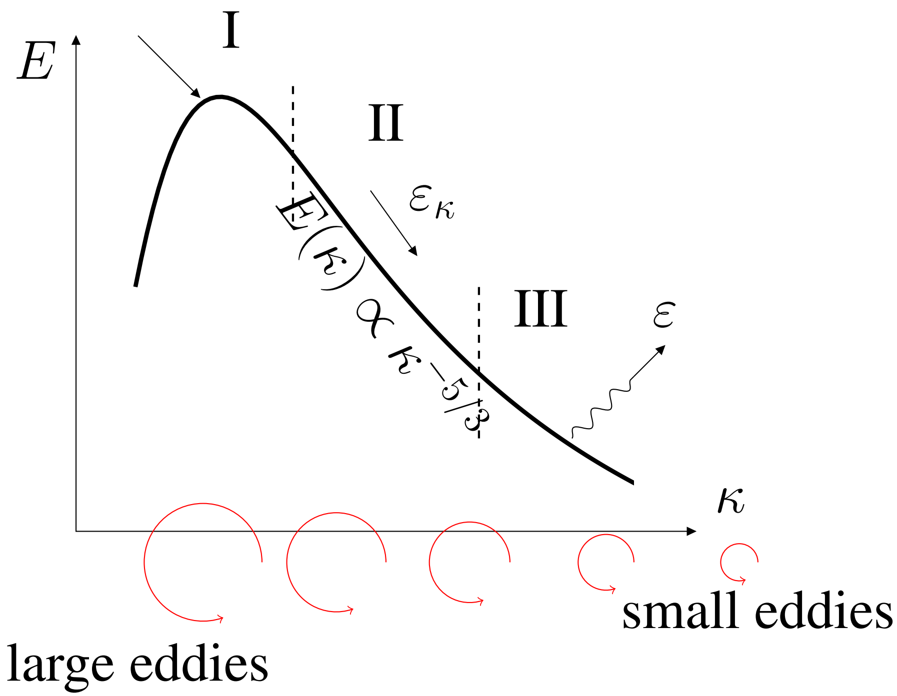

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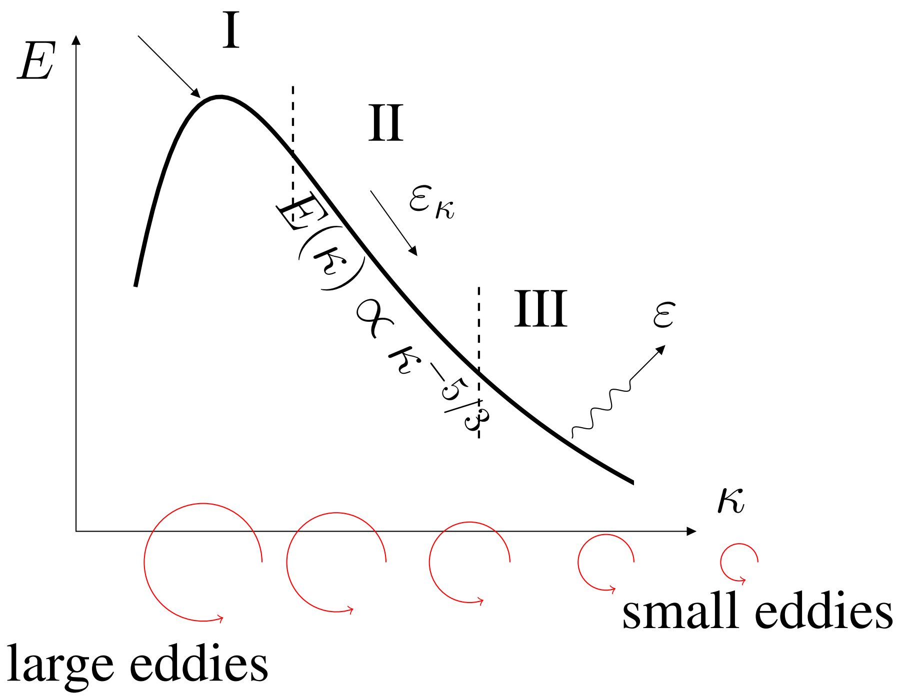

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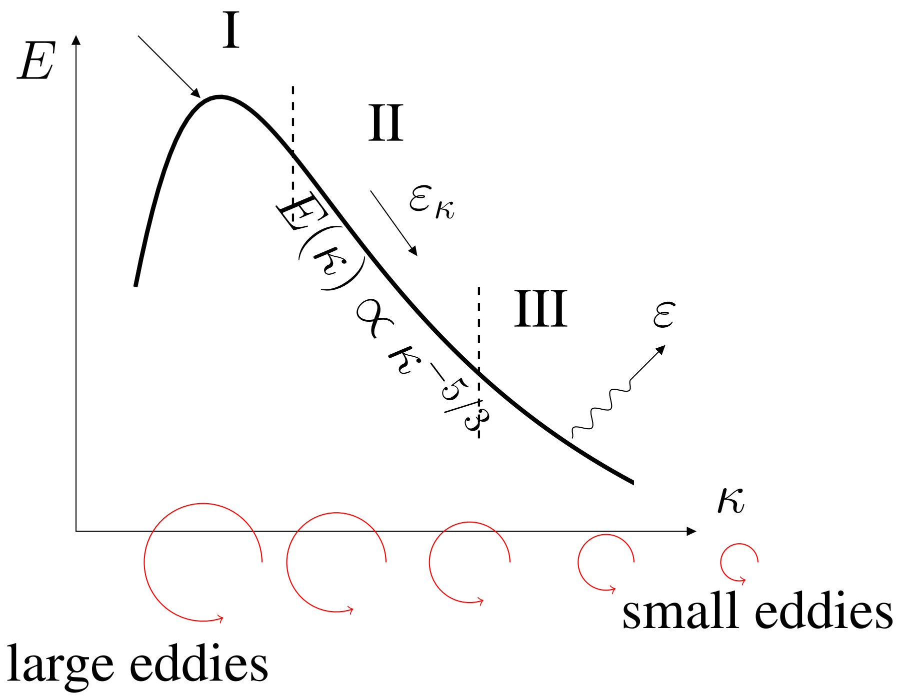

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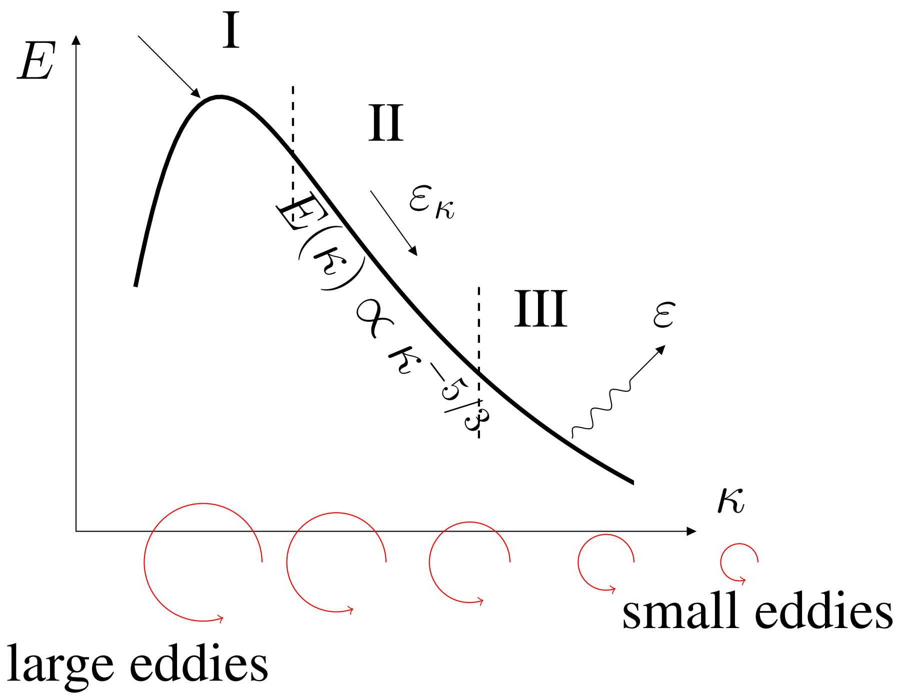

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E

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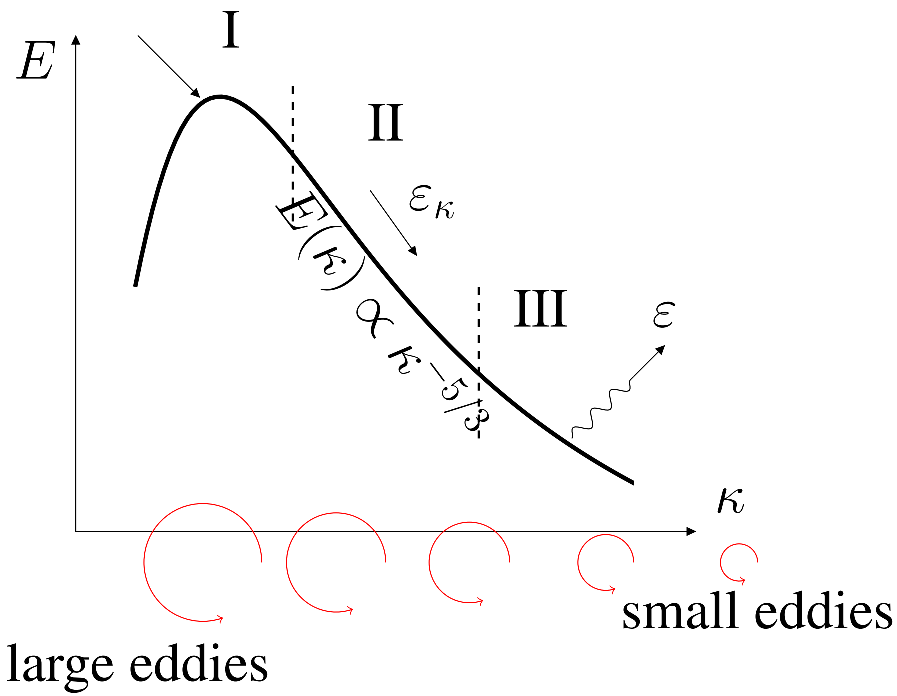

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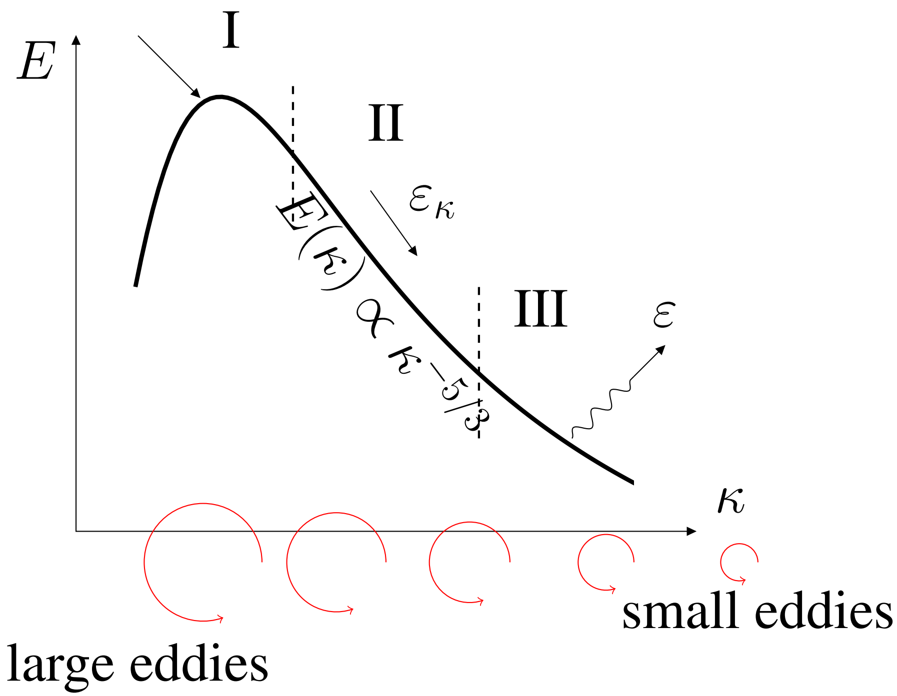

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$$E = \kappa^a$$

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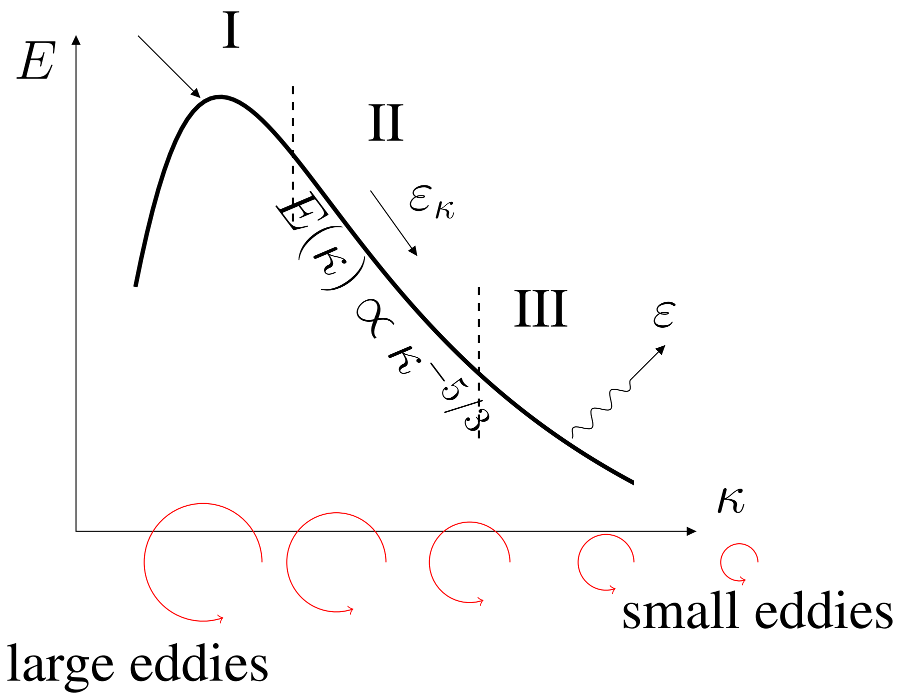

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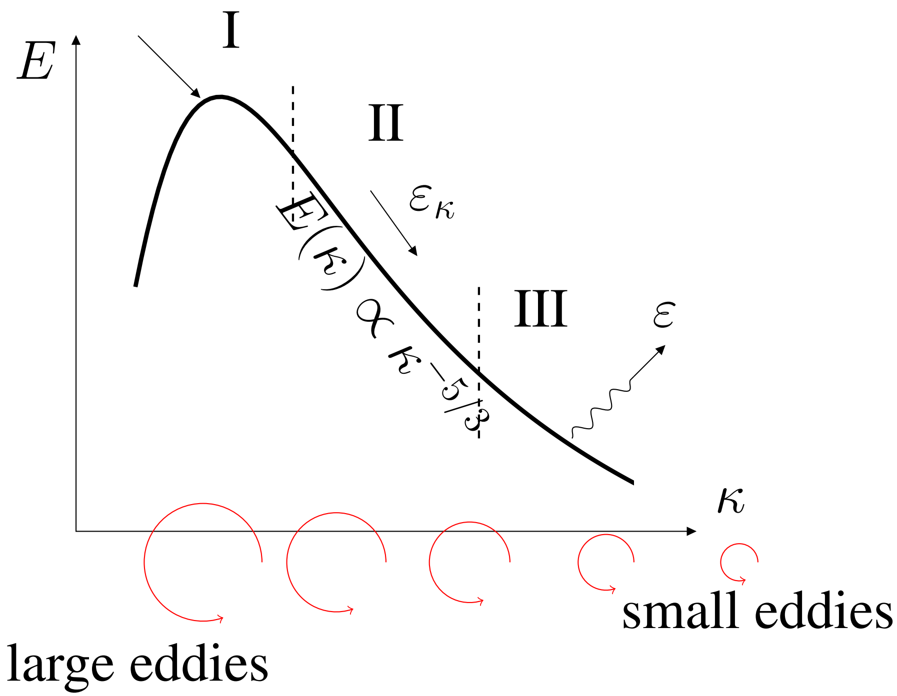

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$$\underset{[m^3/s^2]}{E} = \kappa^a \qquad \varepsilon^b$$

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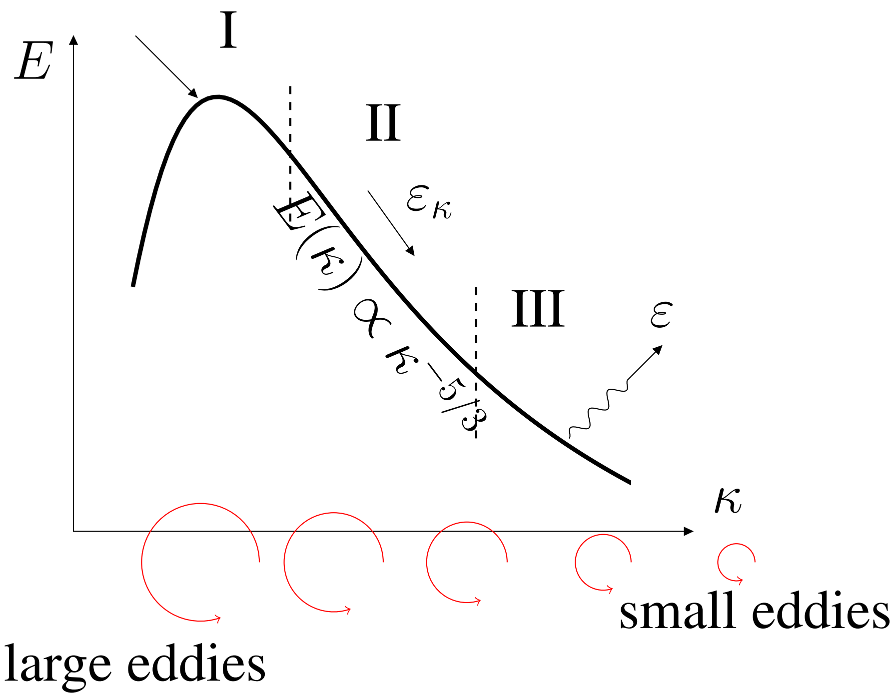

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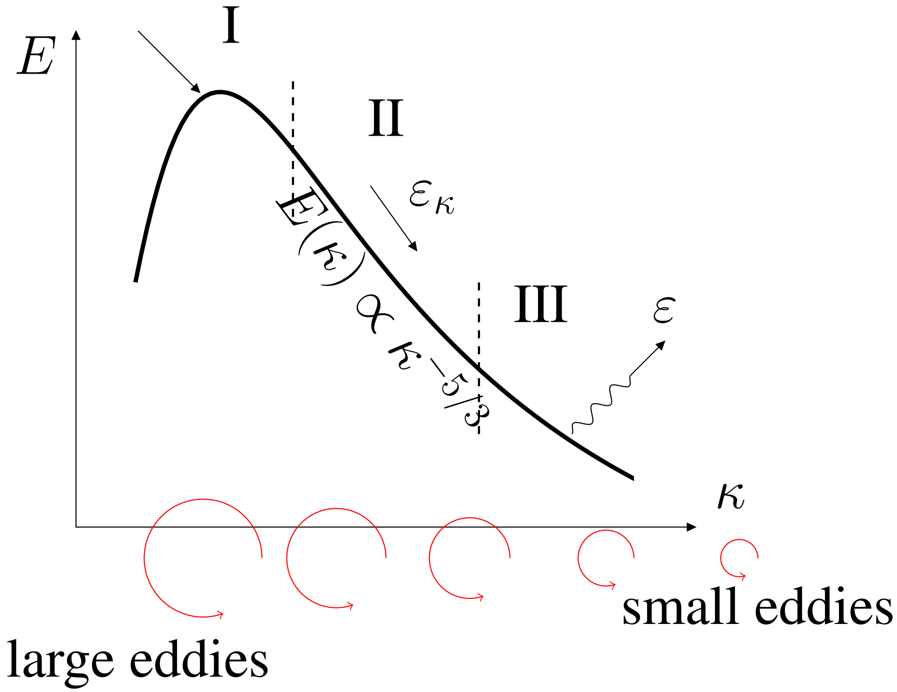

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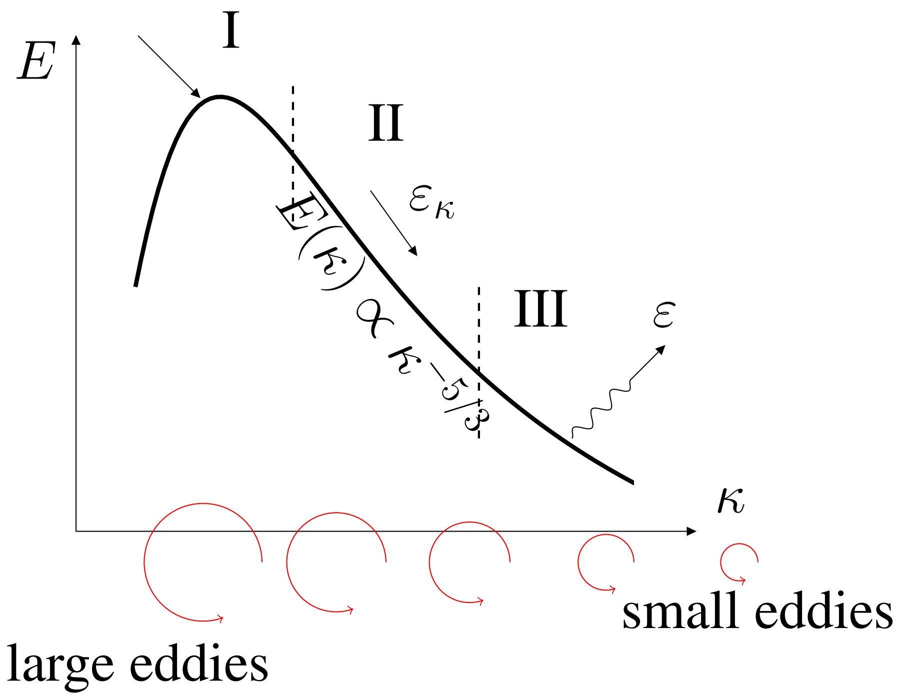

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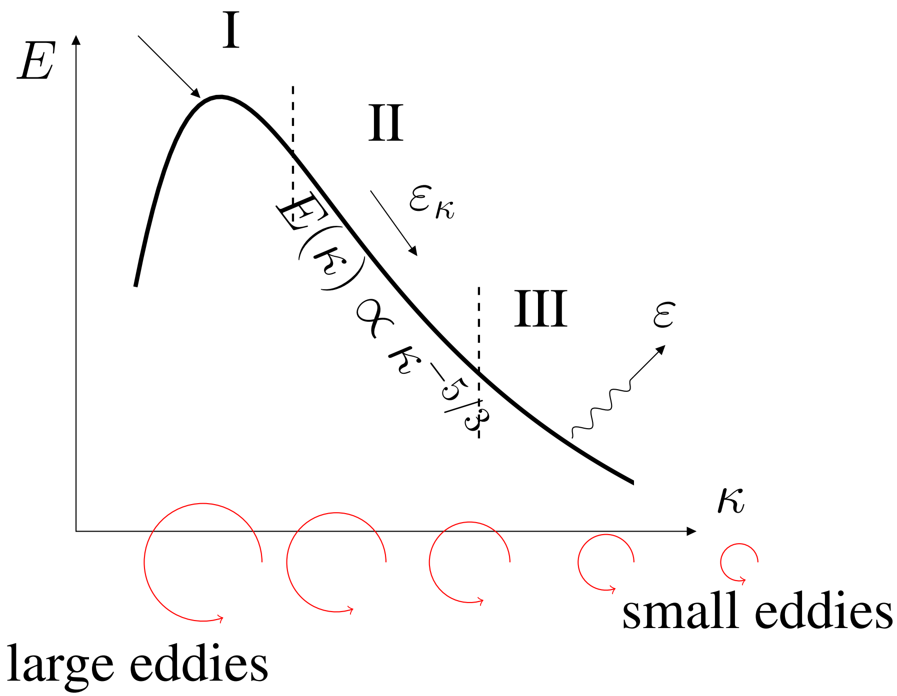

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$$[m]$$

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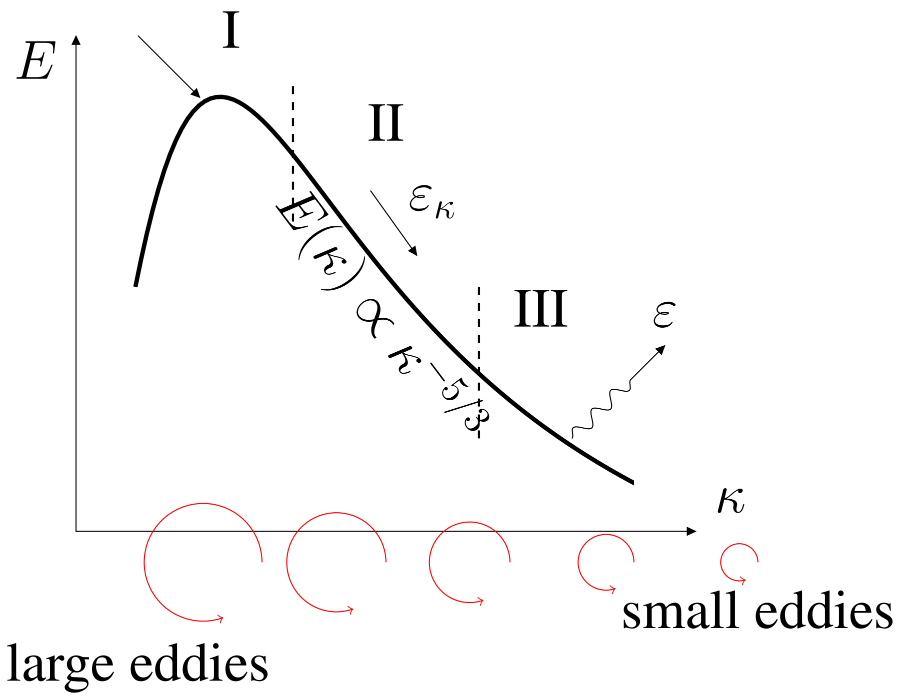

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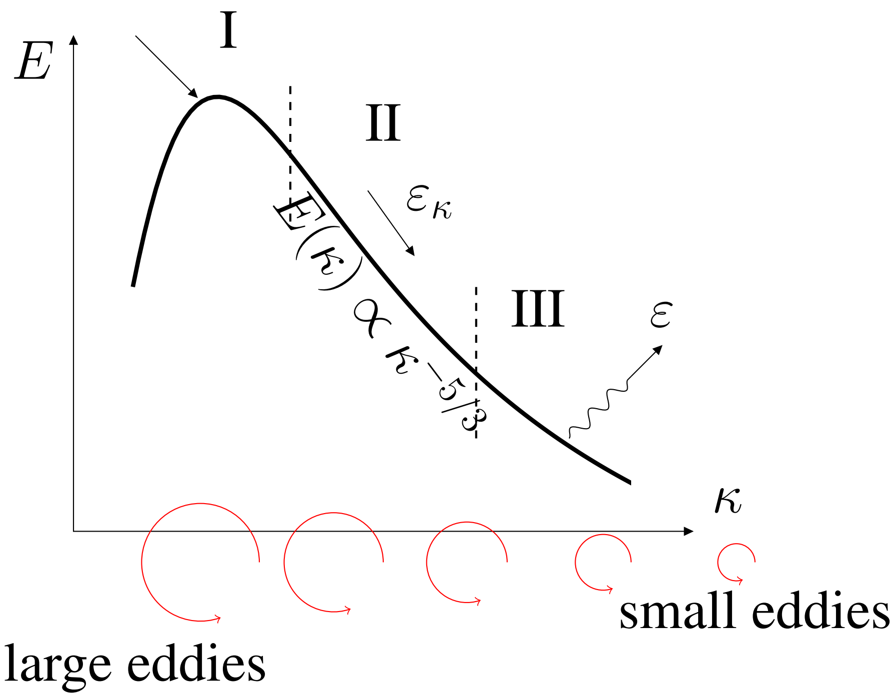

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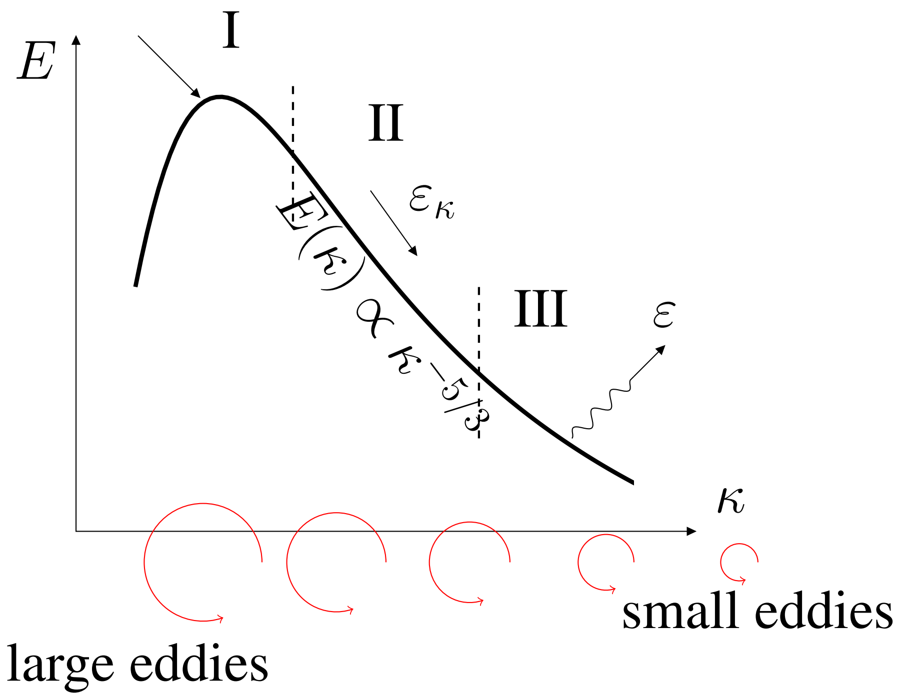

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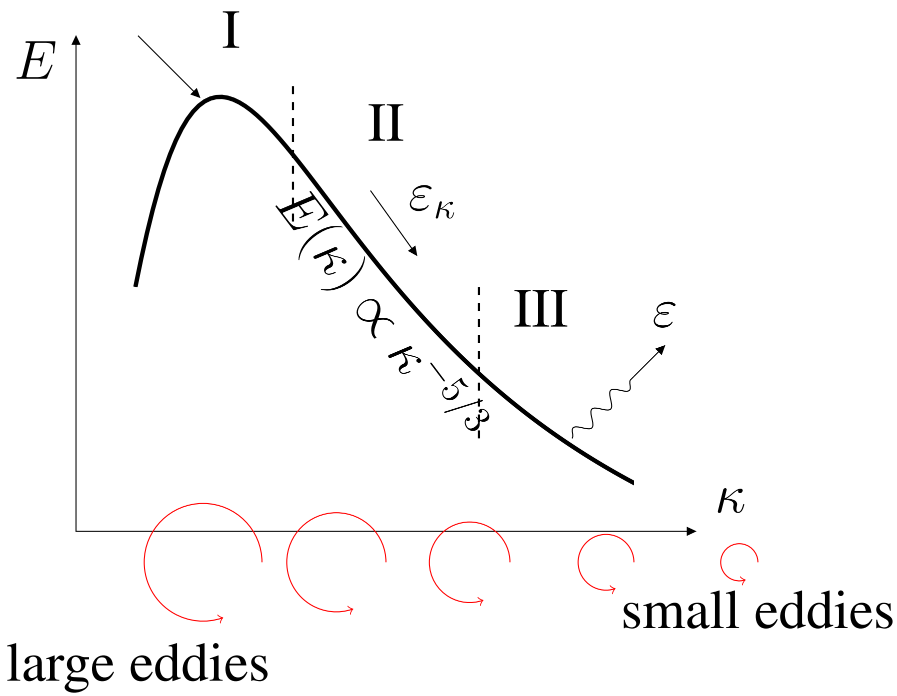

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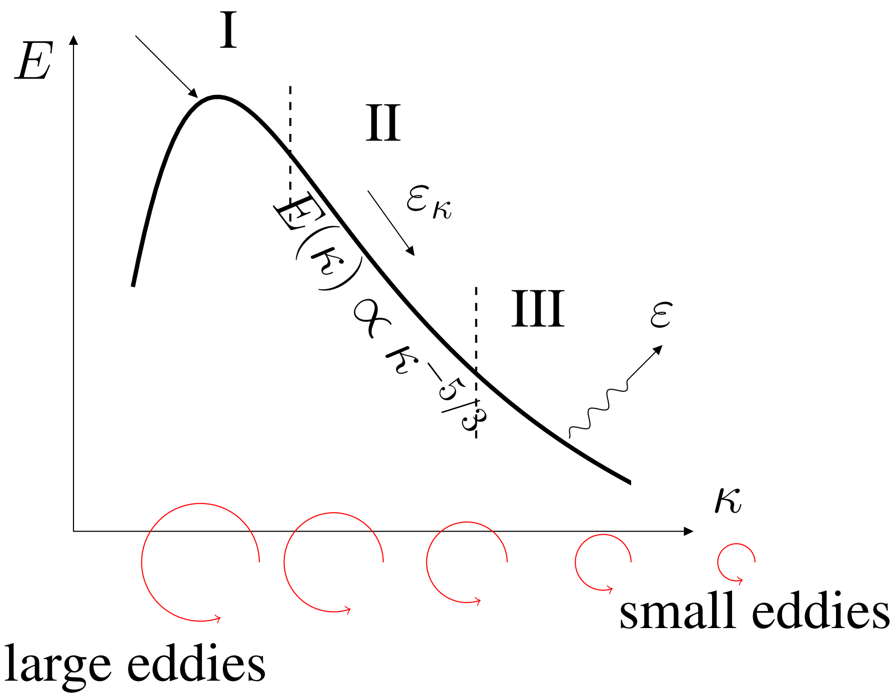

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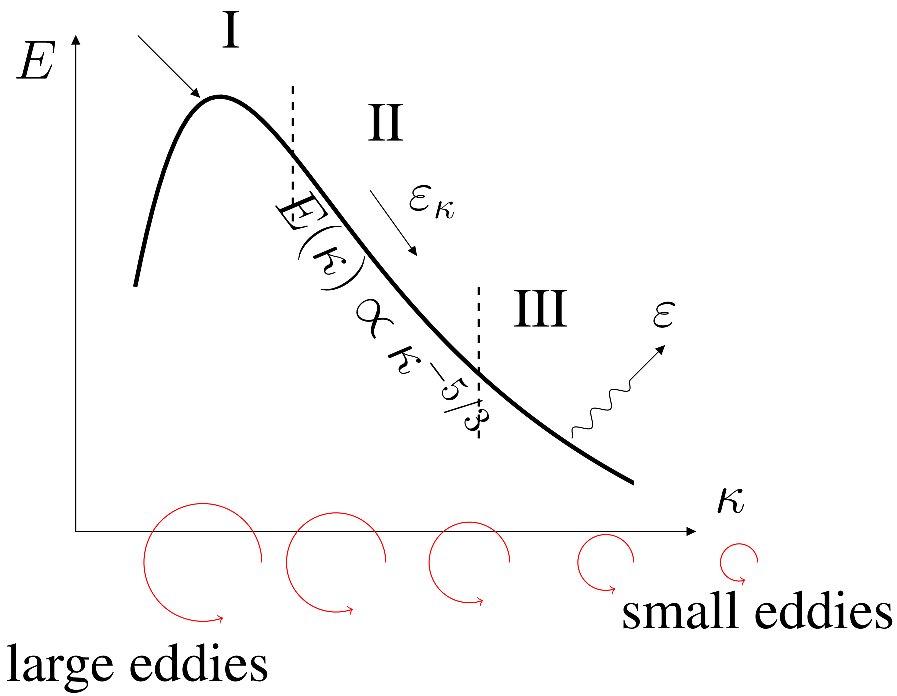

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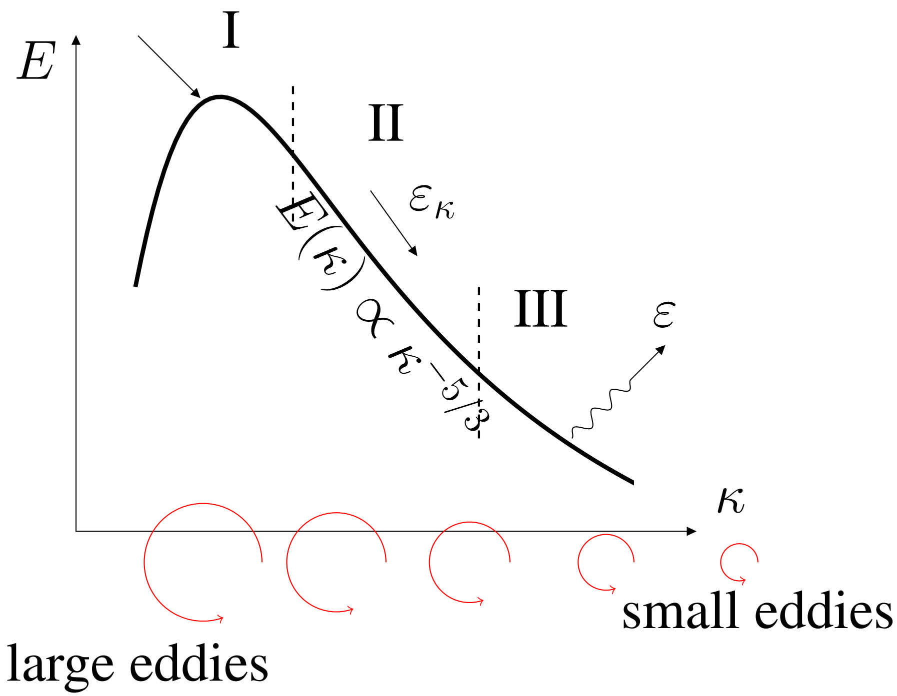

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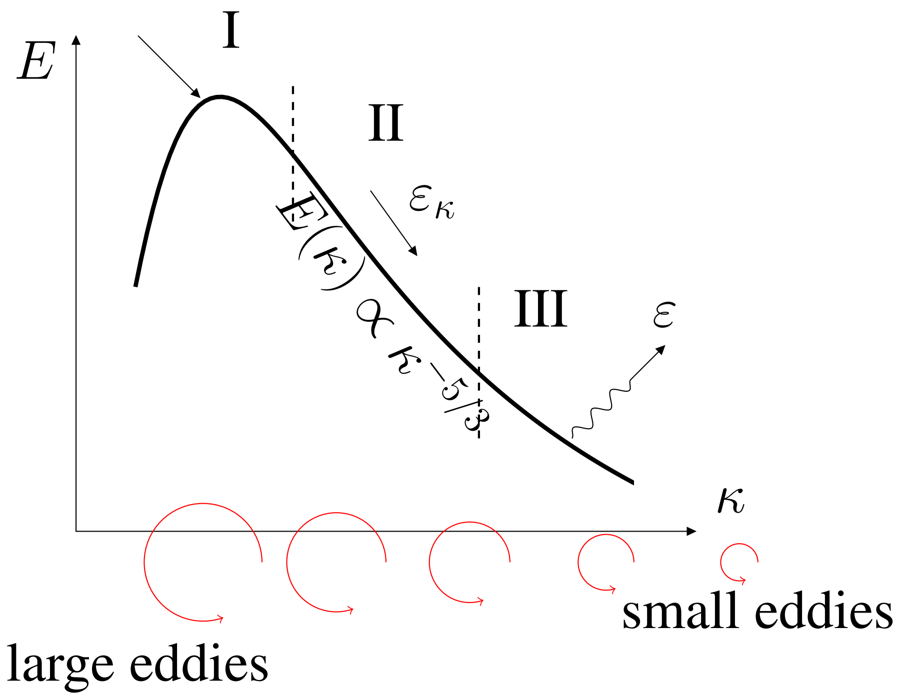

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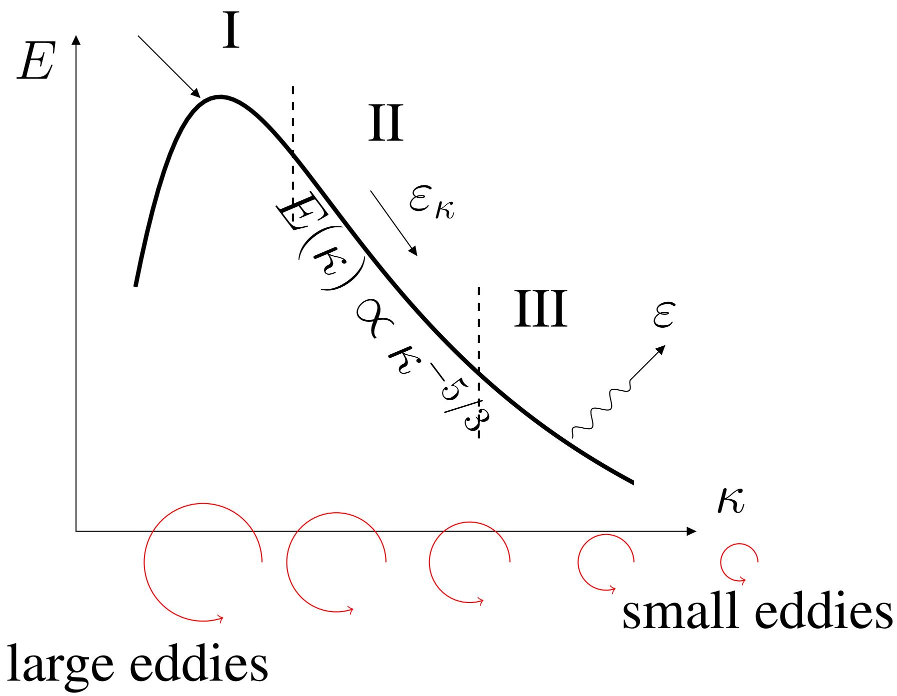

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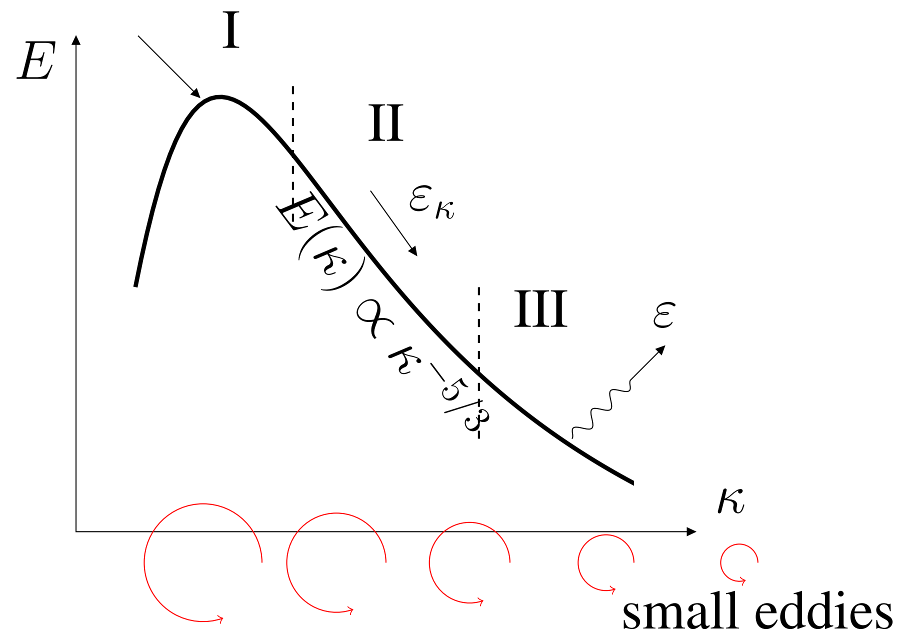

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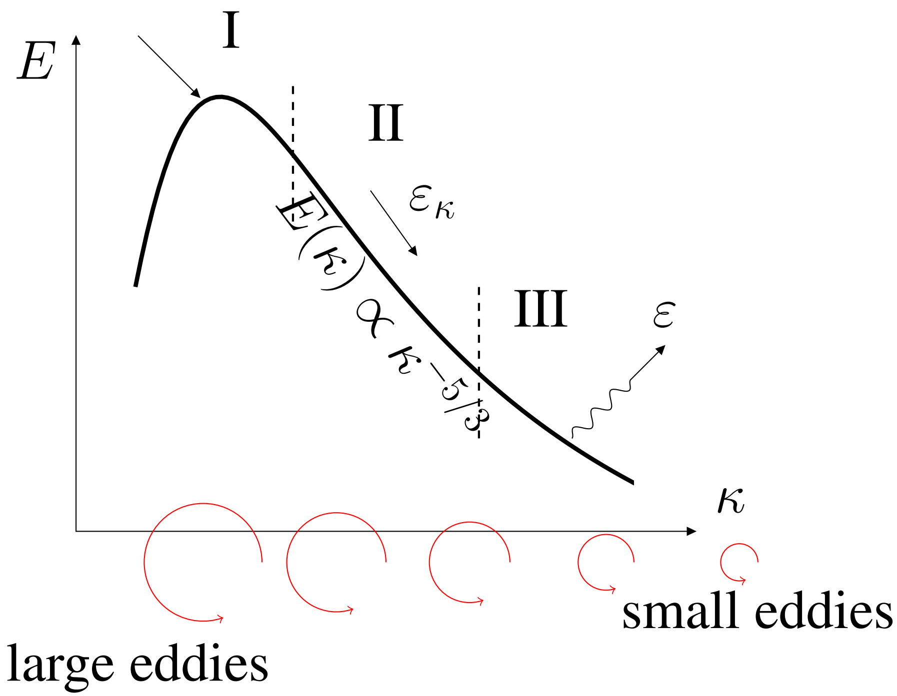

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▶This is called von Kármán spectrum or $-5/3$ law

► Energy transfer from eddy-to-eddy

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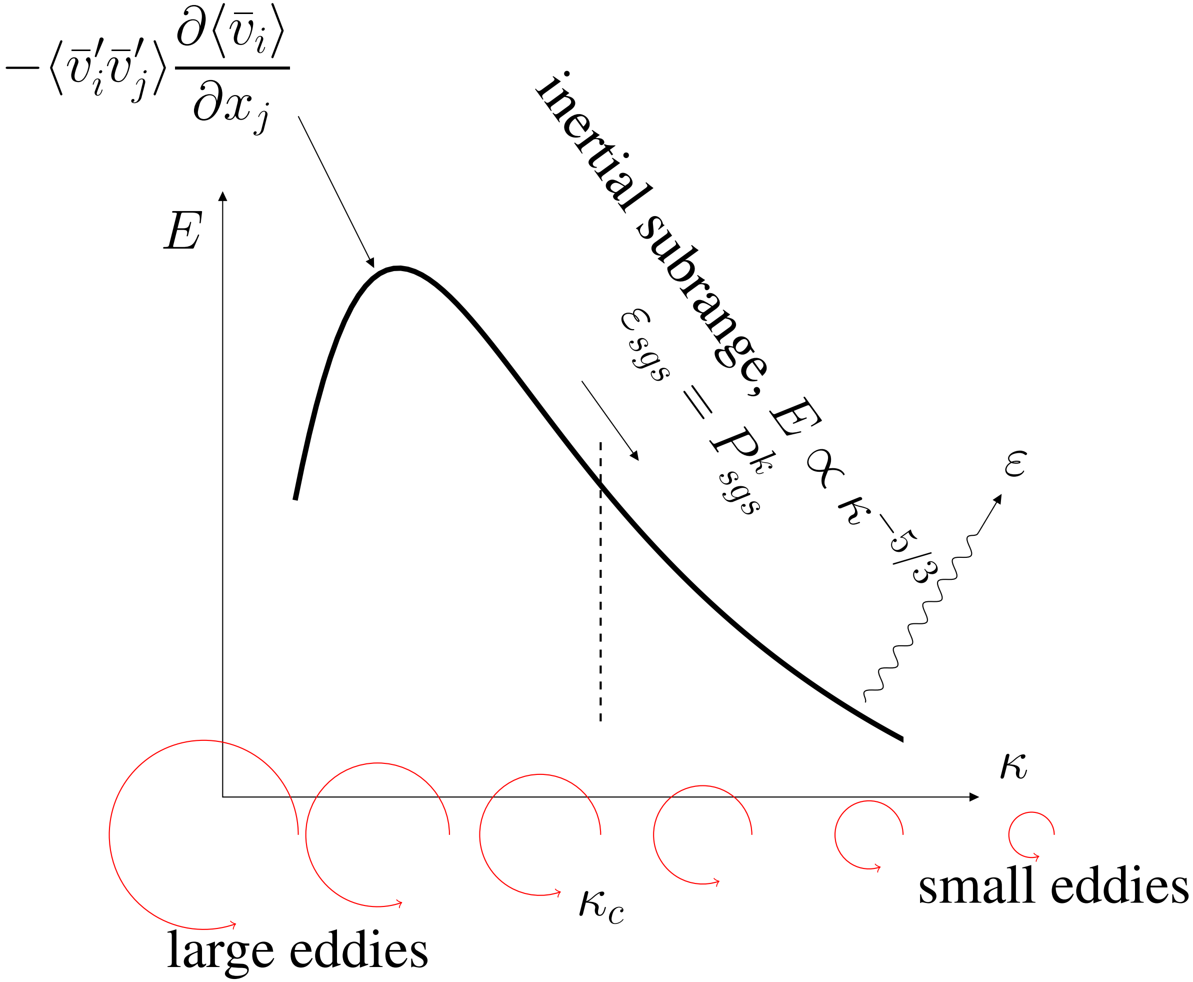
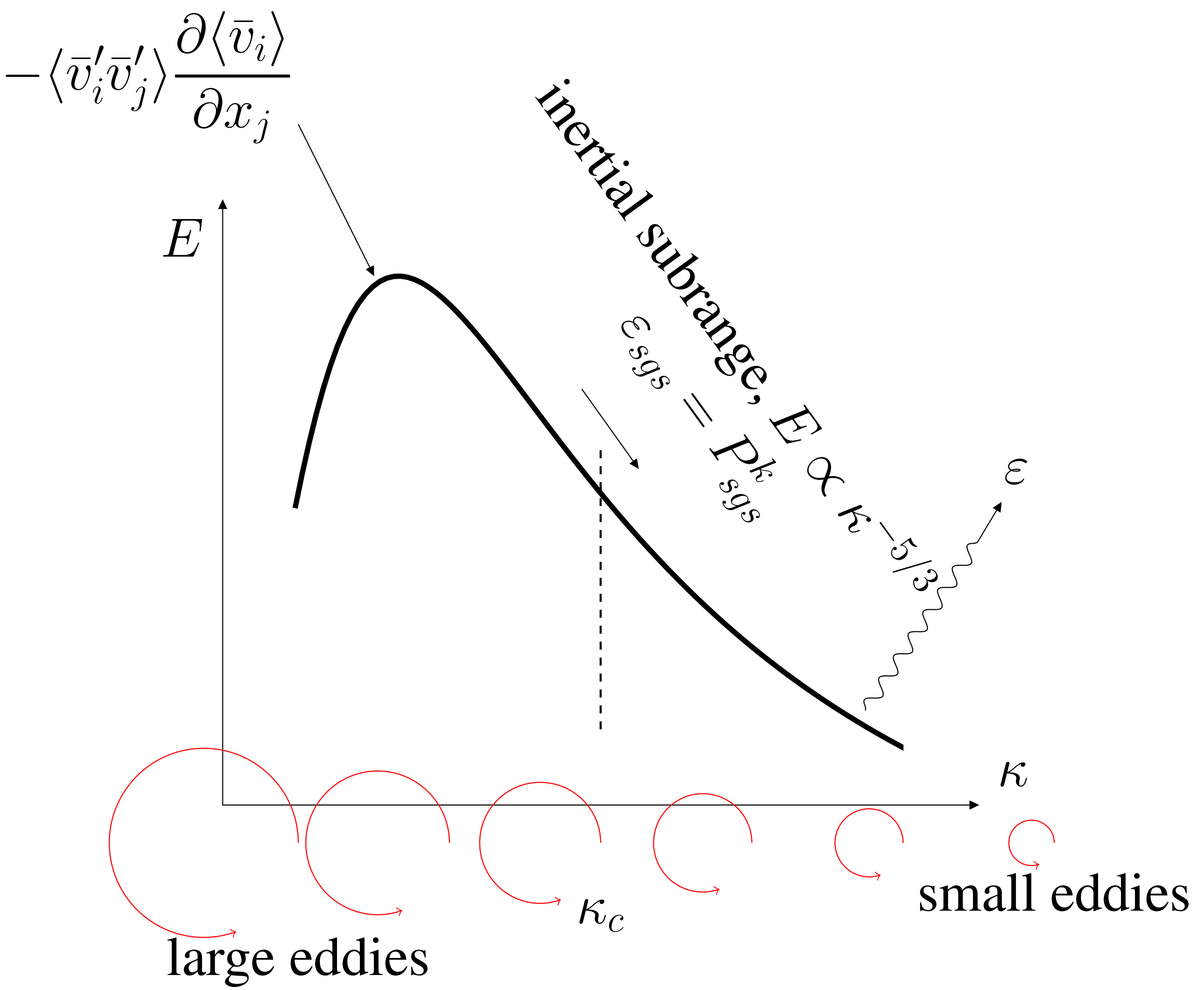
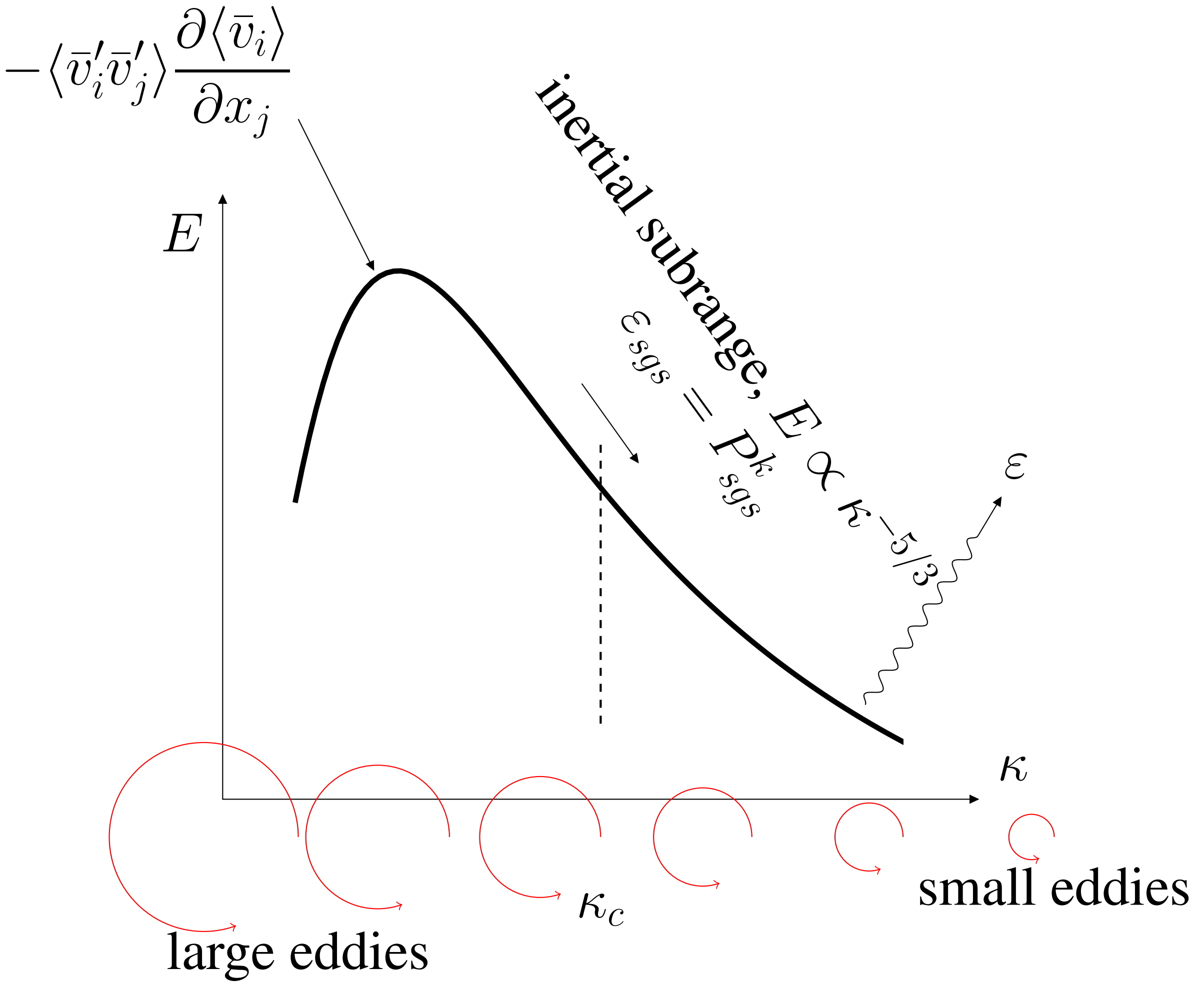


▶Energy transfer from eddy-to-eddy



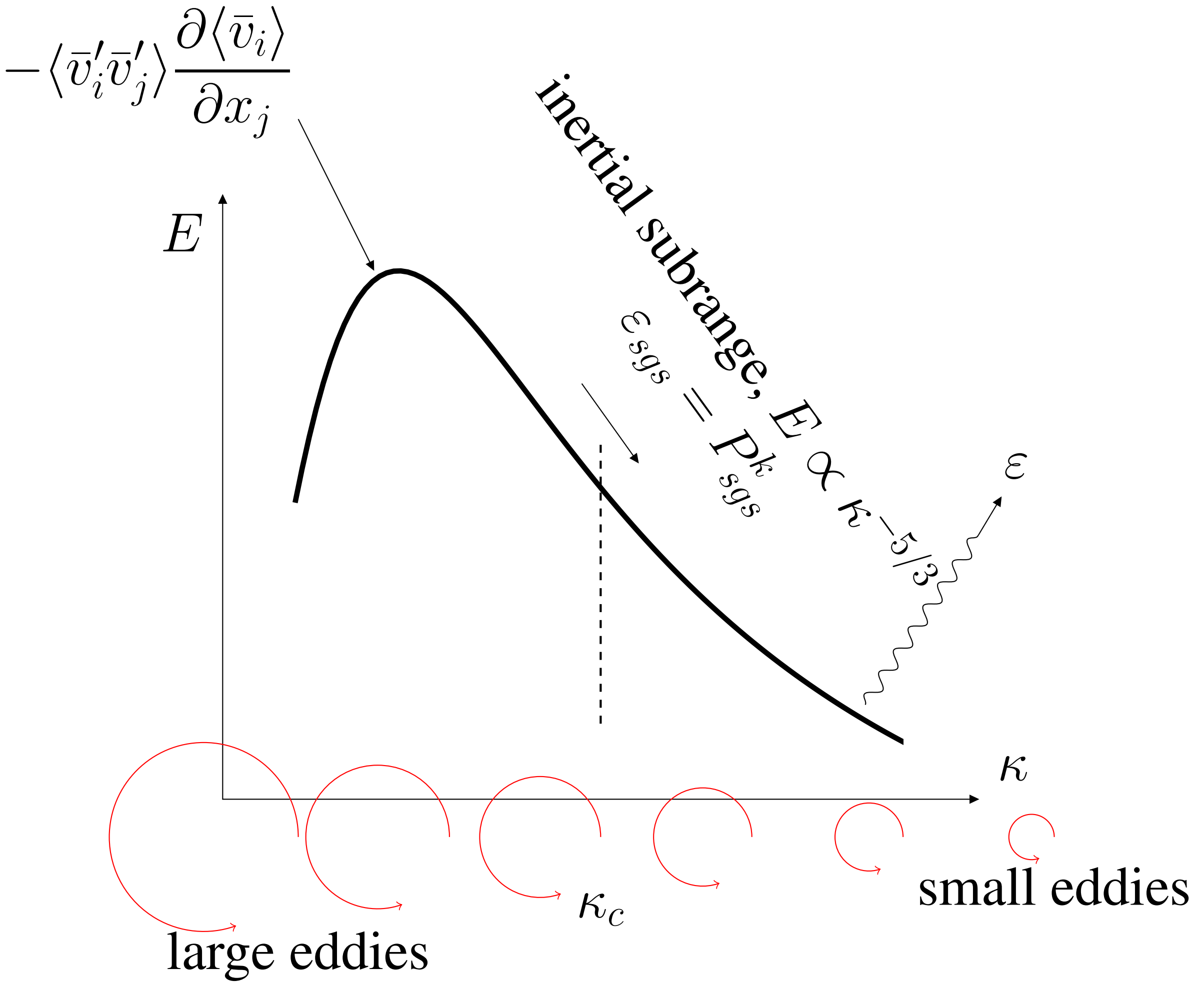
$\varepsilon_\kappa \propto$

▶Energy transfer from eddy-to-eddy



$\varepsilon_\kappa \propto v_\kappa^2/$

▶Energy transfer from eddy-to-eddy



$$\varepsilon_\kappa \propto v_\kappa^2/(\ell_\kappa/v_\kappa) \propto$$

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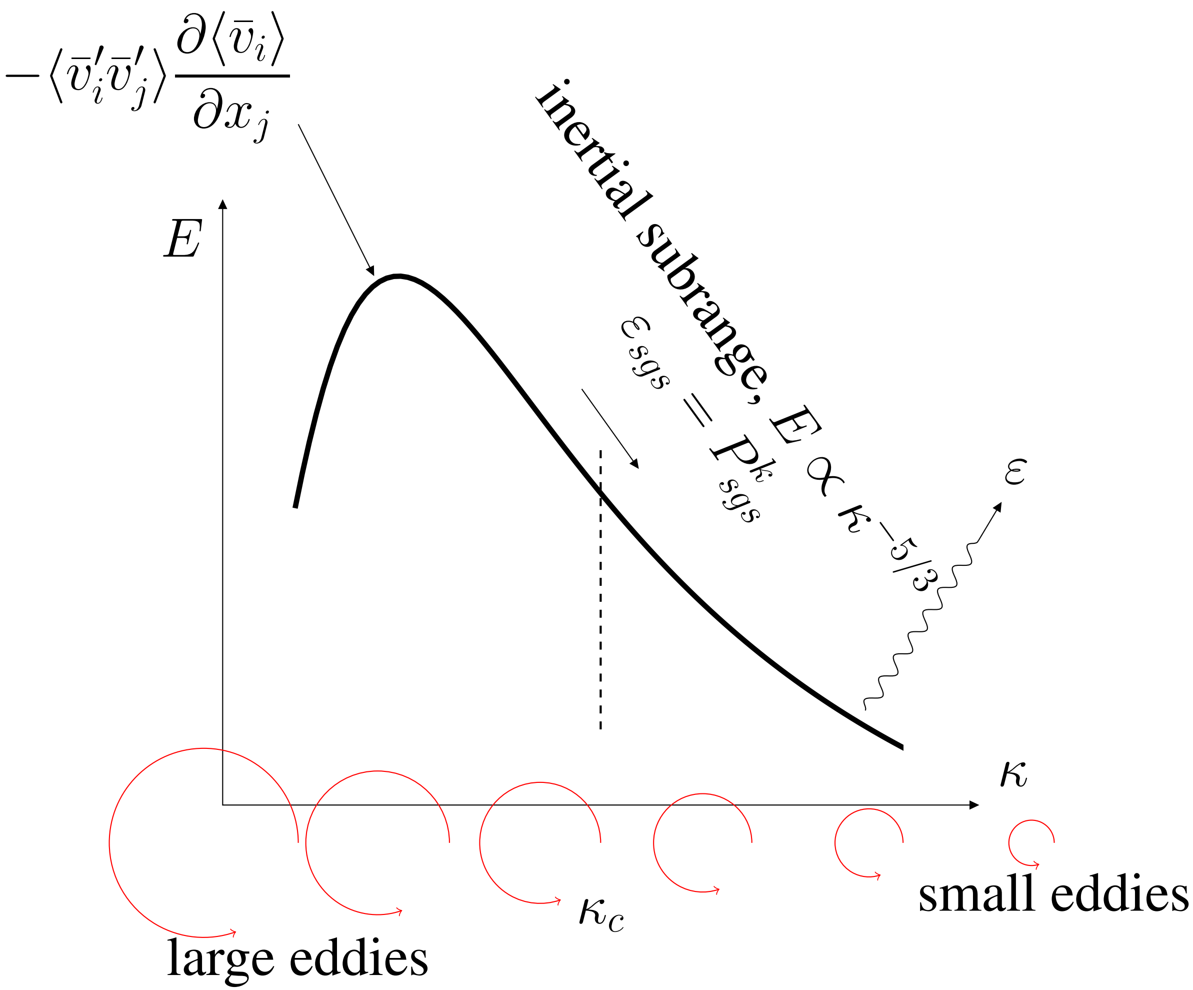

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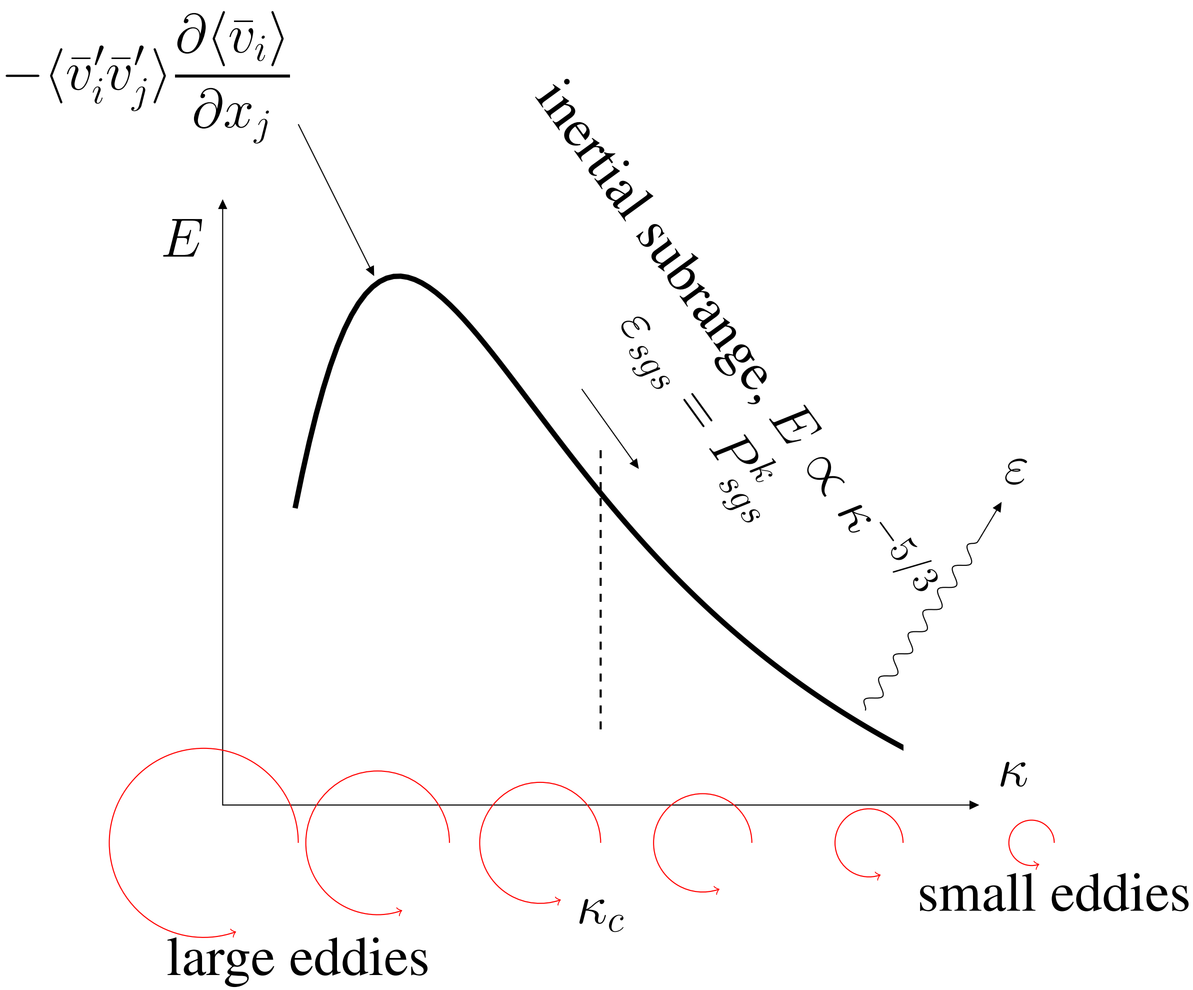

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- The velocity gradient for an eddy can be estimated as

$$\left(\frac{\partial v}{\partial x}\right)_\kappa \propto \frac{v_\kappa}{\ell_\kappa} \propto \left(v_\kappa^2\right)^{1/2} \kappa$$

▶Energy spectrum: recall that k for wavenumber κ is $k \propto E \Delta\kappa$ (see Eq. 35.2). We get

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▶Hence ε increases as κ increases, i.e. ε gets larger for small eddies.

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in RANS:

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$$\langle \Phi \rangle =$$

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$$\langle \Phi \rangle = \frac{1}{2T}\int_{-T}^{T} \Phi(t) dt$$

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$$\langle \Phi \rangle = \frac{1}{2T}\int_{-T}^{T} \Phi(t)dt, \quad \Phi = \langle \Phi \rangle + \Phi'$$

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Momentum equations in DNS:

$$\frac{\partial v_i}{\partial t} + \frac{\partial}{\partial x_j}(v_i v_j) = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 v_i}{\partial x_j \partial x_j} \tag{36.1}$$

Momentum equations in LES:

$$\frac{\partial \bar{v}_i}{\partial t} + \frac{\partial}{\partial x_j}(\bar{v}_i \bar{v}_j) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{v}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j}, \quad \tau_{ij} = \overline{v_i v_j} - \bar{v}_i \bar{v}_j \tag{36.2}$$

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►Filter pressure gradient in Eq. 36.1

Momentum equations in LES:

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$$\overline{\frac{\partial p}{\partial x_i}} = \frac{1}{V}\int_V \frac{\partial p}{\partial x_i} dV \stackrel{?}{=} \frac{\partial}{\partial x_i}\left(\frac{1}{V}\int_V p dV\right) =$$

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$$\overline{\frac{\partial p}{\partial x_i}} = \frac{\partial}{\partial x_i}\left(\frac{1}{V}\int_V p dV\right) + \mathcal{O}\left((\Delta x)^2\right) =$$

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$$\cancel{\overline{\frac{\partial p}{\partial x_i}} = \frac{1}{V}\int_V \frac{\partial p}{\partial x_i} dV \stackrel{?}{=} \frac{\partial}{\partial x_i}\left(\frac{1}{V}\int_V p dV\right) = \frac{\partial \bar{p}}{\partial x_i}}$$

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▶Filter pressure gradient in Eq. 36.1

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▶Filter non-linear term in Eq. 36.1

Momentum equations in LES:

$$\frac{\partial \bar{v}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{v}_i \bar{v}_j\right) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{v}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j}, \qquad \tau_{ij} = \overline{v_i v_j} - \bar{v}_i \bar{v}_j \tag{36.3}$$

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Momentum equations in LES:

$$\frac{\partial \bar{v}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{v}_i \bar{v}_j\right) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{v}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j}, \qquad \tau_{ij} = \overline{v_i v_j} - \bar{v}_i \bar{v}_j \tag{36.3}$$

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►Filter non-linear term in Eq. 36.1

$$\overline{\frac{\partial v_i v_j}{\partial x_j}} = \frac{\partial}{\partial x_j}\left(\overline{v_i v_j}\right) + \mathcal{O}\left((\Delta x)^2\right)$$

Momentum equations in LES:

$$\frac{\partial \bar{v}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{v}_i \bar{v}_j\right) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{v}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j}, \qquad \tau_{ij} = \overline{v_i v_j} - \bar{v}_i \bar{v}_j \tag{36.3}$$

▶Filter pressure gradient in Eq. 36.1

$$\overline{\frac{\partial p}{\partial x_i}} = \frac{1}{V}\int_V \frac{\partial p}{\partial x_i} dV \overset{?}{=} \frac{\partial}{\partial x_i}\left(\frac{1}{V}\int_V p dV\right) = \frac{\partial \bar{p}}{\partial x_i}$$

$$\overline{\frac{\partial p}{\partial x_i}} = \frac{\partial}{\partial x_i}\left(\frac{1}{V}\int_V p dV\right) + \mathcal{O}\left((\Delta x)^2\right) = \frac{\partial \bar{p}}{\partial x_i} + \mathcal{O}\left((\Delta x)^2\right)$$

▶Filter non-linear term in Eq. 36.1

$$\overline{\frac{\partial v_i v_j}{\partial x_j}} = \frac{\partial}{\partial x_j}(\overline{v_i v_j}) + \mathcal{O}\left((\Delta x)^2\right)$$

Left side :

Momentum equations in LES:

$$\frac{\partial \bar{v}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{v}_i \bar{v}_j\right) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{v}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j}, \qquad \tau_{ij} = \overline{v_i v_j} - \bar{v}_i \bar{v}_j \tag{36.3}$$

▶Filter pressure gradient in Eq. 36.1

$$\overline{\frac{\partial p}{\partial x_i}} = \frac{1}{V}\int_V \frac{\partial p}{\partial x_i} dV \stackrel{?}{=} \frac{\partial}{\partial x_i}\left(\frac{1}{V}\int_V p dV\right) = \frac{\partial \bar{p}}{\partial x_i}$$

$$\overline{\frac{\partial p}{\partial x_i}} = \frac{\partial}{\partial x_i}\left(\frac{1}{V}\int_V p dV\right) + \mathcal{O}\left((\Delta x)^2\right) = \frac{\partial \bar{p}}{\partial x_i} + \mathcal{O}\left((\Delta x)^2\right)$$

▶Filter non-linear term in Eq. 36.1

$$\overline{\frac{\partial v_i v_j}{\partial x_j}} = \frac{\partial}{\partial x_j}\left(\overline{v_i v_j}\right) + \mathcal{O}\left((\Delta x)^2\right)$$

$$\text{Left side } : \frac{\partial}{\partial x_j}\left(\overline{v_i v_j}\right) \underbrace{- \frac{\partial}{\partial x_j}\left(\overline{v_i v_j}\right) + \frac{\partial}{\partial x_j}\left(\bar{v}_i \bar{v}_j\right)} =$$

<u>Momentum equations in LES:</u>

$$\frac{\partial \bar{v}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{v}_i \bar{v}_j\right) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{v}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j}, \qquad \tau_{ij} = \overline{v_i v_j} - \bar{v}_i \bar{v}_j \tag{36.3}$$

▶Filter pressure gradient in Eq. 36.1

$$\overline{\frac{\partial p}{\partial x_i}} = \frac{1}{V}\int_V \frac{\partial p}{\partial x_i} dV \stackrel{?}{=} \frac{\partial}{\partial x_i}\left(\frac{1}{V}\int_V p dV\right) = \frac{\partial \bar{p}}{\partial x_i}$$

$$\overline{\frac{\partial p}{\partial x_i}} = \frac{\partial}{\partial x_i}\left(\frac{1}{V}\int_V p dV\right) + \mathcal{O}\left((\Delta x)^2\right) = \frac{\partial \bar{p}}{\partial x_i} + \mathcal{O}\left((\Delta x)^2\right)$$

▶Filter non-linear term in Eq. 36.1

$$\overline{\frac{\partial v_i v_j}{\partial x_j}} = \frac{\partial}{\partial x_j}(\overline{v_i v_j}) + \mathcal{O}\left((\Delta x)^2\right)$$

$$\text{Left side}\ :\ \frac{\partial}{\partial x_j}(\overline{v_i v_j}) \underbrace{- \frac{\partial}{\partial x_j}(\overline{v_i v_j}) + \frac{\partial}{\partial x_j}(\bar{v}_i \bar{v}_j)} = \frac{\partial}{\partial x_j}(\bar{v}_i \bar{v}_j)$$

Momentum equations in LES:

$$\frac{\partial \bar{v}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{v}_i \bar{v}_j\right) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{v}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j}, \qquad \tau_{ij} = \overline{v_i v_j} - \bar{v}_i \bar{v}_j \tag{36.3}$$

▶Filter pressure gradient in Eq. 36.1

$$\overline{\frac{\partial p}{\partial x_i}} = \frac{1}{V}\int_V \frac{\partial p}{\partial x_i} dV \overset{?}{=} \frac{\partial}{\partial x_i}\left(\frac{1}{V}\int_V p dV\right) = \frac{\partial \bar{p}}{\partial x_i}$$

$$\overline{\frac{\partial p}{\partial x_i}} = \frac{\partial}{\partial x_i}\left(\frac{1}{V}\int_V p dV\right) + \mathcal{O}\left((\Delta x)^2\right) = \frac{\partial \bar{p}}{\partial x_i} + \mathcal{O}\left((\Delta x)^2\right)$$

▶Filter non-linear term in Eq. 36.1

$$\overline{\frac{\partial v_i v_j}{\partial x_j}} = \frac{\partial}{\partial x_j}(\overline{v_i v_j}) + \mathcal{O}\left((\Delta x)^2\right)$$

$$\text{Left side} \;:\; \frac{\partial}{\partial x_j}(\overline{v_i v_j}) \underbrace{- \frac{\partial}{\partial x_j}(\overline{v_i v_j}) + \frac{\partial}{\partial x_j}(\bar{v}_i \bar{v}_j)} = \frac{\partial}{\partial x_j}(\bar{v}_i \bar{v}_j)$$

Right side :

Momentum equations in LES:

$$\frac{\partial \bar{v}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{v}_i \bar{v}_j\right) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{v}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j}, \qquad \tau_{ij} = \overline{v_i v_j} - \bar{v}_i \bar{v}_j \tag{36.3}$$

▶Filter pressure gradient in Eq. 36.1

$$\overline{\frac{\partial p}{\partial x_i}} = \frac{1}{V}\int_V \frac{\partial p}{\partial x_i} dV = \frac{\partial}{\partial x_i}\left(\frac{1}{V}\int_V p dV\right) = \frac{\partial \bar{p}}{\partial x_i}$$

$$\overline{\frac{\partial p}{\partial x_i}} = \frac{\partial}{\partial x_i}\left(\frac{1}{V}\int_V p dV\right) + \mathcal{O}\left((\Delta x)^2\right) = \frac{\partial \bar{p}}{\partial x_i} + \mathcal{O}\left((\Delta x)^2\right)$$

▶Filter non-linear term in Eq. 36.1

$$\overline{\frac{\partial v_i v_j}{\partial x_j}} = \frac{\partial}{\partial x_j}(\overline{v_i v_j}) + \mathcal{O}\left((\Delta x)^2\right)$$

$$\text{Left side}: \frac{\partial}{\partial x_j}(\overline{v_i v_j}) \underbrace{- \frac{\partial}{\partial x_j}(\overline{v_i v_j}) + \frac{\partial}{\partial x_j}(\bar{v}_i \bar{v}_j)} = \frac{\partial}{\partial x_j}(\bar{v}_i \bar{v}_j)$$

$$\text{Right side}: \underbrace{- \frac{\partial}{\partial x_j}(\overline{v_i v_j}) + \frac{\partial}{\partial x_j}(\bar{v}_i \bar{v}_j)} =$$

Momentum equations in LES:

$$\frac{\partial \bar{v}_i}{\partial t} + \frac{\partial}{\partial x_j}(\bar{v}_i \bar{v}_j) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{v}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j}, \qquad \tau_{ij} = \overline{v_i v_j} - \bar{v}_i \bar{v}_j \tag{36.3}$$

▶Filter pressure gradient in Eq. 36.1

$$\overline{\frac{\partial p}{\partial x_i}} = \frac{1}{V}\int_V \frac{\partial p}{\partial x_i} dV \overset{?}{=} \frac{\partial}{\partial x_i}\left(\frac{1}{V}\int_V p dV\right) = \frac{\partial \bar{p}}{\partial x_i}$$

$$\overline{\frac{\partial p}{\partial x_i}} = \frac{\partial}{\partial x_i}\left(\frac{1}{V}\int_V p dV\right) + \mathcal{O}\left((\Delta x)^2\right) = \frac{\partial \bar{p}}{\partial x_i} + \mathcal{O}\left((\Delta x)^2\right)$$

▶Filter non-linear term in Eq. 36.1

$$\overline{\frac{\partial v_i v_j}{\partial x_j}} = \frac{\partial}{\partial x_j}(\overline{v_i v_j}) + \mathcal{O}\left((\Delta x)^2\right)$$

$$\text{Left side}: \quad \frac{\partial}{\partial x_j}(\overline{v_i v_j}) \underbrace{- \frac{\partial}{\partial x_j}(\overline{v_i v_j}) + \frac{\partial}{\partial x_j}(\bar{v}_i \bar{v}_j)} = \frac{\partial}{\partial x_j}(\bar{v}_i \bar{v}_j)$$

$$\text{Right side}: \quad \underbrace{-\frac{\partial}{\partial x_j}(\overline{v_i v_j}) + \frac{\partial}{\partial x_j}(\bar{v}_i \bar{v}_j)} = -\frac{\partial \tau_{ij}}{\partial x_j}$$

Momentum equations in LES:

$$\frac{\partial \bar{v}_i}{\partial t} + \frac{\partial}{\partial x_j}(\bar{v}_i \bar{v}_j) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{v}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j}, \qquad \tau_{ij} = \overline{v_i v_j} - \bar{v}_i \bar{v}_j \tag{36.3}$$

▶Filter pressure gradient in Eq. 36.1

$$\overline{\frac{\partial p}{\partial x_i}} = \frac{1}{V}\int_V \frac{\partial p}{\partial x_i} dV \overset{?}{=} \frac{\partial}{\partial x_i}\left(\frac{1}{V}\int_V p dV\right) = \frac{\partial \bar{p}}{\partial x_i}$$

$$\overline{\frac{\partial p}{\partial x_i}} = \frac{\partial}{\partial x_i}\left(\frac{1}{V}\int_V p dV\right) + \mathcal{O}\left((\Delta x)^2\right) = \frac{\partial \bar{p}}{\partial x_i} + \mathcal{O}\left((\Delta x)^2\right)$$

▶Filter non-linear term in Eq. 36.1

$$\overline{\frac{\partial v_i v_j}{\partial x_j}} = \frac{\partial}{\partial x_j}(\overline{v_i v_j}) + \mathcal{O}\left((\Delta x)^2\right)$$

$$\text{Left side} : \frac{\partial}{\partial x_j}(\overline{v_i v_j}) \underbrace{- \frac{\partial}{\partial x_j}(\overline{v_i v_j}) + \frac{\partial}{\partial x_j}(\bar{v}_i \bar{v}_j)} = \frac{\partial}{\partial x_j}(\bar{v}_i \bar{v}_j)$$

$$\text{Right side} : \underbrace{- \frac{\partial}{\partial x_j}(\overline{v_i v_j}) + \frac{\partial}{\partial x_j}(\bar{v}_i \bar{v}_j)} = -\frac{\partial \tau_{ij}}{\partial x_j}$$

▶ $\dfrac{\partial \bar{v}_i}{\partial t} + \dfrac{\partial}{\partial x_j}(\bar{v}_i \bar{v}_j) =$

Momentum equations in LES:

$$\frac{\partial \bar{v}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{v}_i \bar{v}_j\right) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{v}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j}, \qquad \tau_{ij} = \overline{v_i v_j} - \bar{v}_i \bar{v}_j \tag{36.3}$$

▶Filter pressure gradient in Eq. 36.1

$$\overline{\frac{\partial p}{\partial x_i}} = \frac{1}{V}\int_V \frac{\partial p}{\partial x_i} dV \stackrel{?}{=} \frac{\partial}{\partial x_i}\left(\frac{1}{V}\int_V p dV\right) = \frac{\partial \bar{p}}{\partial x_i}$$

$$\overline{\frac{\partial p}{\partial x_i}} = \frac{\partial}{\partial x_i}\left(\frac{1}{V}\int_V p dV\right) + \mathcal{O}\left((\Delta x)^2\right) = \frac{\partial \bar{p}}{\partial x_i} + \mathcal{O}\left((\Delta x)^2\right)$$

▶Filter non-linear term in Eq. 36.1

$$\overline{\frac{\partial v_i v_j}{\partial x_j}} = \frac{\partial}{\partial x_j}\left(\overline{v_i v_j}\right) + \mathcal{O}\left((\Delta x)^2\right)$$

$$\text{Left side}: \frac{\partial}{\partial x_j}\left(\overline{v_i v_j}\right) \underbrace{- \frac{\partial}{\partial x_j}\left(\overline{v_i v_j}\right) + \frac{\partial}{\partial x_j}\left(\bar{v}_i \bar{v}_j\right)} = \frac{\partial}{\partial x_j}\left(\bar{v}_i \bar{v}_j\right)$$

$$\text{Right side}: \underbrace{-\frac{\partial}{\partial x_j}\left(\overline{v_i v_j}\right) + \frac{\partial}{\partial x_j}\left(\bar{v}_i \bar{v}_j\right)} = -\frac{\partial \tau_{ij}}{\partial x_j}$$

▶ $$\frac{\partial \bar{v}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{v}_i \bar{v}_j\right) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{v}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j}$$

Momentum equations in LES:

$$\frac{\partial \bar{v}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{v}_i \bar{v}_j\right) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{v}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j}, \qquad \tau_{ij} = \overline{v_i v_j} - \bar{v}_i \bar{v}_j \tag{36.3}$$

▶Filter pressure gradient in Eq. 36.1

$$\overline{\frac{\partial p}{\partial x_i}} = \frac{1}{V}\int_V \frac{\partial p}{\partial x_i} dV = \frac{\partial}{\partial x_i}\left(\frac{1}{V}\int_V p dV\right) = \frac{\partial \bar{p}}{\partial x_i}$$

$$\overline{\frac{\partial p}{\partial x_i}} = \frac{\partial}{\partial x_i}\left(\frac{1}{V}\int_V p dV\right) + \mathcal{O}\left((\Delta x)^2\right) = \frac{\partial \bar{p}}{\partial x_i} + \mathcal{O}\left((\Delta x)^2\right)$$

▶Filter non-linear term in Eq. 36.1

$$\overline{\frac{\partial v_i v_j}{\partial x_j}} = \frac{\partial}{\partial x_j}(\overline{v_i v_j}) + \mathcal{O}\left((\Delta x)^2\right)$$

$$\text{Left side} : \frac{\partial}{\partial x_j}(\overline{v_i v_j}) \underbrace{- \frac{\partial}{\partial x_j}(\overline{v_i v_j}) + \frac{\partial}{\partial x_j}(\bar{v}_i \bar{v}_j)} = \frac{\partial}{\partial x_j}(\bar{v}_i \bar{v}_j)$$

$$\text{Right side} : \underbrace{- \frac{\partial}{\partial x_j}(\overline{v_i v_j}) + \frac{\partial}{\partial x_j}(\bar{v}_i \bar{v}_j)} = - \frac{\partial \tau_{ij}}{\partial x_j}$$

▶ $$\frac{\partial \bar{v}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{v}_i \bar{v}_j\right) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{v}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j}, \qquad \tau_{ij} = \overline{v_i v_j} - \bar{v}_i \bar{v}_j$$

Momentum equations in LES:

$$\frac{\partial \bar{v}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{v}_i \bar{v}_j\right) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{v}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j}, \qquad \tau_{ij} = \overline{v_i v_j} - \bar{v}_i \bar{v}_j \tag{36.3}$$

▶Filter pressure gradient in Eq. 36.1

$$\overline{\frac{\partial p}{\partial x_i}} = \frac{1}{V}\int_V \frac{\partial p}{\partial x_i} dV \overset{?}{=} \frac{\partial}{\partial x_i}\left(\frac{1}{V}\int_V p dV\right) = \frac{\partial \bar{p}}{\partial x_i}$$

$$\overline{\frac{\partial p}{\partial x_i}} = \frac{\partial}{\partial x_i}\left(\frac{1}{V}\int_V p dV\right) + \mathcal{O}\left((\Delta x)^2\right) = \frac{\partial \bar{p}}{\partial x_i} + \mathcal{O}\left((\Delta x)^2\right)$$

▶Filter non-linear term in Eq. 36.1

$$\overline{\frac{\partial v_i v_j}{\partial x_j}} = \frac{\partial}{\partial x_j}(\overline{v_i v_j}) + \mathcal{O}\left((\Delta x)^2\right)$$

$$\text{Left side} : \frac{\partial}{\partial x_j}(\overline{v_i v_j}) \underbrace{- \frac{\partial}{\partial x_j}(\overline{v_i v_j}) + \frac{\partial}{\partial x_j}(\bar{v}_i \bar{v}_j)} = \frac{\partial}{\partial x_j}(\bar{v}_i \bar{v}_j)$$

$$\text{Right side} : \underbrace{-\frac{\partial}{\partial x_j}(\overline{v_i v_j}) + \frac{\partial}{\partial x_j}(\bar{v}_i \bar{v}_j)} = -\frac{\partial \tau_{ij}}{\partial x_j}$$

▶ $$\frac{\partial \bar{v}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{v}_i \bar{v}_j\right) = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x_i} + \nu \frac{\partial^2 \bar{v}_i}{\partial x_j \partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j}, \qquad \tau_{ij} = \overline{v_i v_j} - \bar{v}_i \bar{v}_j$$

▶Filtering twice (used for turbulence modeling)

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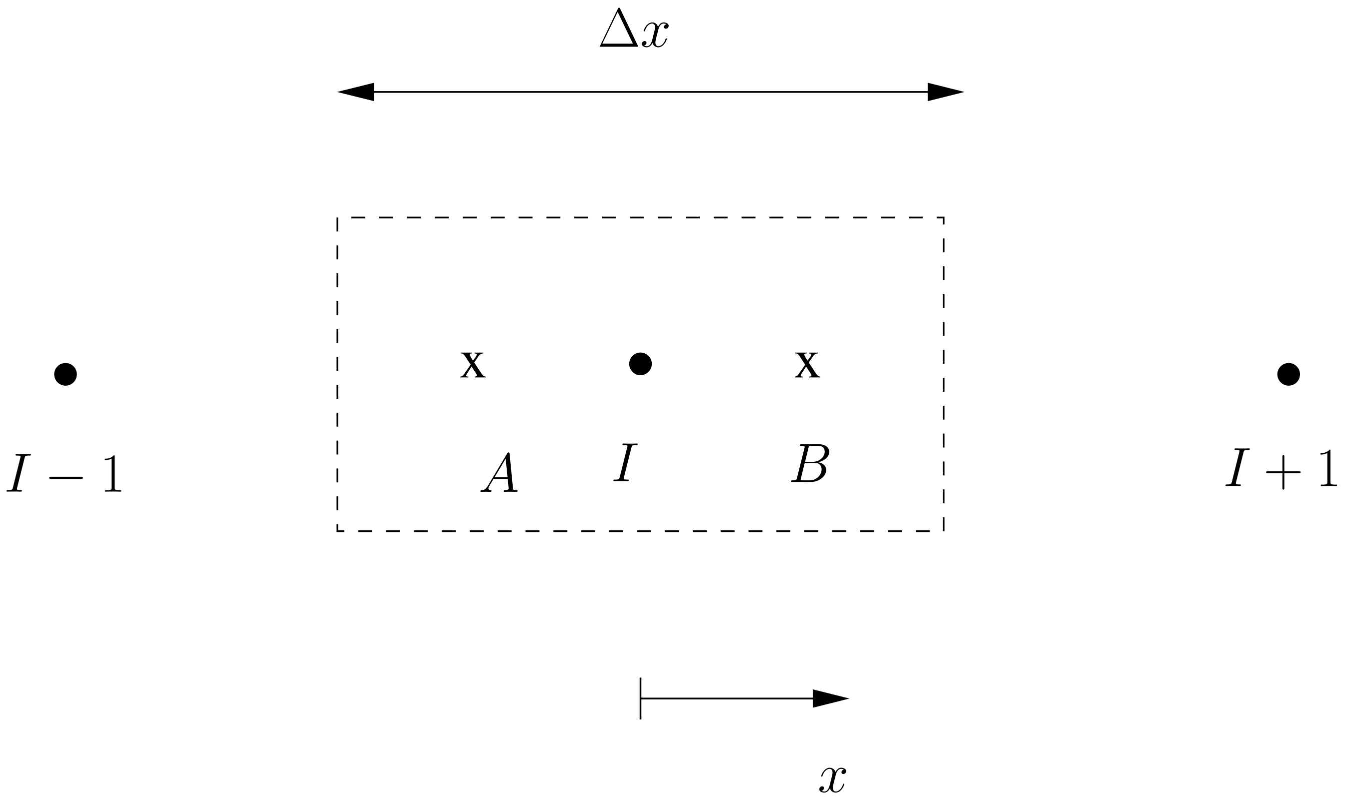
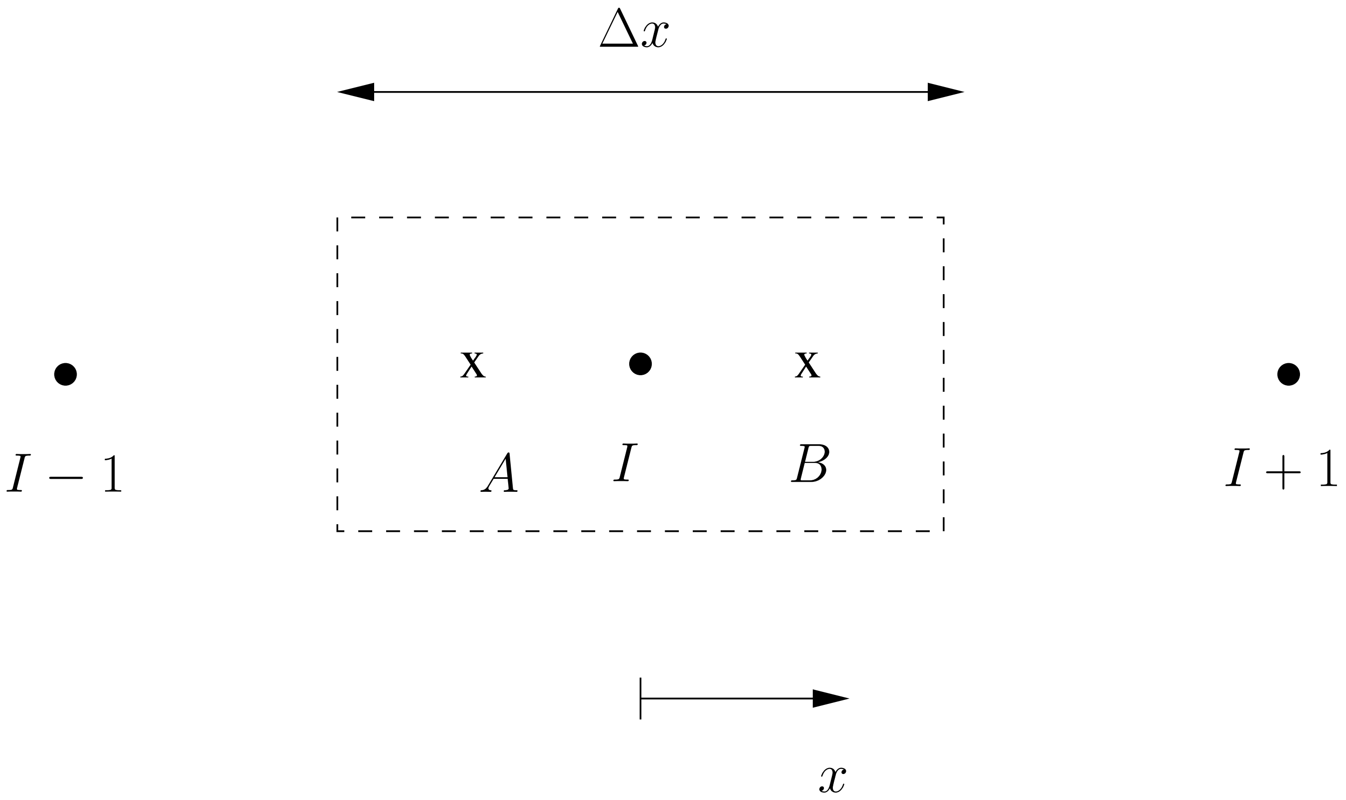


Control volume (dashed lines). $\overline{\overline{v}}_I \neq \bar{v}_I$

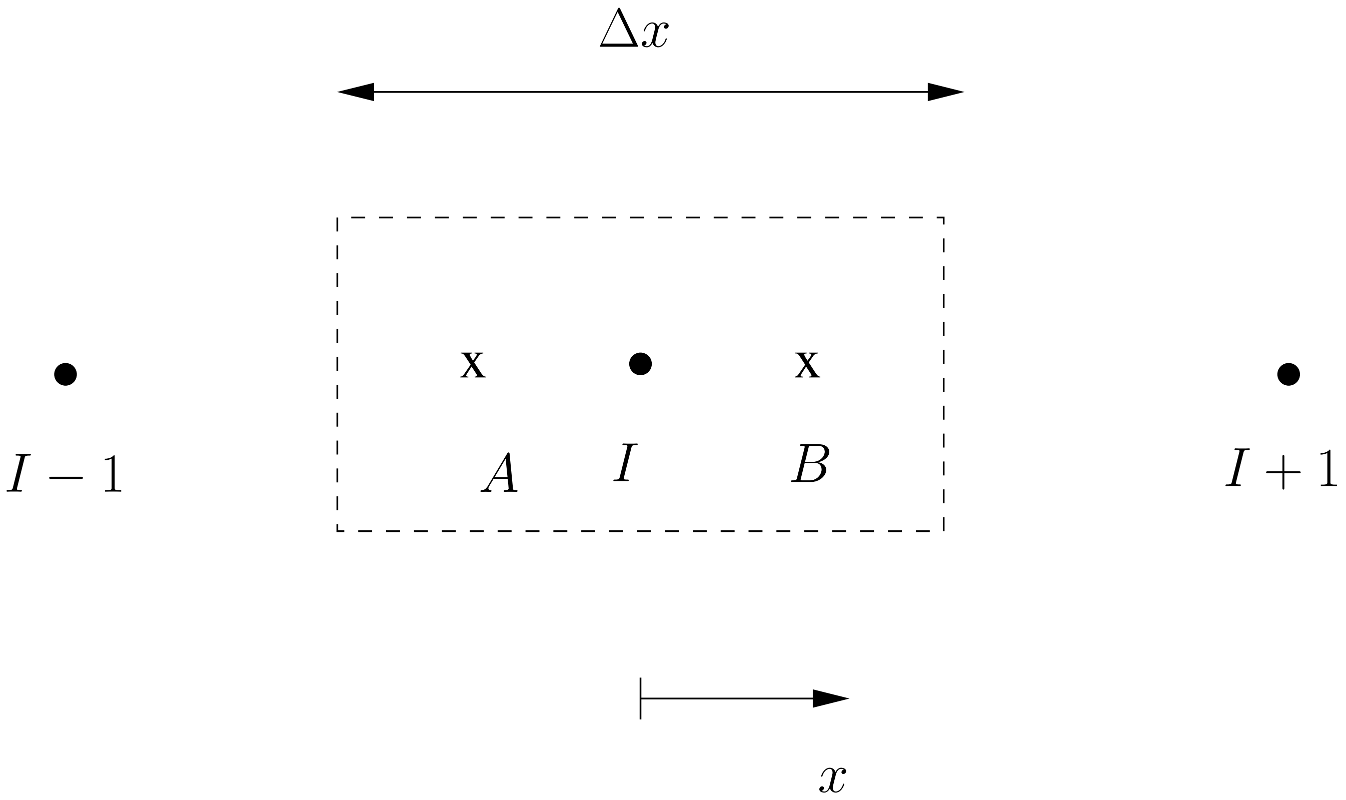
▶Filtering twice (used for turbulence modeling)



Control volume (dashed lines). $\bar{\bar{v}}_I \neq \bar{v}_I$

$$\bar{\bar{v}}_I =$$

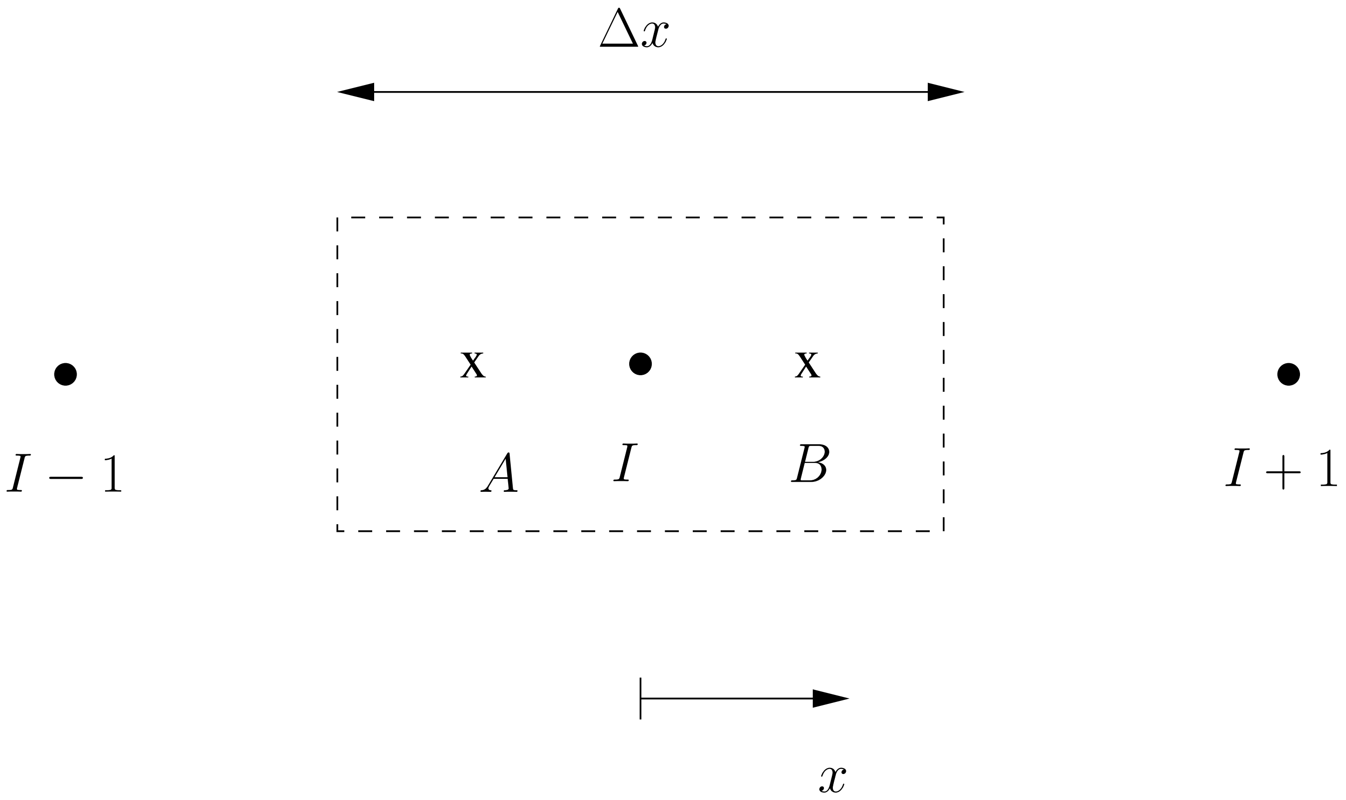
▶Filtering twice (used for turbulence modeling)



Control volume (dashed lines). $\bar{\bar{v}}_I \neq \bar{v}_I$

$$\bar{\bar{v}}_I = \frac{1}{\Delta x}\int_{-\Delta x/2}^{\Delta x/2} \bar{v}(\xi) d\xi =$$

▶Filtering twice (used for turbulence modeling)

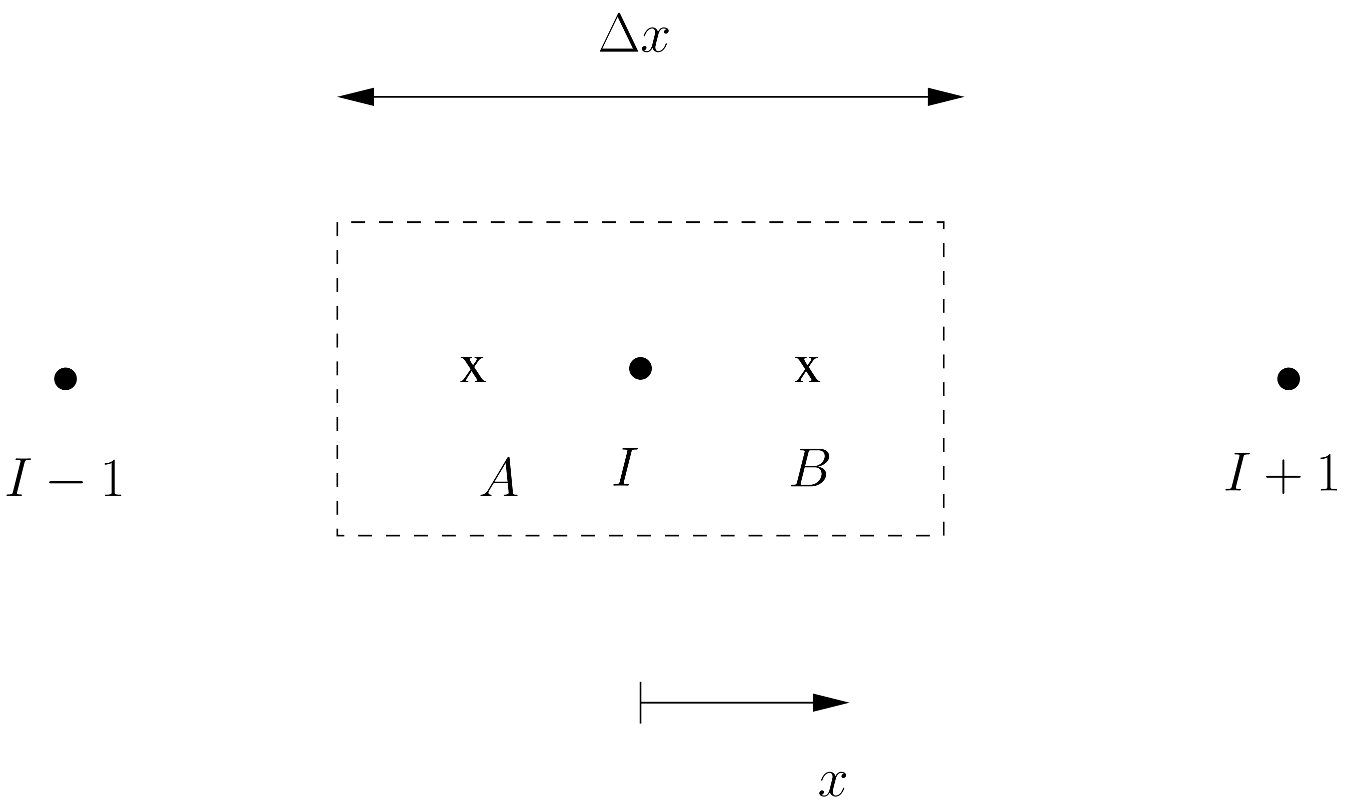


Control volume (dashed lines). $\bar{\bar{v}}_I \neq \bar{v}_I$

$$\bar{\bar{v}}_I = \frac{1}{\Delta x}\int_{-\Delta x/2}^{\Delta x/2} \bar{v}(\xi)d\xi = \frac{1}{\Delta x}\left(\int_{-\Delta x/2}^{0} \bar{v}(\xi)d\xi + \int_{0}^{\Delta x/2} \bar{v}(\xi)d\xi\right) =$$

$$=$$

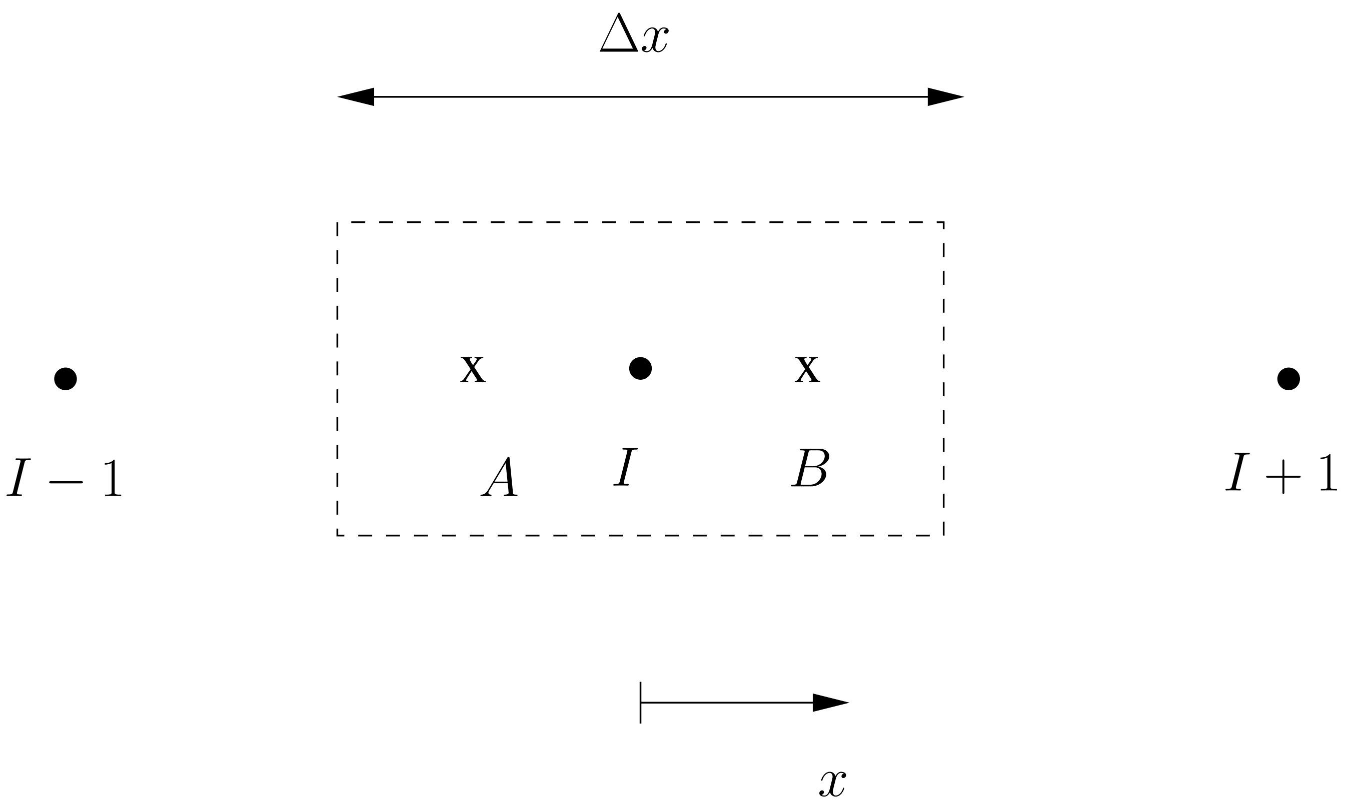
▶Filtering twice (used for turbulence modeling)



Control volume (dashed lines). $\overline{\overline{v}}_I \neq \bar{v}_I$

$$
\begin{aligned}
\overline{\overline{v}}_I &= \frac{1}{\Delta x}\int_{-\Delta x/2}^{\Delta x/2} \bar{v}(\xi)d\xi = \frac{1}{\Delta x}\left(\int_{-\Delta x/2}^{0} \bar{v}(\xi)d\xi + \int_{0}^{\Delta x/2} \bar{v}(\xi)d\xi\right) = \\
&= \frac{1}{\Delta x}\left(\frac{\Delta x}{2}\bar{v}_A + \frac{\Delta x}{2}\bar{v}_B\right) =
\end{aligned}
$$

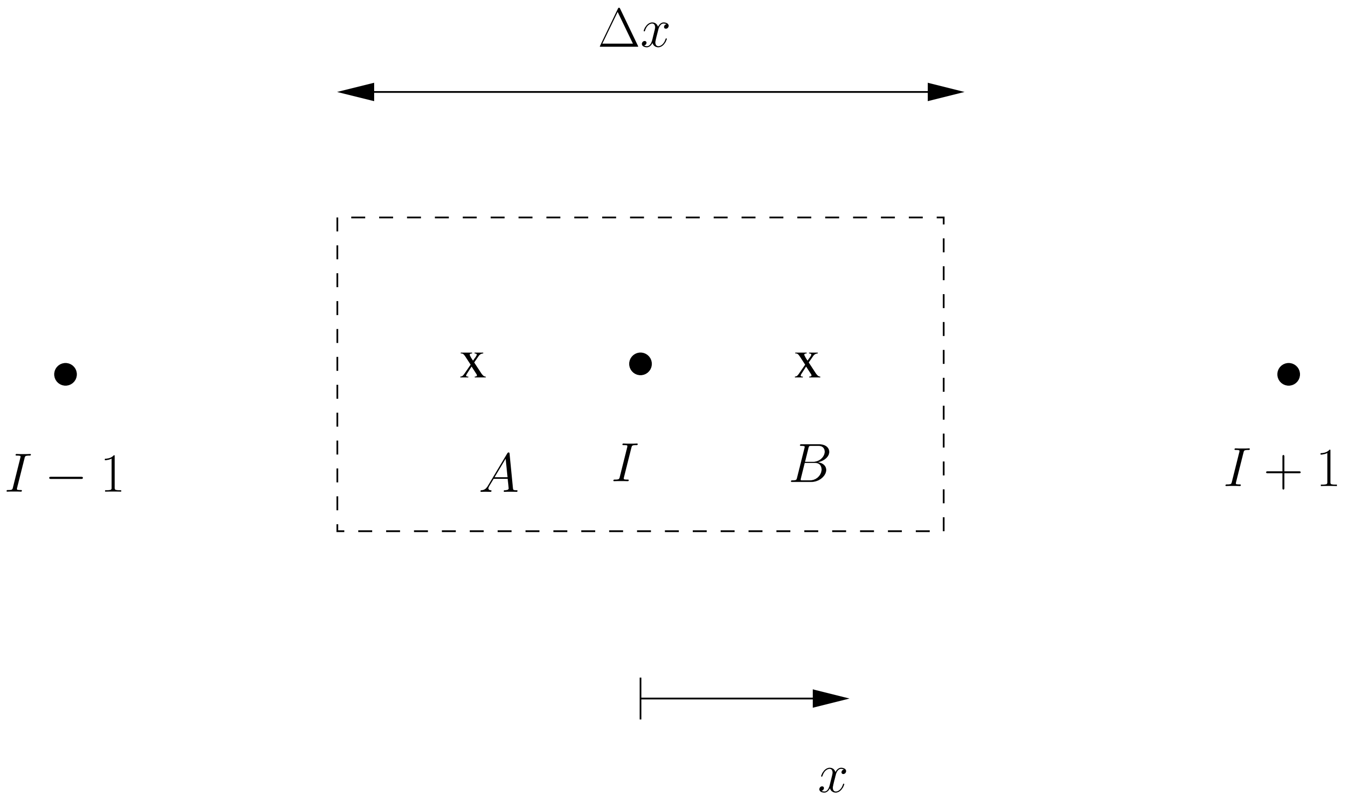
►Filtering twice (used for turbulence modeling)



Control volume (dashed lines). $\bar{\bar{v}}_I \neq \bar{v}_I$

$$
\begin{aligned}
\bar{\bar{v}}_I &= \frac{1}{\Delta x}\int_{-\Delta x/2}^{\Delta x/2} \bar{v}(\xi)d\xi = \frac{1}{\Delta x}\left(\int_{-\Delta x/2}^{0} \bar{v}(\xi)d\xi + \int_{0}^{\Delta x/2} \bar{v}(\xi)d\xi\right) = \\
&= \frac{1}{\Delta x}\left(\frac{\Delta x}{2}\bar{v}_A + \frac{\Delta x}{2}\bar{v}_B\right) = \frac{1}{2}\left[\left(\frac{1}{4}\bar{v}_{I-1} + \frac{3}{4}\bar{v}_I\right) + \left(\frac{3}{4}\bar{v}_I + \frac{1}{4}\bar{v}_{I+1}\right)\right] =
\end{aligned}
$$

▶Filtering twice (used for turbulence modeling)

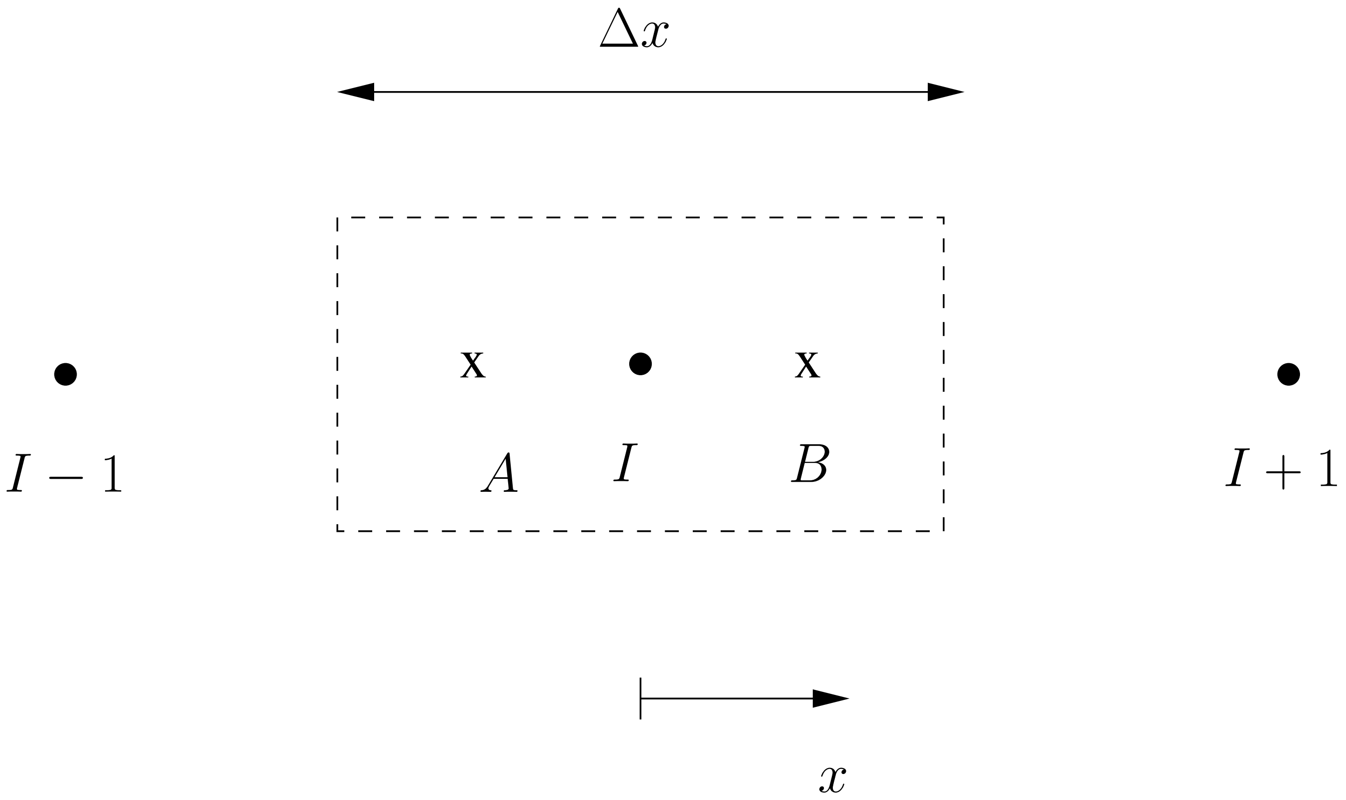


Control volume (dashed lines). $\bar{\bar{v}}_I \neq \bar{v}_I$

$$\bar{\bar{v}}_I = \frac{1}{\Delta x}\int_{-\Delta x/2}^{\Delta x/2} \bar{v}(\xi)d\xi = \frac{1}{\Delta x}\left(\int_{-\Delta x/2}^{0} \bar{v}(\xi)d\xi + \int_{0}^{\Delta x/2} \bar{v}(\xi)d\xi\right) =$$

$$= \frac{1}{\Delta x}\left(\frac{\Delta x}{2}\bar{v}_A + \frac{\Delta x}{2}\bar{v}_B\right) = \frac{1}{2}\left[\left(\frac{1}{4}\bar{v}_{I-1} + \frac{3}{4}\bar{v}_I\right) + \left(\frac{3}{4}\bar{v}_I + \frac{1}{4}\bar{v}_{I+1}\right)\right] = \frac{1}{8}\left(\bar{v}_{I-1} + 6\bar{v}_I + \bar{v}_{I+1}\right)$$

▶Filtering twice (used for turbulence modeling)



Control volume (dashed lines). $\overline{\overline{v}}_I \neq \bar{v}_I$

$$\overline{\overline{v}}_I = \frac{1}{\Delta x}\int_{-\Delta x/2}^{\Delta x/2} \bar{v}(\xi)d\xi = \frac{1}{\Delta x}\left(\int_{-\Delta x/2}^{0} \bar{v}(\xi)d\xi + \int_{0}^{\Delta x/2} \bar{v}(\xi)d\xi\right) =$$

$$= \frac{1}{\Delta x}\left(\frac{\Delta x}{2}\bar{v}_A + \frac{\Delta x}{2}\bar{v}_B\right) = \frac{1}{2}\left[\left(\frac{1}{4}\bar{v}_{I-1} + \frac{3}{4}\bar{v}_I\right) + \left(\frac{3}{4}\bar{v}_I + \frac{1}{4}\bar{v}_{I+1}\right)\right] = \frac{1}{8}\left(\bar{v}_{I-1} + 6\bar{v}_I + \bar{v}_{I+1}\right) \neq \bar{v}_I$$

¶See Section 18.3, Resolved & SGS scales (GS & SGS)

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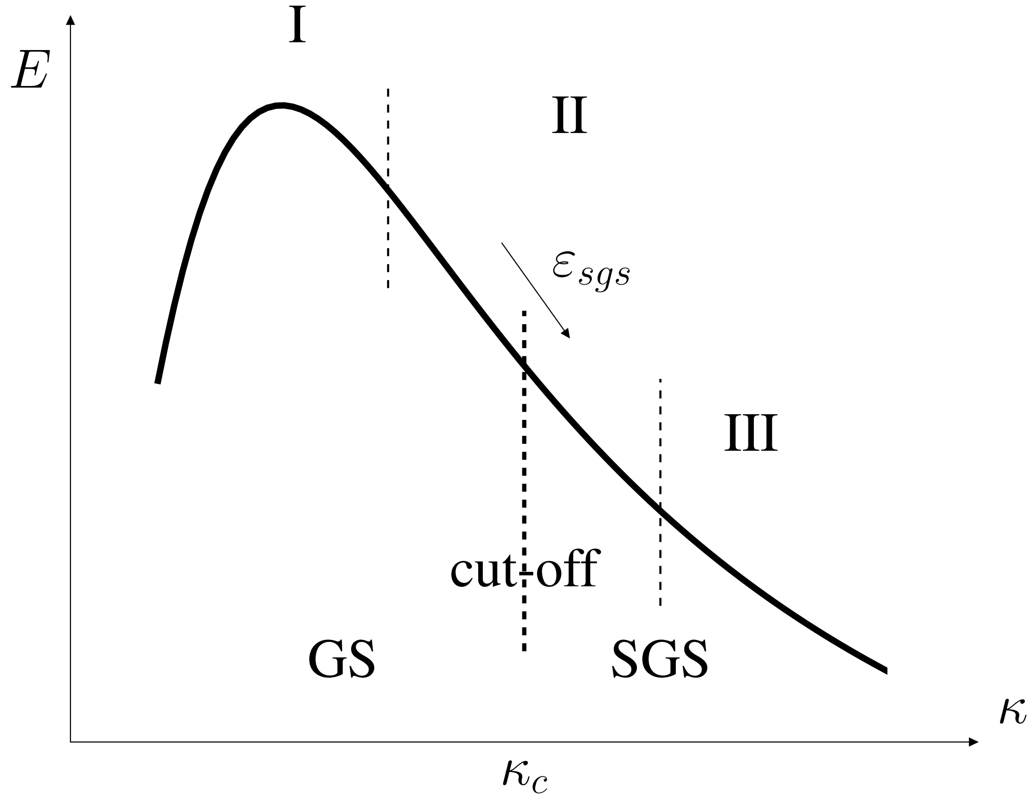
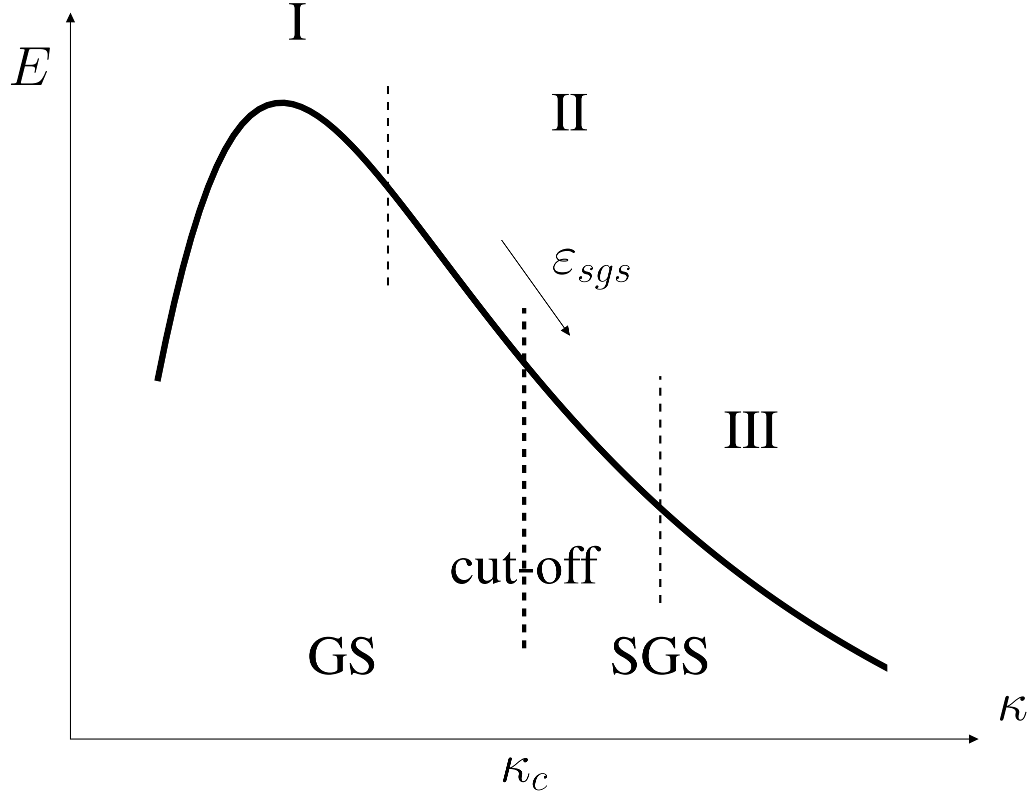


$\kappa \leq \kappa_c$: Grid (=resolved) Scales; $\kappa > \kappa_c$= Sub-Grid Scales

¶See Section 18.3, Resolved & SGS scales (GS & SGS)



$\kappa \leq \kappa_c$: Grid (=resolved) Scales; $\kappa > \kappa_c$= Sub-Grid Scales

¶See Section 18.5, Highest resolved wavenumbers

►A Fourier series (see Appendix H)

¶See Section 18.5, Highest resolved wavenumbers

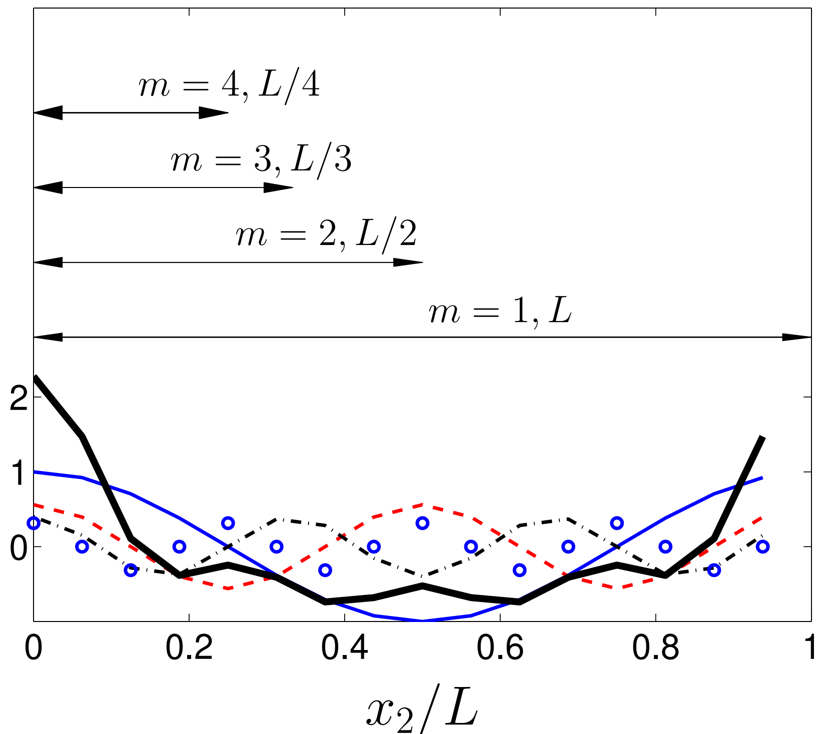
►A Fourier series (see Appendix H)

$$v_1'(x) = \sum_{n=-\infty}^{\infty} c_n \exp(\imath \kappa_n x_1))$$

¶See Section 18.5, Highest resolved wavenumbers

▶A Fourier series (see Appendix H)

$$v_1'(x) = \sum_{n=-\infty}^{\infty} c_n \exp(\imath \kappa_n x_1)) \quad \text{only} \quad \text{symmetric} \quad \text{part,} \quad \text{i.e.} \quad \text{real}$$

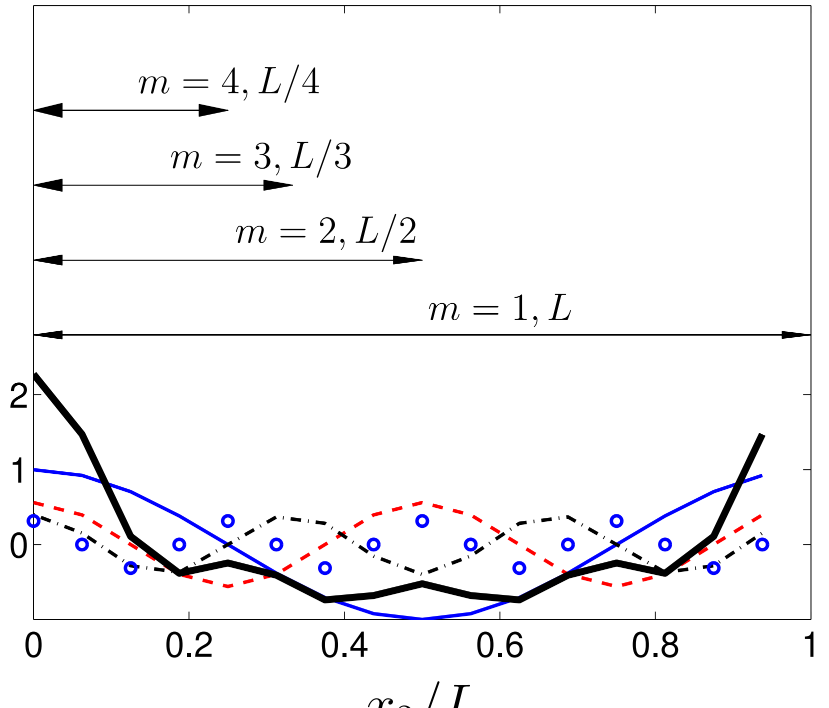


v_2' vs. x_2/L. —: term 1 ($m = 1$); - - -: term 2 ($m = 2$); -·-·-: term 3 ($m = 3$); ○: term 4 ($m = 4$); ▬: v_2'
Matlab code is given in Section I.3.

¶See Section 18.5, Highest resolved wavenumbers

►A Fourier series (see Appendix H)

$$v_1'(x) = \sum_{n=-\infty}^{\infty} c_n \exp(\imath \kappa_n x_1)) \quad \text{only symmetric part, i.e. real}$$



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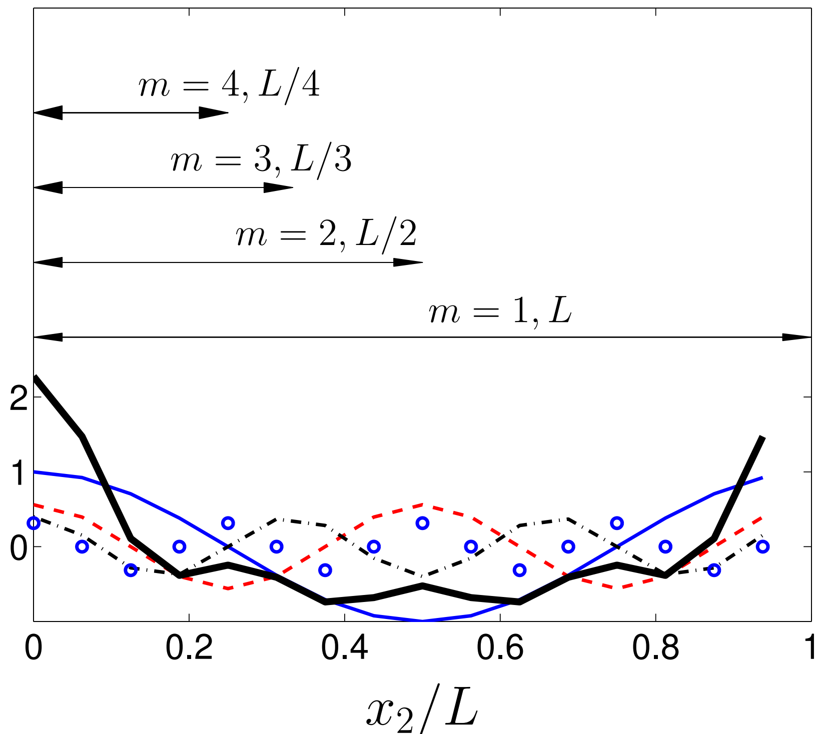
Matlab code is given in Section I.3.

►We construct v_2' as a sum of four Fourier components

¶See Section 18.5, Highest resolved wavenumbers

▶A Fourier series (see Appendix H)

$$v_1'(x) = \sum_{n=-\infty}^{\infty} c_n \exp(\imath \kappa_n x_1)) \quad \text{only symmetric part, i.e. real}$$



v_2' vs. x_2/L. —: term 1 ($m=1$); - -: term 2 ($m=2$); -·-: term 3 ($m=3$); ○: term 4 ($m=4$); ━: v_2'
Matlab code is given in Section I.3.

▶We construct v_2' as a sum of four Fourier components

$$v_2'(x_2) = b_1 \cos\left(\frac{2\pi}{L/1}x_2\right) + b_2 \cos\left(\frac{2\pi}{L/2}x_2\right) + b_3 \cos\left(\frac{2\pi}{L/3}x_2\right) + b_4 \cos\left(\frac{2\pi}{L/4}x_2\right)$$

- On the previous slide, we showed a couple of different wave number (Fourier) modes.

$$v'_1 = \sin(\kappa_c x_1)$$

- How large wave numbers (i.e. how short wavelengths) can we resolve in an LES?

▶ On the previous slide, we showed a couple of different wave number (Fourier) modes.

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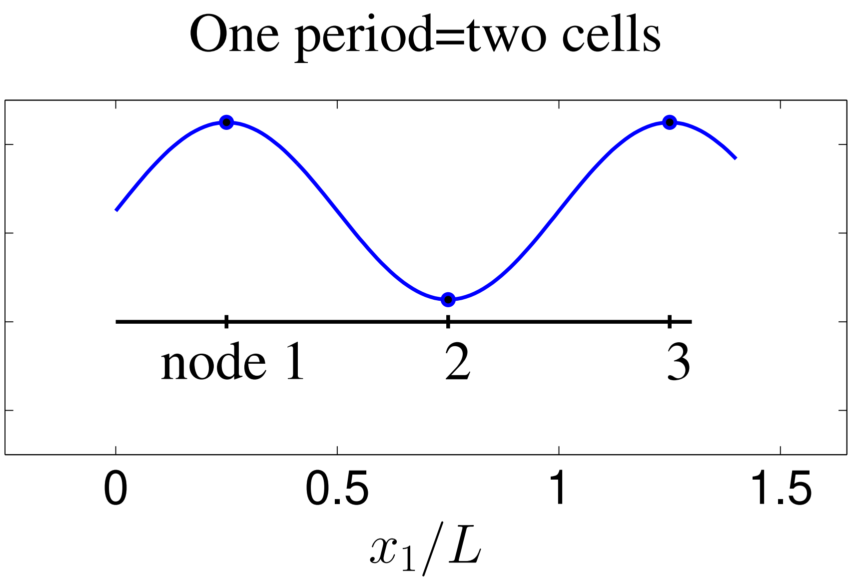

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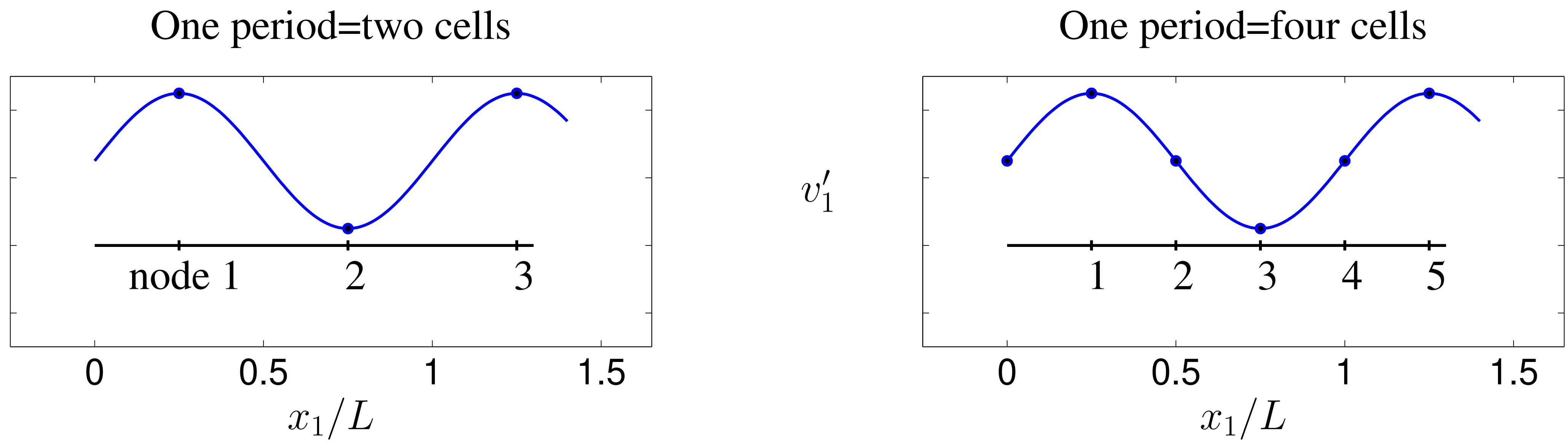


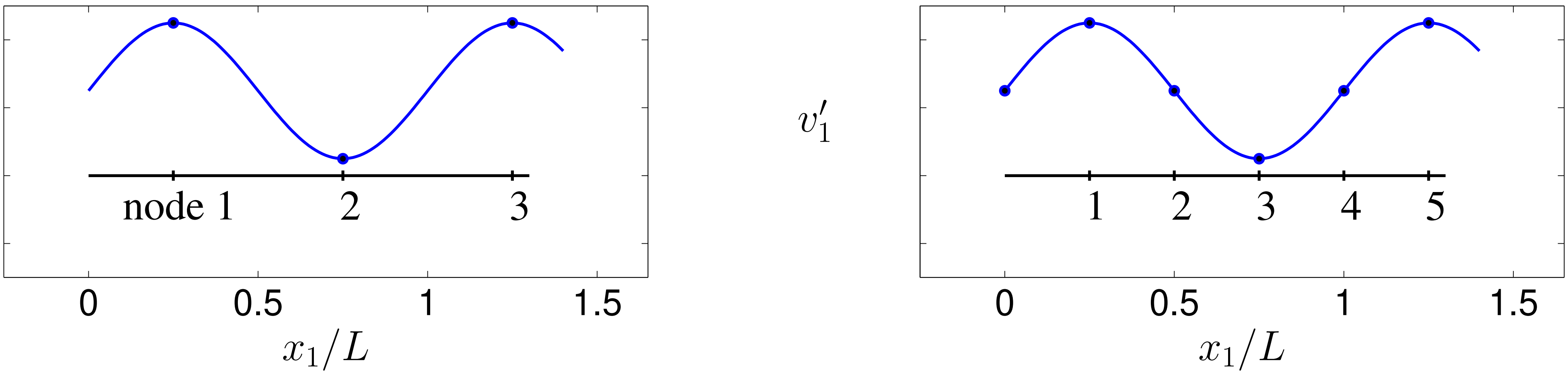
► On the previous slide, we showed a couple of different wave number (Fourier) modes.

$$v_1' = \sin(\kappa_c x_1)$$

► How large wave numbers (i.e. how short wavelengths) can we resolve in an LES?

One period=two cells

One period=four cells



two cells :

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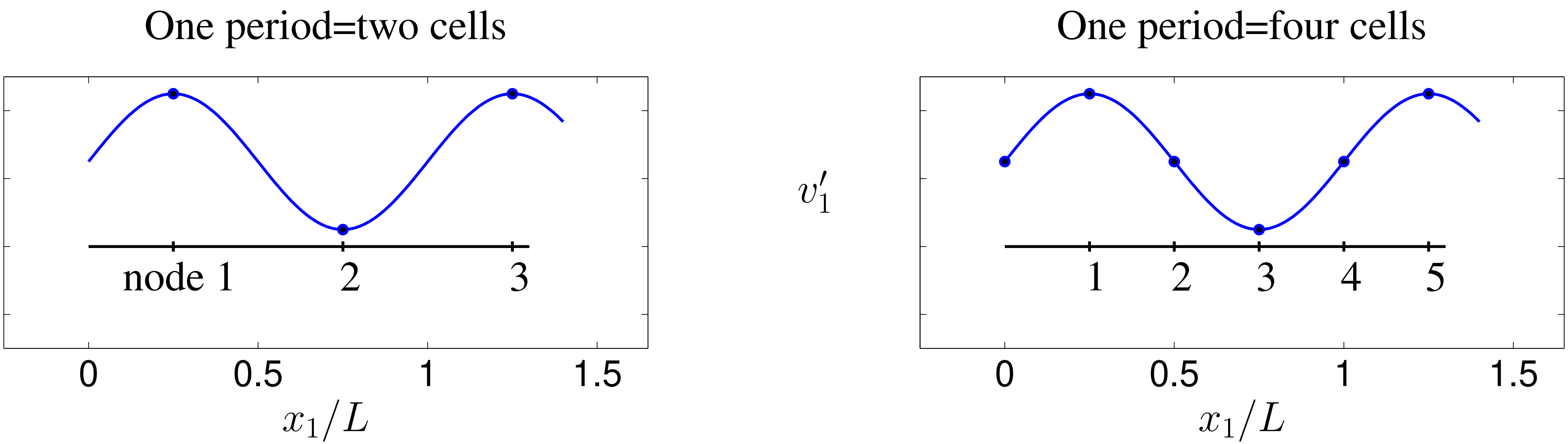

$$\text{two cells} \; : \; \kappa_c 2\Delta x_1 =$$

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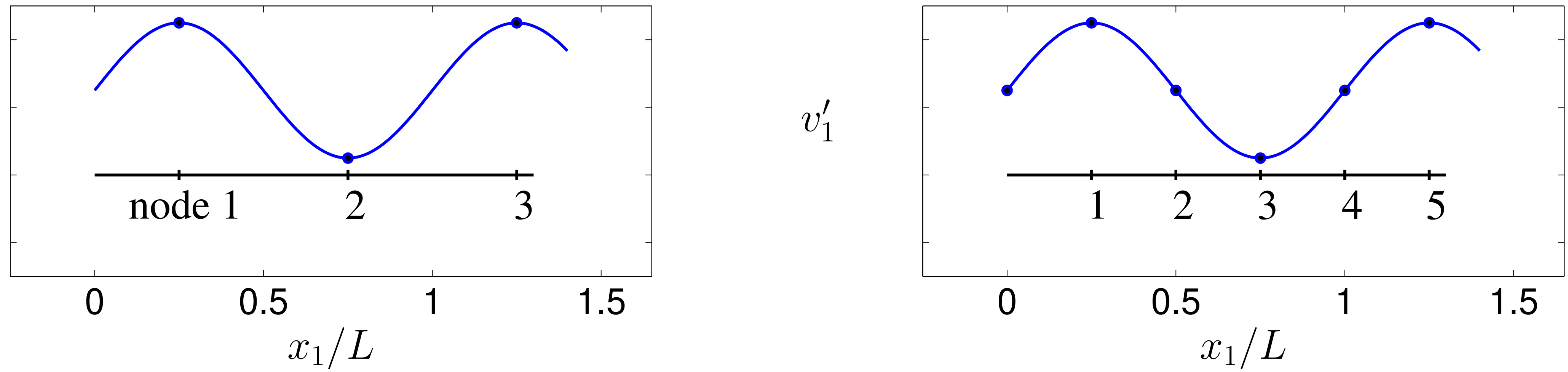

two cells : $\kappa_c 2\Delta x_1 = 2\pi$

- On the previous slide, we showed a couple of different wave number (Fourier) modes.

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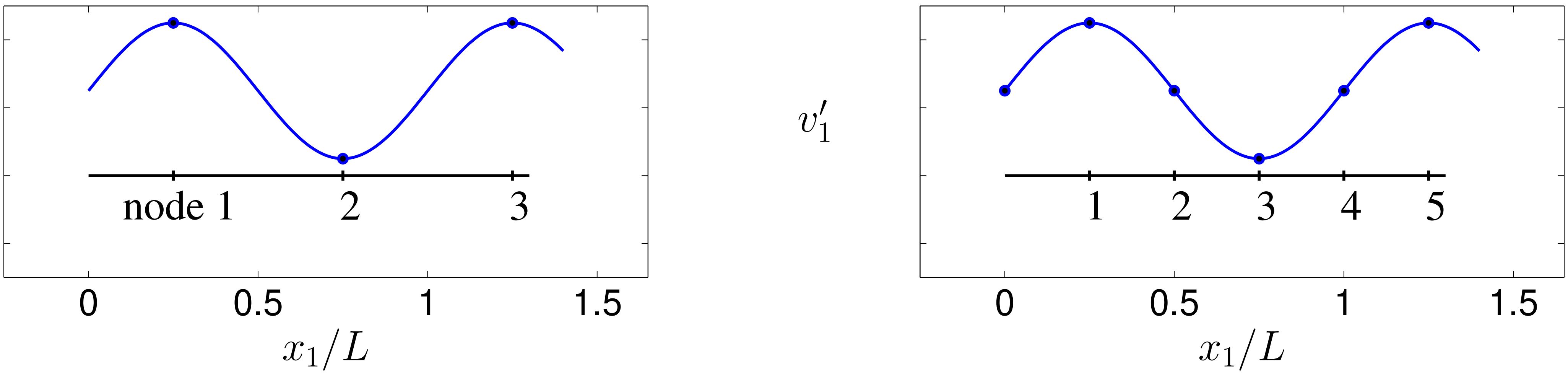
- How large wave numbers (i.e. how short wavelengths) can we resolve in an LES?

One period=two cells | One period=four cells



two cells : $\kappa_c 2\Delta x_1 = 2\pi \Rightarrow \kappa_c =$

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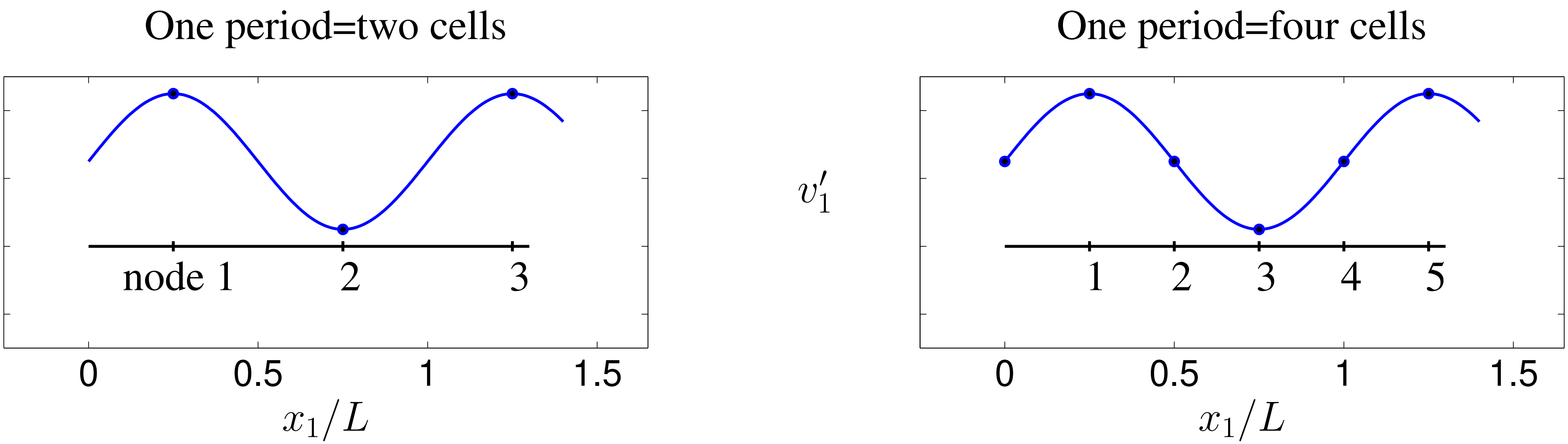

$$\text{two cells}\ :\ \kappa_c 2\Delta x_1 = 2\pi \Rightarrow \kappa_c = 2\pi/(2\Delta x_1) =$$

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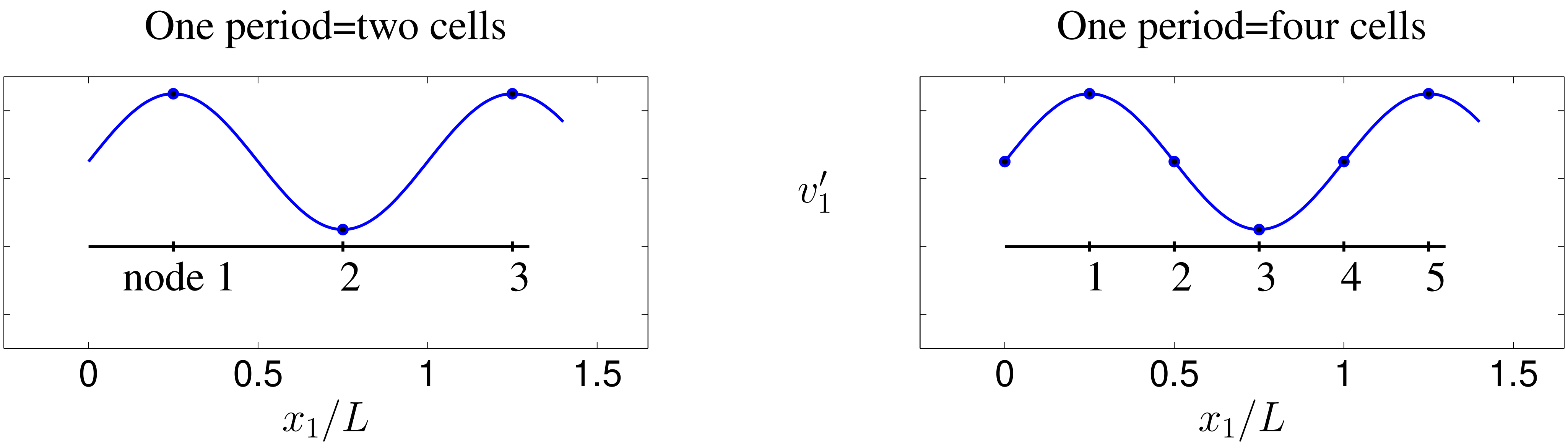

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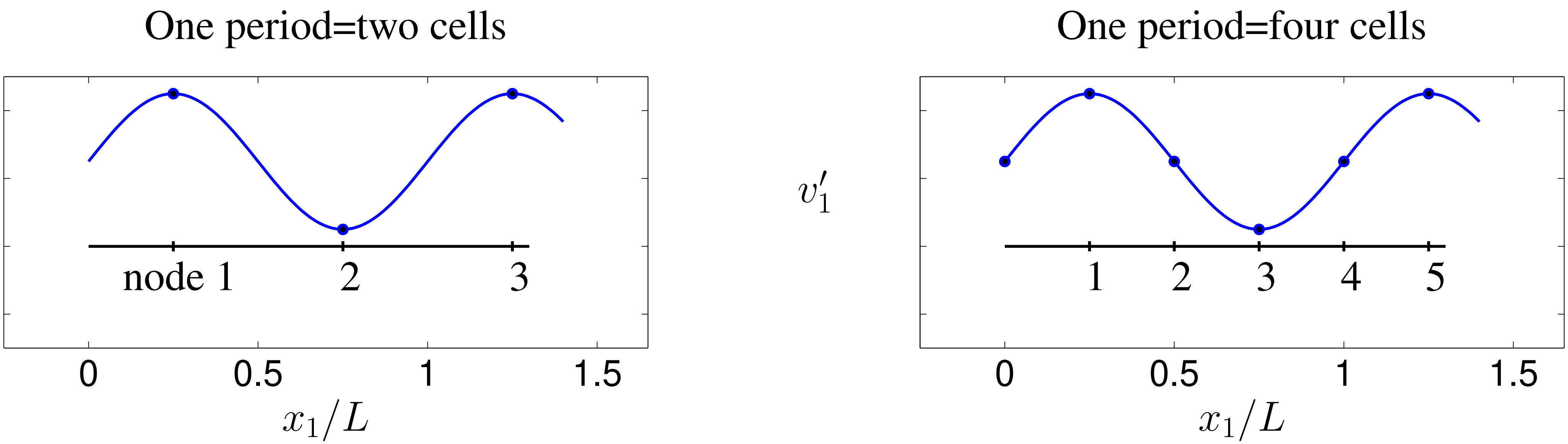

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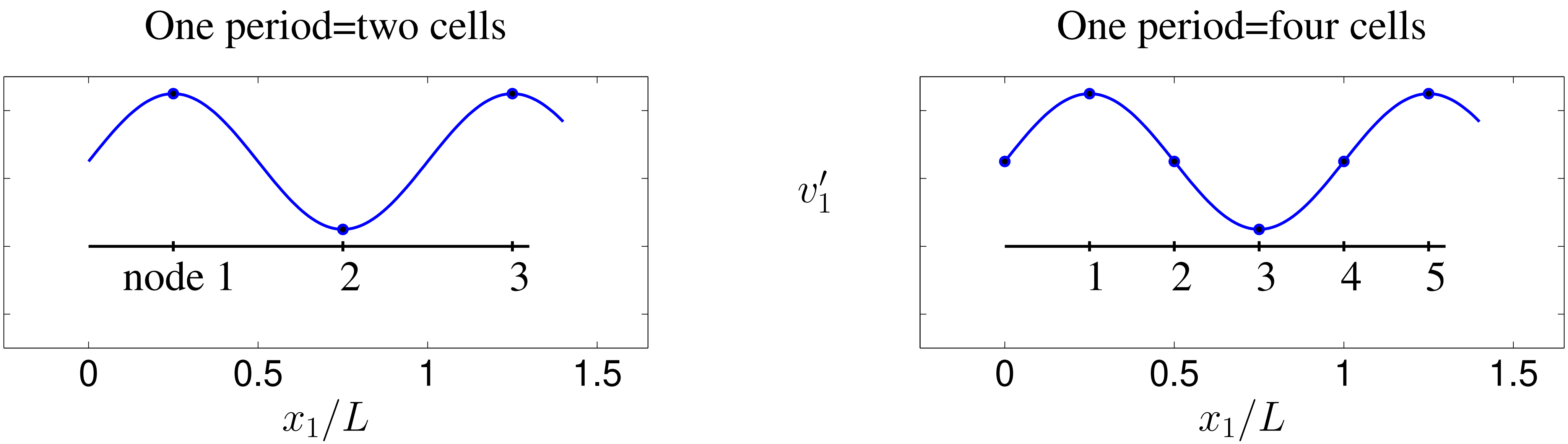

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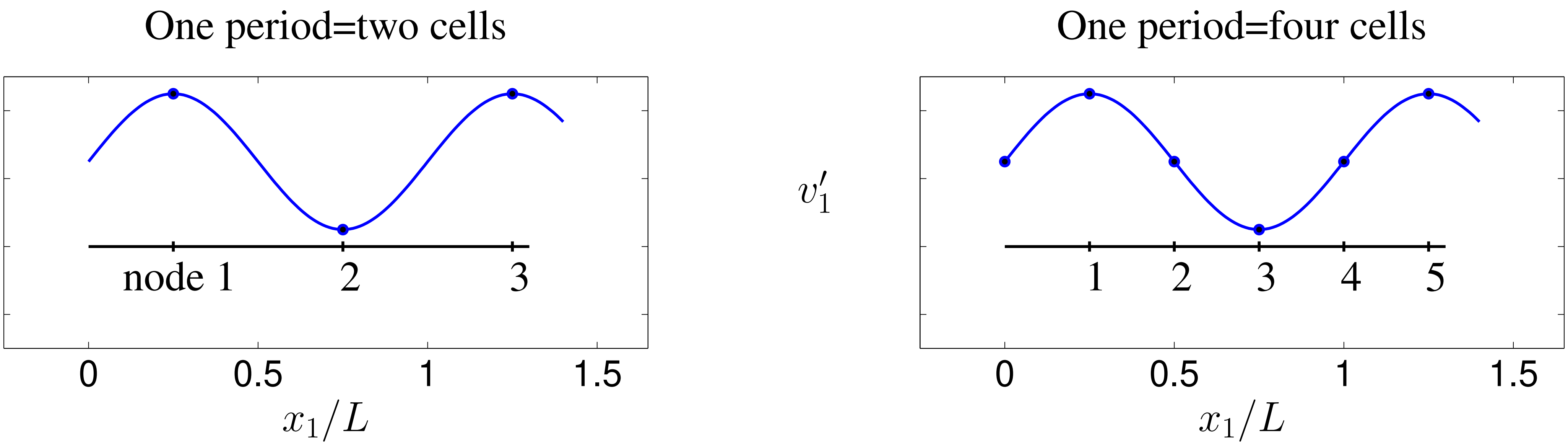

$$\text{two cells} \; : \; \kappa_c 2\Delta x_1 = 2\pi \Rightarrow \kappa_c = 2\pi/(2\Delta x_1) = \pi/\Delta x_1$$

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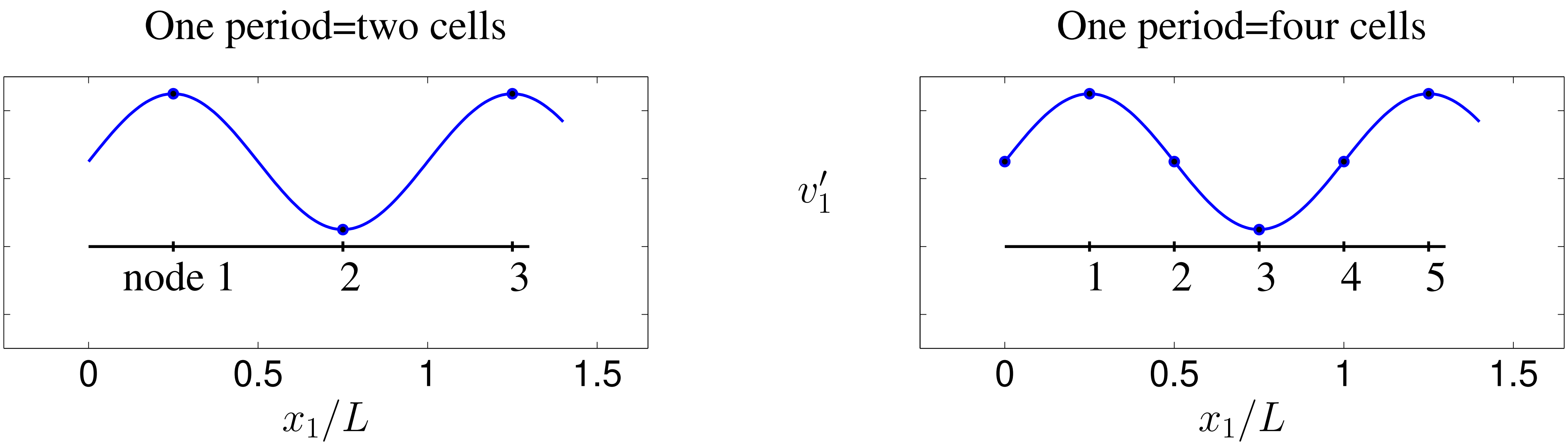

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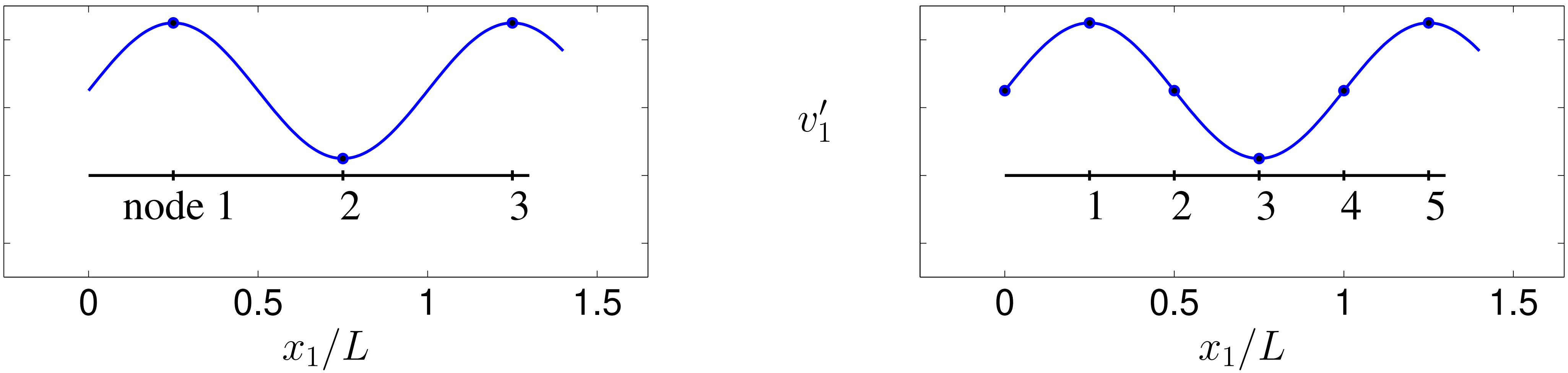

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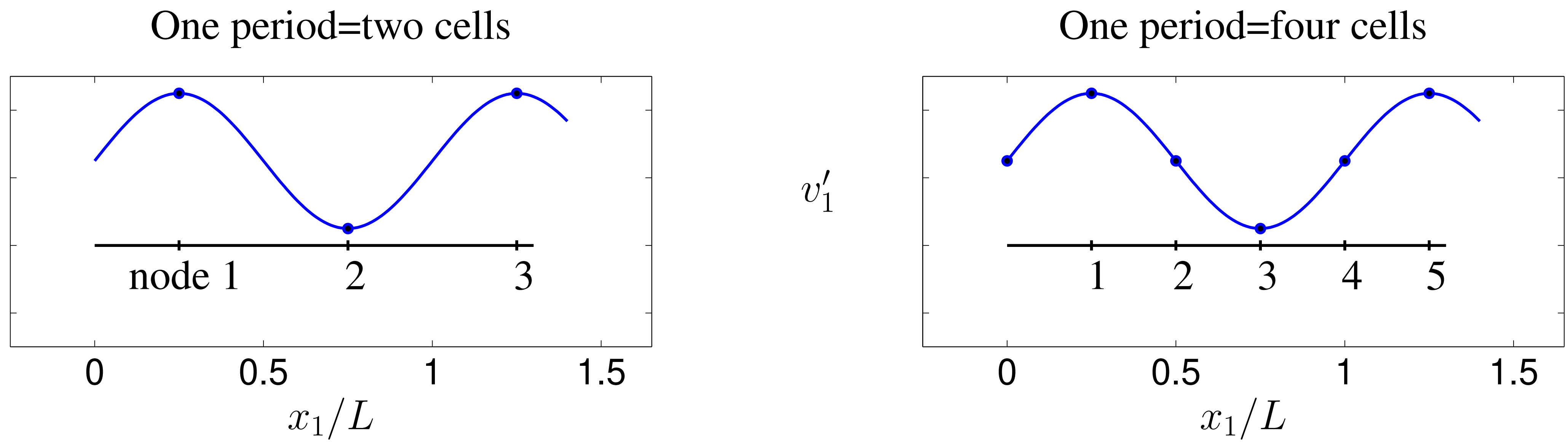

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¶See Section 18.6, Subgrid model
▶Smagorinsky Subgrid model

¶See Section 18.6, Subgrid model

►Smagorinsky Subgrid model

$$\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} =$$

¶See Section 18.6, Subgrid model

►Smagorinsky Subgrid model

$$\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} = -\nu_{sgs}\left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i}\right) =$$

¶See Section 18.6, Subgrid model

►Smagorinsky Subgrid model

$$\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} = -\nu_{sgs}\left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i}\right) = -2\nu_{sgs}\bar{s}_{ij}$$

¶See Section 18.6, Subgrid model

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$$\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} = -\nu_{sgs}\left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i}\right) = -2\nu_{sgs}\bar{s}_{ij}$$

$$\nu_{sgs} \propto$$

¶See Section 18.6, Subgrid model

▶Smagorinsky Subgrid model

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$$\nu_{sgs} \;\propto\; v'\ell \propto$$

¶See Section 18.6, Subgrid model

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$$\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} \;=\; -\nu_{sgs}\left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i}\right) = -2\nu_{sgs}\bar{s}_{ij}$$

$$\nu_{sgs} \;\propto\; v'\ell \propto \left(\Delta x_1 \frac{\partial \bar{v}_1}{\partial x_1}\right)\Delta x_1 \propto$$

¶See Section 18.6, Subgrid model

►Smagorinsky Subgrid model

$$\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} = -\nu_{sgs}\left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i}\right) = -2\nu_{sgs}\bar{s}_{ij}$$

$$\nu_{sgs} \propto v'\ell \propto \left(\Delta x_1 \frac{\partial \bar{v}_1}{\partial x_1}\right)\Delta x_1 \propto \Delta x_1^2 \left(\frac{\partial \bar{v}_1}{\partial x_1}\right) \propto$$

¶See Section 18.6, Subgrid model

►Smagorinsky Subgrid model

$$\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} = -\nu_{sgs}\left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i}\right) = -2\nu_{sgs}\bar{s}_{ij}$$

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¶See Section 18.6, Subgrid model

▶Smagorinsky Subgrid model

$$\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} = -\nu_{sgs}\left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i}\right) = -2\nu_{sgs}\bar{s}_{ij}$$

$$\nu_{sgs} \propto v'\ell \propto \left(\Delta x_1 \frac{\partial \bar{v}_1}{\partial x_1}\right)\Delta x_1 \propto \Delta x_1^2 \left(\frac{\partial \bar{v}_1}{\partial x_1}\right) \propto (C_S\Delta)^2\sqrt{2\bar{s}_{ij}\bar{s}_{ij}}$$

¶See Section 18.6, Subgrid model

►Smagorinsky Subgrid model

$$\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} = -\nu_{sgs}\left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i}\right) = -2\nu_{sgs}\bar{s}_{ij}$$

$$\nu_{sgs} \propto v'\ell \propto \left(\Delta x_1 \frac{\partial \bar{v}_1}{\partial x_1}\right)\Delta x_1 \propto \Delta x_1^2 \left(\frac{\partial \bar{v}_1}{\partial x_1}\right) \propto (C_S\Delta)^2\sqrt{2\bar{s}_{ij}\bar{s}_{ij}} = (C_S\Delta)^2\left|\bar{s}\right|$$

¶See Section 18.6, Subgrid model

▶Smagorinsky Subgrid model

$$
\begin{aligned}
\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} &= -\nu_{sgs}\left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i}\right) = -2\nu_{sgs}\bar{s}_{ij} \\
\nu_{sgs} &\propto v'\ell \propto \left(\Delta x_1 \frac{\partial \bar{v}_1}{\partial x_1}\right)\Delta x_1 \propto \Delta x_1^2\left(\frac{\partial \bar{v}_1}{\partial x_1}\right) \propto (C_S\Delta)^2\sqrt{2\bar{s}_{ij}\bar{s}_{ij}} = (C_S\Delta)^2\left|\bar{s}\right| \\
\Delta &= (\Delta V_{IJK})^{1/3}
\end{aligned}
$$

¶See Section 18.6, Subgrid model

▶Smagorinsky Subgrid model

$$\begin{aligned}
\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} &= -\nu_{sgs}\left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i}\right) = -2\nu_{sgs}\bar{s}_{ij} \\
\nu_{sgs} &\propto v'\ell \propto \left(\Delta x_1 \frac{\partial \bar{v}_1}{\partial x_1}\right)\Delta x_1 \propto \Delta x_1^2 \left(\frac{\partial \bar{v}_1}{\partial x_1}\right) \propto (C_S\Delta)^2 \sqrt{2\bar{s}_{ij}\bar{s}_{ij}} = (C_S\Delta)^2 |\bar{s}| \\
\Delta &= (\Delta V_{IJK})^{1/3}
\end{aligned}$$

$|\bar{s}|$ stems from the production term in the k eq., $|\bar{s}^2| = 2\bar{s}_{ij}\bar{s}_{ij}$

¶See Section 18.21, One-equation k_{sgs} model

¶See Section 18.21, One-equation k_{sgs} model

$$\frac{\partial k_{sgs}}{\partial t} + \frac{\partial}{\partial x_j}(\bar{v}_j k_{sgs}) = \frac{\partial}{\partial x_j}\left[(\nu + \nu_{sgs})\frac{\partial k_{sgs}}{\partial x_j}\right] + P_{k_{sgs}} - \varepsilon$$

¶See Section 18.21, One-equation k_{sgs} model

$$\frac{\partial k_{sgs}}{\partial t} + \frac{\partial}{\partial x_j}(\bar{v}_j k_{sgs}) = \frac{\partial}{\partial x_j}\left[(\nu + \nu_{sgs})\frac{\partial k_{sgs}}{\partial x_j}\right] + P_{k_{sgs}} - \varepsilon$$

ν_{sgs}

¶See Section 18.21, One-equation k_{sgs} model

$$\frac{\partial k_{sgs}}{\partial t} + \frac{\partial}{\partial x_j}(\bar{v}_j k_{sgs}) = \frac{\partial}{\partial x_j}\left[(\nu + \nu_{sgs})\frac{\partial k_{sgs}}{\partial x_j}\right] + P_{k_{sgs}} - \varepsilon$$

$$\nu_{sgs} \propto \ell v' =$$

¶See Section 18.21, One-equation k_{sgs} model

$$\frac{\partial k_{sgs}}{\partial t} + \frac{\partial}{\partial x_j}(\bar{v}_j k_{sgs}) \; = \; \frac{\partial}{\partial x_j}\left[(\nu + \nu_{sgs})\frac{\partial k_{sgs}}{\partial x_j}\right] + P_{k_{sgs}} - \varepsilon$$

$$\nu_{sgs} \propto \ell v' = c_k \Delta k_{sgs}^{1/2}$$

¶See Section 18.21, One-equation k_{sgs} model

$$\frac{\partial k_{sgs}}{\partial t} + \frac{\partial}{\partial x_j}(\bar{v}_j k_{sgs}) \;=\; \frac{\partial}{\partial x_j}\left[(\nu + \nu_{sgs})\frac{\partial k_{sgs}}{\partial x_j}\right] + P_{k_{sgs}} - \varepsilon$$

$$\nu_{sgs} \propto \ell v' = c_k \Delta k_{sgs}^{1/2}, \qquad P_{k_{sgs}} =$$

¶See Section 18.21, One-equation k_{sgs} model

$$\frac{\partial k_{sgs}}{\partial t}+\frac{\partial}{\partial x_j}(\bar{v}_j k_{sgs}) = \frac{\partial}{\partial x_j}\left[(\nu+\nu_{sgs})\frac{\partial k_{sgs}}{\partial x_j}\right]+P_{k_{sgs}}-\varepsilon$$

$$\nu_{sgs}\propto \ell v' = c_k \Delta k_{sgs}^{1/2}, \quad P_{k_{sgs}} = 2\nu_{sgs}\bar{s}_{ij}\bar{s}_{ij}$$

¶See Section 18.21, One-equation k_{sgs} model

$$\frac{\partial k_{sgs}}{\partial t}+\frac{\partial}{\partial x_j}(\bar{v}_j k_{sgs}) \;=\; \frac{\partial}{\partial x_j}\left[(\nu+\nu_{sgs})\frac{\partial k_{sgs}}{\partial x_j}\right]+P_{k_{sgs}}-\varepsilon$$

$$\nu_{sgs}\propto \ell v' = c_k\Delta k_{sgs}^{1/2}, \quad P_{k_{sgs}} \;=\; 2\nu_{sgs}\bar{s}_{ij}\bar{s}_{ij}, \quad \varepsilon \propto$$

¶See Section 18.21, One-equation k_{sgs} model

$$\frac{\partial k_{sgs}}{\partial t} + \frac{\partial}{\partial x_j}(\bar{v}_j k_{sgs}) = \frac{\partial}{\partial x_j}\left[(\nu + \nu_{sgs})\frac{\partial k_{sgs}}{\partial x_j}\right] + P_{k_{sgs}} - \varepsilon$$

$$\nu_{sgs} \propto \ell v' = c_k \Delta k_{sgs}^{1/2}, \quad P_{k_{sgs}} = 2\nu_{sgs}\bar{s}_{ij}\bar{s}_{ij}, \quad \varepsilon \propto \frac{v'^3}{\ell} =$$

¶See Section 18.21, One-equation k_{sgs} model

$$\frac{\partial k_{sgs}}{\partial t}+\frac{\partial}{\partial x_j}(\bar{v}_j k_{sgs}) = \frac{\partial}{\partial x_j}\left[(\nu+\nu_{sgs})\frac{\partial k_{sgs}}{\partial x_j}\right]+P_{k_{sgs}}-\varepsilon$$

$$\nu_{sgs}\propto \ell v' = c_k \Delta k_{sgs}^{1/2}, \quad P_{k_{sgs}} = 2\nu_{sgs}\bar{s}_{ij}\bar{s}_{ij}, \quad \varepsilon \propto \frac{v'^3}{\ell} = C_\varepsilon \frac{k_{sgs}^{3/2}}{\Delta}$$

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$$\frac{\partial k_{sgs}}{\partial t}+\frac{\partial}{\partial x_j}(\bar{v}_j k_{sgs}) = \frac{\partial}{\partial x_j}\left[(\nu+\nu_{sgs})\frac{\partial k_{sgs}}{\partial x_j}\right]+P_{k_{sgs}}-\varepsilon$$

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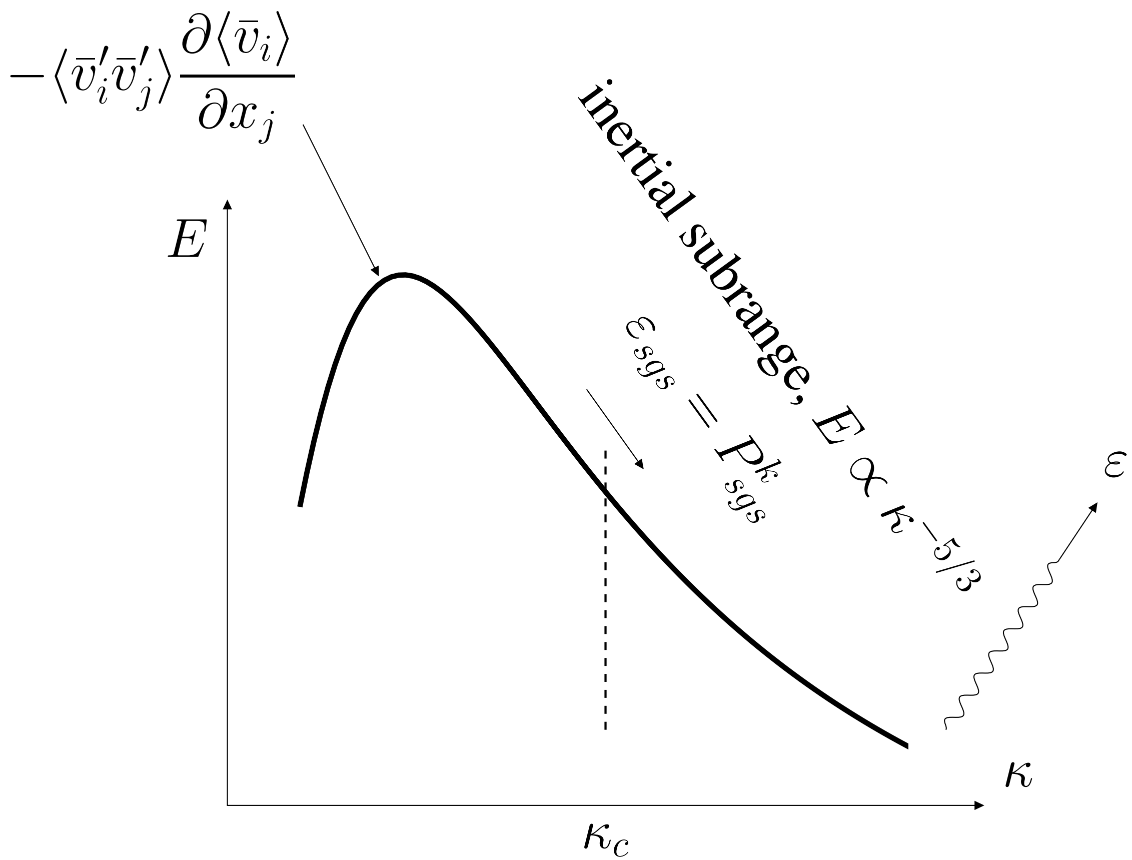
¶See Section 18.8, Energy path

¶See Section 18.21, One-equation k_{sgs} model

$$\frac{\partial k_{sgs}}{\partial t} + \frac{\partial}{\partial x_j}(\bar{v}_j k_{sgs}) = \frac{\partial}{\partial x_j}\left[(\nu + \nu_{sgs})\frac{\partial k_{sgs}}{\partial x_j}\right] + P_{k_{sgs}} - \varepsilon$$

$$\nu_{sgs} \propto \ell v' = c_k \Delta k_{sgs}^{1/2}, \quad P_{k_{sgs}} = 2\nu_{sgs}\bar{s}_{ij}\bar{s}_{ij}, \quad \varepsilon \propto \frac{v'^3}{\ell} = C_\varepsilon \frac{k_{sgs}^{3/2}}{\Delta}$$

¶See Section 18.8, Energy path



¶See Section 18.9, SGS kinetic energy

¶See Section 18.9, SGS kinetic energy

$$v_i =$$

¶See Section 18.9, SGS kinetic energy

$$v_i = \langle v_i \rangle + v_i'$$

¶See Section 18.9, SGS kinetic energy

$$v_i = \langle v_i \rangle + v_i', \quad v_i = \bar{v}_i + v_i'' =$$

¶See Section 18.9, SGS kinetic energy

$$v_i = \langle v_i \rangle + v_i', \quad v_i = \bar{v}_i + v_i'' = \langle \bar{v}_i \rangle + \bar{v}_i' + v_i''$$

¶See Section 18.9, SGS kinetic energy

$$v_i = \langle v_i \rangle + v'_i, \quad v_i = \bar{v}_i + v''_i = \langle \bar{v}_i \rangle + \bar{v}'_i + v''_i$$

$$k \equiv$$

¶See Section 18.9, SGS kinetic energy

$$
\begin{aligned}
v_i &= \langle v_i \rangle + v_i', \quad v_i = \bar{v}_i + v_i'' = \langle \bar{v}_i \rangle + \bar{v}_i' + v_i'' \\
k &\equiv \frac{1}{2} \langle v_i' v_i' \rangle
\end{aligned}
$$

¶See Section 18.9, SGS kinetic energy

$$v_i = \langle v_i \rangle + v_i', \quad v_i = \bar{v}_i + v_i'' = \langle \bar{v}_i \rangle + \bar{v}_i' + v_i''$$

$$k \equiv \frac{1}{2}\langle v_i' v_i' \rangle = \int_0^\infty E(\kappa) d\kappa$$

¶See Section 18.9, SGS kinetic energy

$$v_i = \langle v_i \rangle + v_i', \quad v_i = \bar{v}_i + v_i'' = \langle \bar{v}_i \rangle + \bar{v}_i' + v_i''$$

$$k \equiv \frac{1}{2}\langle v_i' v_i' \rangle = \int_0^\infty E(\kappa) d\kappa, \quad k_{sgs} \equiv$$

¶See Section 18.9, SGS kinetic energy

$$
\begin{aligned}
v_i &= \langle v_i \rangle + v_i', \quad v_i = \bar{v}_i + v_i'' = \langle \bar{v}_i \rangle + \bar{v}_i' + v_i'' \\
k &\equiv \frac{1}{2}\langle v_i' v_i' \rangle = \int_0^\infty E(\kappa) d\kappa, \quad k_{sgs} \equiv \frac{1}{2}\langle v_i'' v_i'' \rangle =
\end{aligned}
$$

¶See Section 18.9, SGS kinetic energy

$$
\begin{aligned}
v_i &= \langle v_i \rangle + v_i', \quad v_i = \bar{v}_i + v_i'' = \langle \bar{v}_i \rangle + \bar{v}_i' + v_i'' \\
k &\equiv \frac{1}{2}\langle v_i' v_i' \rangle = \int_0^\infty E(\kappa) d\kappa, \quad k_{sgs} \equiv \frac{1}{2}\langle v_i'' v_i'' \rangle = \int_{\kappa_c}^\infty E(\kappa) d\kappa
\end{aligned}
$$

¶See Section 18.9, SGS kinetic energy

$$
\begin{aligned}
v_i &= \langle v_i \rangle + v_i', \quad v_i = \bar{v}_i + v_i'' = \langle \bar{v}_i \rangle + \bar{v}_i' + v_i'' \\
k &\equiv \frac{1}{2} \langle v_i' v_i' \rangle = \int_0^\infty E(\kappa) d\kappa, \quad k_{sgs} \equiv \frac{1}{2} \langle v_i'' v_i'' \rangle = \int_{\kappa_c}^\infty E(\kappa) d\kappa \\
\bar{k} &\equiv
\end{aligned}
$$

¶See Section 18.9, SGS kinetic energy

$$
\begin{aligned}
v_i &= \langle v_i \rangle + v_i', \quad v_i = \bar{v}_i + v_i'' = \langle \bar{v}_i \rangle + \bar{v}_i' + v_i'' \\
k &\equiv \frac{1}{2}\langle v_i' v_i' \rangle = \int_0^\infty E(\kappa) d\kappa, \quad k_{sgs} \equiv \frac{1}{2}\langle v_i'' v_i'' \rangle = \int_{\kappa_c}^\infty E(\kappa) d\kappa \\
\bar{k} &\equiv \frac{1}{2}\langle \bar{v}_i' \bar{v}_i' \rangle =
\end{aligned}
$$

¶See Section 18.9, SGS kinetic energy

$$
\begin{aligned}
v_i &= \langle v_i \rangle + v_i', \quad v_i = \bar{v}_i + v_i'' = \langle \bar{v}_i \rangle + \bar{v}_i' + v_i'' \\
k &\equiv \frac{1}{2}\langle v_i' v_i' \rangle = \int_0^\infty E(\kappa) d\kappa, \quad k_{sgs} \equiv \frac{1}{2}\langle v_i'' v_i'' \rangle = \int_{\kappa_c}^\infty E(\kappa) d\kappa \\
\bar{k} &\equiv \frac{1}{2}\langle \bar{v}_i' \bar{v}_i' \rangle = \int_0^{\kappa_c} E(\kappa) d\kappa
\end{aligned}
$$

¶See Section 18.9, SGS kinetic energy

$$
\begin{aligned}
v_i &= \langle v_i \rangle + v_i', \quad v_i = \bar{v}_i + v_i'' = \langle \bar{v}_i \rangle + \bar{v}_i' + v_i'' \\
k &\equiv \frac{1}{2}\langle v_i' v_i' \rangle = \int_0^\infty E(\kappa) d\kappa, \quad k_{sgs} \equiv \frac{1}{2}\langle v_i'' v_i'' \rangle = \int_{\kappa_c}^\infty E(\kappa) d\kappa \\
\bar{k} &\equiv \frac{1}{2}\langle \bar{v}_i' \bar{v}_i' \rangle = \int_0^{\kappa_c} E(\kappa) d\kappa, \quad \bar{K} \equiv
\end{aligned}
$$

¶See Section 18.9, SGS kinetic energy

$$
\begin{aligned}
v_i &= \langle v_i \rangle + v_i', \quad v_i = \bar{v}_i + v_i'' = \langle \bar{v}_i \rangle + \bar{v}_i' + v_i'' \\
k &\equiv \frac{1}{2}\langle v_i' v_i' \rangle = \int_0^\infty E(\kappa) d\kappa, \quad k_{sgs} \equiv \frac{1}{2}\langle v_i'' v_i'' \rangle = \int_{\kappa_c}^\infty E(\kappa) d\kappa \\
\bar{k} &\equiv \frac{1}{2}\langle \bar{v}_i' \bar{v}_i' \rangle = \int_0^{\kappa_c} E(\kappa) d\kappa, \quad \bar{K} \equiv \frac{1}{2}\langle \bar{v}_i \rangle \langle \bar{v}_i \rangle
\end{aligned}
$$

¶See Section 18.9, SGS kinetic energy

$$
\begin{aligned}
v_i &= \langle v_i \rangle + v_i', \quad v_i = \bar{v}_i + v_i'' = \langle \bar{v}_i \rangle + \bar{v}_i' + v_i'' \\
k &\equiv \frac{1}{2}\langle v_i' v_i' \rangle = \int_0^\infty E(\kappa) d\kappa, \quad k_{sgs} \equiv \frac{1}{2}\langle v_i'' v_i'' \rangle = \int_{\kappa_c}^\infty E(\kappa) d\kappa \\
\bar{k} &\equiv \frac{1}{2}\langle \bar{v}_i' \bar{v}_i' \rangle = \int_0^{\kappa_c} E(\kappa) d\kappa, \quad \bar{K} \equiv \frac{1}{2}\langle \bar{v}_i \rangle \langle \bar{v}_i \rangle
\end{aligned}
$$

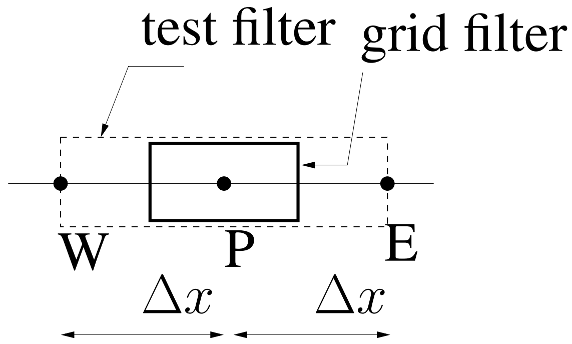
On-line Lecture 8

¶See Section 18.11, The dynamic model

On-line Lecture 8

¶See Section 18.11, The dynamic model

►The dynamic model. C is computed. Test filter, $\widehat{\Delta} = 2\Delta$

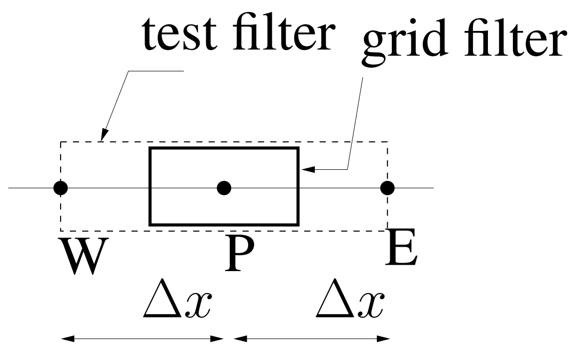


Control volume for grid and test filter.

On-line Lecture 8

¶See Section 18.11, The dynamic model

► The dynamic model. C is computed. Test filter, $\widehat{\Delta} = 2\Delta$



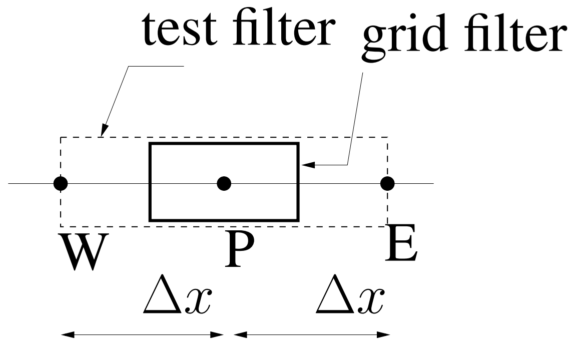
Control volume for grid and test filter.

► First, grid and test filter the Navier-Stokes (DNS)

On-line Lecture 8

¶See Section 18.11, The dynamic model

▶The dynamic model. C is computed. Test filter, $\widehat{\Delta} = 2\Delta$



Control volume for grid and test filter.

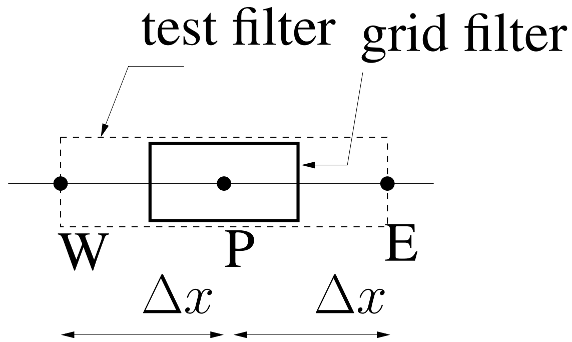
▶First, grid and test filter the Navier-Stokes (DNS)

$$\frac{\partial \widehat{\bar{v}}_i}{\partial t} +$$

On-line Lecture 8

¶See Section 18.11, The dynamic model

▶The dynamic model. C is computed. Test filter, $\widehat{\Delta} = 2\Delta$



Control volume for grid and test filter.

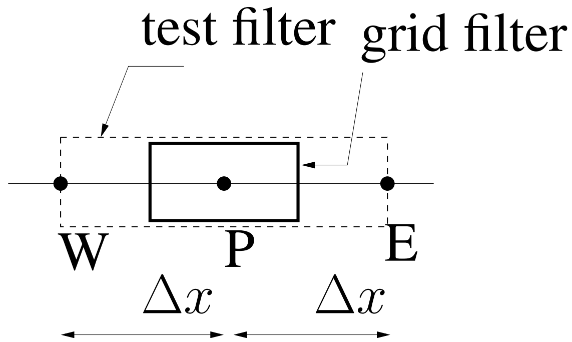
▶First, grid and test filter the Navier-Stokes (DNS)

$$\frac{\partial \widehat{\bar{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right) =$$

On-line Lecture 8

¶See Section 18.11, The dynamic model

▶The dynamic model. C is computed. Test filter, $\widehat{\Delta} = 2\Delta$



Control volume for grid and test filter.

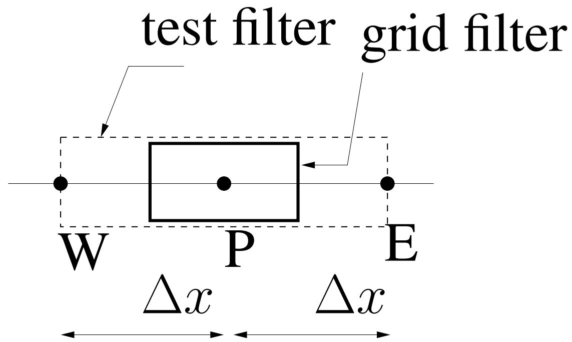
▶First, grid and test filter the Navier-Stokes (DNS)

$$\frac{\partial \widehat{\bar{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right) = -\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i} +$$

On-line Lecture 8

¶See Section 18.11, The dynamic model

▶The dynamic model. C is computed. Test filter, $\widehat{\Delta} = 2\Delta$



Control volume for grid and test filter.

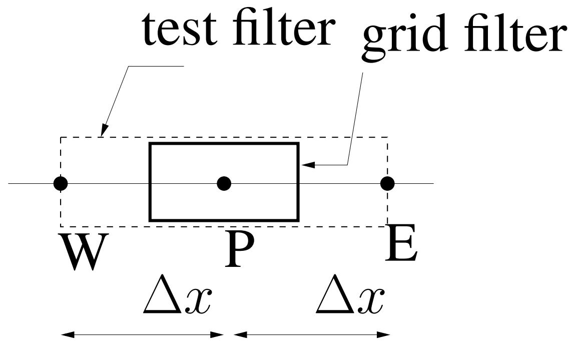
▶First, grid and test filter the Navier-Stokes (DNS)

$$\frac{\partial \widehat{\bar{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right) = -\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\bar{v}}_i}{\partial x_j \partial x_j}$$

On-line Lecture 8

¶See Section 18.11, The dynamic model

▶The dynamic model. C is computed. Test filter, $\widehat{\Delta}=2\Delta$



Control volume for grid and test filter.

▶First, grid and test filter the Navier-Stokes (DNS)

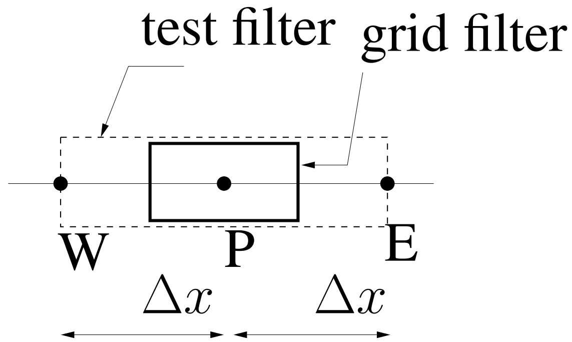
$$\frac{\partial \widehat{\bar{v}}_i}{\partial t}+\frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right) = -\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i}+\nu\frac{\partial^2 \widehat{\bar{v}}_i}{\partial x_j \partial x_j}$$

Left side :

On-line Lecture 8

¶See Section 18.11, The dynamic model

▶The dynamic model. C is computed. Test filter, $\widehat{\Delta} = 2\Delta$



Control volume for grid and test filter.

▶First, grid and test filter the Navier-Stokes (DNS)

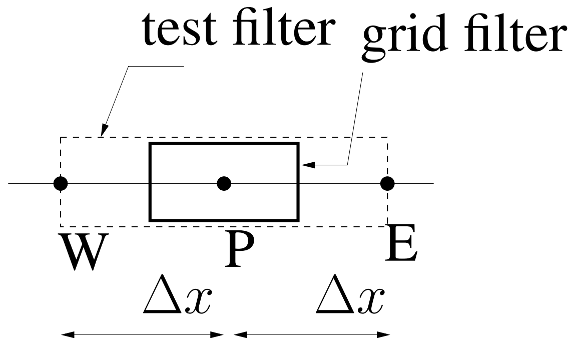
$$\frac{\partial \widehat{\bar{v}}_i}{\partial t}+\frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right) = -\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i}+\nu\frac{\partial^2 \widehat{\bar{v}}_i}{\partial x_j \partial x_j}$$

$$\text{Left side} \quad : \quad \frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right)$$

On-line Lecture 8

¶See Section 18.11, The dynamic model

►The dynamic model. C is computed. Test filter, $\widehat{\Delta} = 2\Delta$



Control volume for grid and test filter.

►First, grid and test filter the Navier-Stokes (DNS)

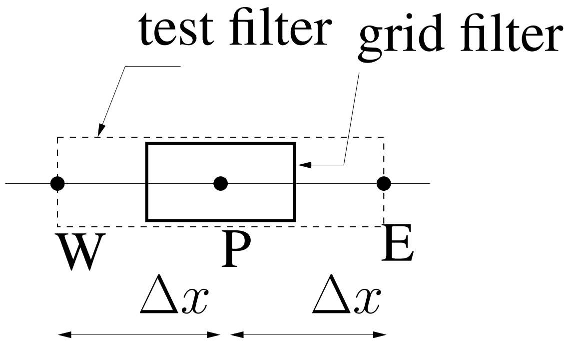
$$\frac{\partial \widehat{\bar{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right) = -\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\bar{v}}_i}{\partial x_j \partial x_j}$$

$$\text{Left side} \; : \; \frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right) - \frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right) +$$

On-line Lecture 8

¶See Section 18.11, The dynamic model

▶The dynamic model. C is computed. Test filter, $\widehat{\Delta} = 2\Delta$



Control volume for grid and test filter.

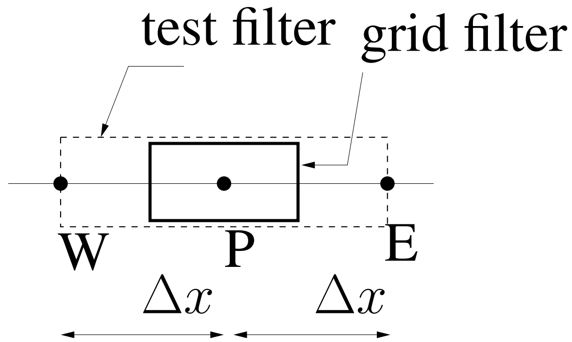
▶First, grid and test filter the Navier-Stokes (DNS)

$$
\begin{aligned}
\frac{\partial \widehat{\bar{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right) &= -\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\bar{v}}_i}{\partial x_j \partial x_j} \\
\text{Left side} &: \frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right) - \frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right) + \frac{\partial}{\partial x_j}\left(\widehat{\bar{v}}_i \widehat{\bar{v}}_j\right) =
\end{aligned}
$$

On-line Lecture 8

¶See Section 18.11, The dynamic model

► The dynamic model. C is computed. Test filter, $\widehat{\Delta} = 2\Delta$



Control volume for grid and test filter.

► First, grid and test filter the Navier-Stokes (DNS)

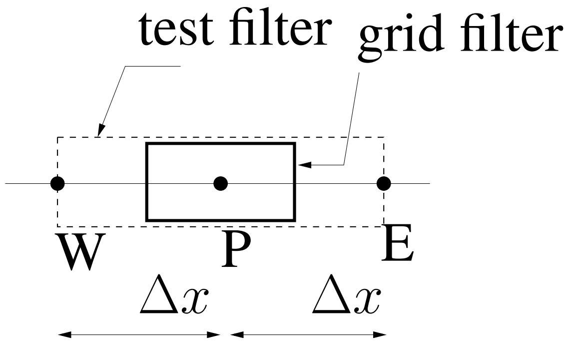
$$\frac{\partial \widehat{\bar{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right) = -\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\bar{v}}_i}{\partial x_j \partial x_j}$$

$$\text{Left side} \;:\; \frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right) - \frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right) + \frac{\partial}{\partial x_j}\left(\widehat{\bar{v}}_i \widehat{\bar{v}}_j\right) = \frac{\partial}{\partial x_j}\left(\widehat{\bar{v}}_i \widehat{\bar{v}}_j\right)$$

On-line Lecture 8

¶See Section 18.11, The dynamic model

▶The dynamic model. C is computed. Test filter, $\widehat{\Delta} = 2\Delta$



Control volume for grid and test filter.

▶First, grid and test filter the Navier-Stokes (DNS)

$$\frac{\partial \widehat{\bar{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right) = -\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\bar{v}}_i}{\partial x_j \partial x_j}$$

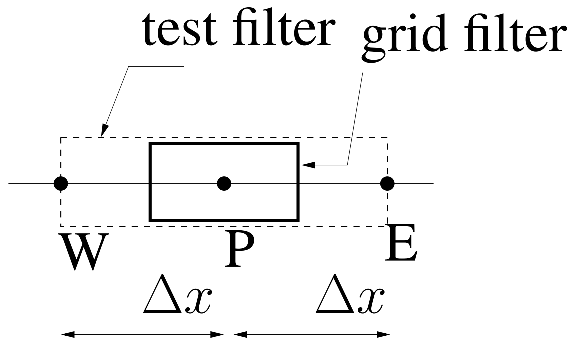
$$\text{Left side} \quad : \quad \frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right) - \frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right) + \frac{\partial}{\partial x_j}\left(\widehat{\bar{v}}_i \widehat{\bar{v}}_j\right) = \frac{\partial}{\partial x_j}\left(\widehat{\bar{v}}_i \widehat{\bar{v}}_j\right)$$

$$\text{Right side} \quad :$$

On-line Lecture 8

¶See Section 18.11, The dynamic model

▶The dynamic model. C is computed. Test filter, $\widehat{\Delta} = 2\Delta$



Control volume for grid and test filter.

▶First, grid and test filter the Navier-Stokes (DNS)

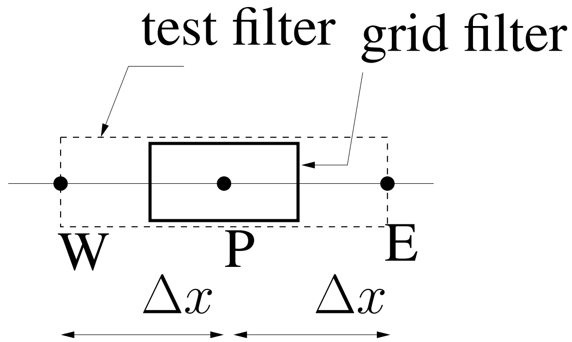
$$
\begin{aligned}
\frac{\partial \widehat{\bar{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right) &= -\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\bar{v}}_i}{\partial x_j \partial x_j} \\
\text{Left side} \quad : \quad \frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right) - \frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right) + \frac{\partial}{\partial x_j}\left(\widehat{\bar{v}}_i \widehat{\bar{v}}_j\right) &= \frac{\partial}{\partial x_j}\left(\widehat{\bar{v}}_i \widehat{\bar{v}}_j\right) \\
\text{Right side} \quad : \quad -\frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right) + \frac{\partial}{\partial x_j}\left(\widehat{\bar{v}}_i \widehat{\bar{v}}_j\right)
\end{aligned}
$$

▶We get

On-line Lecture 8

¶See Section 18.11, The dynamic model

▶The dynamic model. C is computed. Test filter, $\widehat{\Delta} = 2\Delta$



Control volume for grid and test filter.

▶First, grid and test filter the Navier-Stokes (DNS)

$$\frac{\partial \widehat{\bar{v}}_i}{\partial t}+\frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right) = -\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i}+\nu\frac{\partial^2 \widehat{\bar{v}}_i}{\partial x_j \partial x_j}$$

$$\text{Left side} : \frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right)-\frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right)+\frac{\partial}{\partial x_j}\left(\widehat{\bar{v}}_i\widehat{\bar{v}}_j\right)=\frac{\partial}{\partial x_j}\left(\widehat{\bar{v}}_i\widehat{\bar{v}}_j\right)$$

$$\text{Right side} : -\frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right)+\frac{\partial}{\partial x_j}\left(\widehat{\bar{v}}_i\widehat{\bar{v}}_j\right)$$

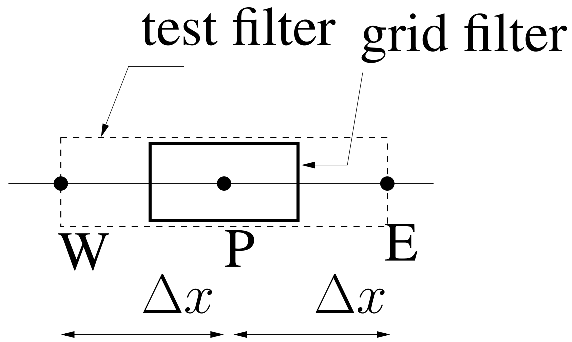
▶We get

$$\frac{\partial \widehat{\bar{v}}_i}{\partial t}+\frac{\partial}{\partial x_j}\left(\widehat{\bar{v}}_i\widehat{\bar{v}}_j\right) = -\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i}+\nu\frac{\partial^2 \widehat{\bar{v}}_i}{\partial x_j \partial x_j}-\frac{\partial T_{ij}}{\partial x_j}, \quad T_{ij}=\widehat{\overline{v_i v_j}}-\widehat{\bar{v}}_i\widehat{\bar{v}}_j \tag{37.1}$$

On-line Lecture 8

¶See Section 18.11, The dynamic model

▶The dynamic model. C is computed. Test filter, $\widehat{\Delta} = 2\Delta$



Control volume for grid and test filter.

▶First, grid and test filter the Navier-Stokes (DNS)

$$
\begin{aligned}
\frac{\partial \widehat{\bar{v}}_i}{\partial t}+\frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right) &= -\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i}+\nu\frac{\partial^2 \widehat{\bar{v}}_i}{\partial x_j \partial x_j} \\
\text{Left side} &: \frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right)-\frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right)+\frac{\partial}{\partial x_j}\left(\widehat{\bar{v}}_i \widehat{\bar{v}}_j\right)=\frac{\partial}{\partial x_j}\left(\widehat{\bar{v}}_i \widehat{\bar{v}}_j\right) \\
\text{Right side} &: -\frac{\partial}{\partial x_j}\left(\widehat{\overline{v_i v_j}}\right)+\frac{\partial}{\partial x_j}\left(\widehat{\bar{v}}_i \widehat{\bar{v}}_j\right)
\end{aligned}
$$

▶We get

$$
\frac{\partial \widehat{\bar{v}}_i}{\partial t}+\frac{\partial}{\partial x_j}\left(\widehat{\bar{v}}_i \widehat{\bar{v}}_j\right)=-\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i}+\nu\frac{\partial^2 \widehat{\bar{v}}_i}{\partial x_j \partial x_j}-\frac{\partial T_{ij}}{\partial x_j}, \quad T_{ij}=\widehat{\overline{v_i v_j}}-\widehat{\bar{v}}_i \widehat{\bar{v}}_j \tag{37.1}
$$

$$\frac{\partial \widehat{\overline{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\overline{v}}_i \widehat{\overline{v}}_j\right) = -\frac{1}{\rho}\frac{\partial \widehat{\overline{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\overline{v}}_i}{\partial x_j \partial x_j} - \frac{\partial T_{ij}}{\partial x_j}, \quad T_{ij} = \widehat{\overline{v_i v_j}} - \widehat{\overline{v}}_i \widehat{\overline{v}}_j \tag{37.1}$$

►Second, we test filter the LES equations

$$\frac{\partial \widehat{\bar{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\bar{v}}_i \widehat{\bar{v}}_j\right) = -\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\bar{v}}_i}{\partial x_j \partial x_j} - \frac{\partial T_{ij}}{\partial x_j}, \quad T_{ij} = \widehat{\overline{v_i v_j}} - \widehat{\bar{v}}_i \widehat{\bar{v}}_j \tag{37.1}$$

►Second, we test filter the LES equations

$$\frac{\partial \widehat{\bar{v}}_i}{\partial t} + \frac{\partial \widehat{\overline{v_i v_j}}}{\partial x_j}$$

$$\frac{\partial \widehat{\bar{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\bar{v}}_i \widehat{\bar{v}}_j\right) = -\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\bar{v}}_i}{\partial x_j \partial x_j} - \frac{\partial T_{ij}}{\partial x_j}, \quad T_{ij} = \widehat{\overline{v_i v_j}} - \widehat{\bar{v}}_i \widehat{\bar{v}}_j \tag{37.1}$$

►Second, we test filter the LES equations

$$\frac{\partial \widehat{\bar{v}}_i}{\partial t} + \frac{\partial \widehat{\bar{v}_i \bar{v}_j}}{\partial x_j} =$$

$$\frac{\partial \widehat{\overline{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\overline{v}}_i \widehat{\overline{v}}_j\right) = -\frac{1}{\rho}\frac{\partial \widehat{\overline{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\overline{v}}_i}{\partial x_j \partial x_j} - \frac{\partial T_{ij}}{\partial x_j}, \quad T_{ij} = \widehat{\overline{v_i v_j}} - \widehat{\overline{v}}_i \widehat{\overline{v}}_j \tag{37.1}$$

►Second, we test filter the LES equations

$$\frac{\partial \widehat{\overline{v}}_i}{\partial t} + \frac{\partial \widehat{\overline{v}_i \overline{v}_j}}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \widehat{\overline{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\overline{v}}_i}{\partial x_j \partial x_j} - \frac{\partial \widehat{\tau}_{ij}}{\partial x_j}$$

$$\frac{\partial \widehat{\overline{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\overline{v}}_i \widehat{\overline{v}}_j\right) = -\frac{1}{\rho}\frac{\partial \widehat{\overline{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\overline{v}}_i}{\partial x_j \partial x_j} - \frac{\partial T_{ij}}{\partial x_j}, \quad T_{ij} = \widehat{\overline{v_i v_j}} - \widehat{\overline{v}}_i \widehat{\overline{v}}_j \tag{37.1}$$

►Second, we test filter the LES equations

$$\frac{\partial \widehat{\overline{v}}_i}{\partial t} + \frac{\partial \widehat{\overline{v}_i \overline{v}_j}}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \widehat{\overline{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\overline{v}}_i}{\partial x_j \partial x_j} - \frac{\partial \widehat{\tau}_{ij}}{\partial x_j}$$

Left side :

$$\frac{\partial \widehat{\overline{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\overline{v}}_i \widehat{\overline{v}}_j\right) = -\frac{1}{\rho}\frac{\partial \widehat{\overline{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\overline{v}}_i}{\partial x_j \partial x_j} - \frac{\partial T_{ij}}{\partial x_j}, \quad T_{ij} = \widehat{\overline{v_i v_j}} - \widehat{\overline{v}}_i \widehat{\overline{v}}_j \tag{37.1}$$

►Second, we test filter the LES equations

$$\frac{\partial \widehat{\overline{v}}_i}{\partial t} + \frac{\partial \widehat{\overline{v}_i \overline{v}_j}}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \widehat{\overline{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\overline{v}}_i}{\partial x_j \partial x_j} - \frac{\partial \widehat{\tau}_{ij}}{\partial x_j}$$

$$\text{Left side} \quad : \quad \frac{\partial \widehat{\overline{v}_i \overline{v}_j}}{\partial x_j} -$$

$$\frac{\partial \widehat{\overline{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\overline{v}}_i \widehat{\overline{v}}_j\right) = -\frac{1}{\rho}\frac{\partial \widehat{\overline{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\overline{v}}_i}{\partial x_j \partial x_j} - \frac{\partial T_{ij}}{\partial x_j}, \quad T_{ij} = \widehat{\overline{v_i v_j}} - \widehat{\overline{v}}_i \widehat{\overline{v}}_j \tag{37.1}$$

►Second, we test filter the LES equations

$$\frac{\partial \widehat{\overline{v}}_i}{\partial t} + \frac{\partial \widehat{\overline{v}_i \overline{v}_j}}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \widehat{\overline{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\overline{v}}_i}{\partial x_j \partial x_j} - \frac{\partial \widehat{\tau}_{ij}}{\partial x_j}$$

$$\text{Left side} \quad : \quad \frac{\partial \widehat{\overline{v}_i \overline{v}_j}}{\partial x_j} - \frac{\partial \widehat{\overline{v}_i \overline{v}_j}}{\partial x_j} +$$

$$\frac{\partial \widehat{\overline{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\overline{v}}_i \widehat{\overline{v}}_j\right) = -\frac{1}{\rho}\frac{\partial \widehat{\overline{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\overline{v}}_i}{\partial x_j \partial x_j} - \frac{\partial T_{ij}}{\partial x_j}, \quad T_{ij} = \widehat{\overline{v_i v_j}} - \widehat{\overline{v}}_i \widehat{\overline{v}}_j \tag{37.1}$$

▶Second, we test filter the LES equations

$$\frac{\partial \widehat{\overline{v}}_i}{\partial t} + \frac{\partial \widehat{\overline{v}_i \overline{v}_j}}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \widehat{\overline{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\overline{v}}_i}{\partial x_j \partial x_j} - \frac{\partial \widehat{\tau}_{ij}}{\partial x_j}$$

$$\text{Left side} \quad : \quad \frac{\partial \widehat{\overline{v}_i \overline{v}_j}}{\partial x_j} - \frac{\partial \widehat{\overline{v}_i \overline{v}_j}}{\partial x_j} + \frac{\partial \widehat{\overline{v}}_i \widehat{\overline{v}}_j}{\partial x_j} =$$

$$\frac{\partial \widehat{\overline{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\overline{v}}_i \widehat{\overline{v}}_j\right) = -\frac{1}{\rho}\frac{\partial \widehat{\overline{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\overline{v}}_i}{\partial x_j \partial x_j} - \frac{\partial T_{ij}}{\partial x_j}, \quad T_{ij} = \widehat{\overline{v_i v_j}} - \widehat{\overline{v}}_i \widehat{\overline{v}}_j \tag{37.1}$$

►Second, we test filter the LES equations

$$\frac{\partial \widehat{\overline{v}}_i}{\partial t} + \frac{\partial \widehat{\overline{v}_i \overline{v}_j}}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \widehat{\overline{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\overline{v}}_i}{\partial x_j \partial x_j} - \frac{\partial \widehat{\tau}_{ij}}{\partial x_j}$$

$$\text{Left side} \quad : \quad \frac{\partial \widehat{\overline{v}_i \overline{v}_j}}{\partial x_j} - \frac{\partial \widehat{\overline{v}_i \overline{v}_j}}{\partial x_j} + \frac{\partial \widehat{\overline{v}}_i \widehat{\overline{v}}_j}{\partial x_j} = \frac{\partial \widehat{\overline{v}}_i \widehat{\overline{v}}_j}{\partial x_j}$$

$$\frac{\partial \widehat{\overline{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\overline{v}}_i \widehat{\overline{v}}_j\right) = -\frac{1}{\rho}\frac{\partial \widehat{\overline{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\overline{v}}_i}{\partial x_j \partial x_j} - \frac{\partial T_{ij}}{\partial x_j}, \quad T_{ij} = \widehat{\overline{v_i v_j}} - \widehat{\overline{v}}_i \widehat{\overline{v}}_j \tag{37.1}$$

▶Second, we test filter the LES equations

$$\frac{\partial \widehat{\overline{v}}_i}{\partial t} + \frac{\partial \widehat{\overline{v}_i \overline{v}_j}}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \widehat{\overline{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\overline{v}}_i}{\partial x_j \partial x_j} - \frac{\partial \widehat{\tau}_{ij}}{\partial x_j}$$

$$\text{Left side} \quad : \quad \frac{\partial \widehat{\overline{v}_i \overline{v}_j}}{\partial x_j} - \frac{\partial \widehat{\overline{v}_i \overline{v}_j}}{\partial x_j} + \frac{\partial \widehat{\overline{v}}_i \widehat{\overline{v}}_j}{\partial x_j} = \frac{\partial \widehat{\overline{v}}_i \widehat{\overline{v}}_j}{\partial x_j}$$

$$\text{Right side} \quad :$$

$$\frac{\partial \widehat{\bar{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\bar{v}}_i \widehat{\bar{v}}_j\right) = -\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\bar{v}}_i}{\partial x_j \partial x_j} - \frac{\partial T_{ij}}{\partial x_j}, \quad T_{ij} = \widehat{\overline{v_i v_j}} - \widehat{\bar{v}}_i \widehat{\bar{v}}_j \tag{37.1}$$

► Second, we test filter the LES equations

$$\frac{\partial \widehat{\bar{v}}_i}{\partial t} + \frac{\partial \widehat{\bar{v}_i \bar{v}_j}}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\bar{v}}_i}{\partial x_j \partial x_j} - \frac{\partial \widehat{\tau}_{ij}}{\partial x_j}$$

$$\text{Left side} : \frac{\partial \widehat{\bar{v}_i \bar{v}_j}}{\partial x_j} - \frac{\partial \widehat{\bar{v}_i \bar{v}_j}}{\partial x_j} + \frac{\partial \widehat{\bar{v}}_i \widehat{\bar{v}}_j}{\partial x_j} = \frac{\partial \widehat{\bar{v}}_i \widehat{\bar{v}}_j}{\partial x_j}$$

$$\text{Right side} : \underbrace{-\frac{\partial \widehat{\bar{v}_i \bar{v}_j}}{\partial x_j} + \frac{\partial \widehat{\bar{v}}_i \widehat{\bar{v}}_j}{\partial x_j}}_{-\partial \mathcal{L}_{ij}/\partial x_j}$$

► We get

$$\frac{\partial \widehat{\overline{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\overline{v}}_i \widehat{\overline{v}}_j\right) = -\frac{1}{\rho}\frac{\partial \widehat{\overline{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\overline{v}}_i}{\partial x_j \partial x_j} - \frac{\partial T_{ij}}{\partial x_j}, \quad T_{ij} = \widehat{\overline{v_i v_j}} - \widehat{\overline{v}}_i \widehat{\overline{v}}_j \tag{37.1}$$

► Second, we test filter the LES equations

$$\begin{aligned}
\frac{\partial \widehat{\overline{v}}_i}{\partial t} + \frac{\partial \widehat{\overline{v}_i \overline{v}_j}}{\partial x_j} &= -\frac{1}{\rho}\frac{\partial \widehat{\overline{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\overline{v}}_i}{\partial x_j \partial x_j} - \frac{\partial \widehat{\tau}_{ij}}{\partial x_j} \\
\text{Left side} \quad &: \quad \frac{\partial \widehat{\overline{v}_i \overline{v}_j}}{\partial x_j} - \frac{\partial \widehat{\overline{v}_i \overline{v}_j}}{\partial x_j} + \frac{\partial \widehat{\overline{v}}_i \widehat{\overline{v}}_j}{\partial x_j} = \frac{\partial \widehat{\overline{v}}_i \widehat{\overline{v}}_j}{\partial x_j} \\
\text{Right side} \quad &: \quad \underbrace{-\frac{\partial \widehat{\overline{v}_i \overline{v}_j}}{\partial x_j} + \frac{\partial \widehat{\overline{v}}_i \widehat{\overline{v}}_j}{\partial x_j}}_{-\partial \mathcal{L}_{ij}/\partial x_j}
\end{aligned}$$

► We get

$$\frac{\partial \widehat{\overline{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\overline{v}}_i \widehat{\overline{v}}_j\right) = -\frac{1}{\rho}\frac{\partial \widehat{\overline{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\overline{v}}_i}{\partial x_j \partial x_j} - \frac{\partial \widehat{\tau}_{ij}}{\partial x_j} - \frac{\partial \mathcal{L}_{ij}}{\partial x_j} \tag{37.2}$$

$$\frac{\partial \widehat{\bar{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\bar{v}}_i \widehat{\bar{v}}_j\right) = -\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\bar{v}}_i}{\partial x_j \partial x_j} - \frac{\partial T_{ij}}{\partial x_j}, \quad T_{ij} = \widehat{\overline{v_i v_j}} - \widehat{\bar{v}}_i \widehat{\bar{v}}_j \tag{37.1}$$

►Second, we test filter the LES equations

$$\begin{aligned}
\frac{\partial \widehat{\bar{v}}_i}{\partial t} + \frac{\partial \widehat{\bar{v}_i \bar{v}_j}}{\partial x_j} &= -\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\bar{v}}_i}{\partial x_j \partial x_j} - \frac{\partial \widehat{\tau}_{ij}}{\partial x_j} \\
\text{Left side} &: \frac{\partial \widehat{\bar{v}_i \bar{v}_j}}{\partial x_j} - \frac{\partial \widehat{\bar{v}_i \bar{v}_j}}{\partial x_j} + \frac{\partial \widehat{\bar{v}}_i \widehat{\bar{v}}_j}{\partial x_j} = \frac{\partial \widehat{\bar{v}}_i \widehat{\bar{v}}_j}{\partial x_j} \\
\text{Right side} &: \underbrace{-\frac{\partial \widehat{\bar{v}_i \bar{v}_j}}{\partial x_j} + \frac{\partial \widehat{\bar{v}}_i \widehat{\bar{v}}_j}{\partial x_j}}_{-\partial \mathcal{L}_{ij}/\partial x_j}
\end{aligned}$$

►We get

$$\frac{\partial \widehat{\bar{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\bar{v}}_i \widehat{\bar{v}}_j\right) = -\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\bar{v}}_i}{\partial x_j \partial x_j} - \frac{\partial \widehat{\tau}_{ij}}{\partial x_j} - \frac{\partial \mathcal{L}_{ij}}{\partial x_j} \tag{37.2}$$

Identification of Eqs. 37.1 and 37.2 gives

$$\tag{37.3}$$

$$\frac{\partial \widehat{\bar{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\bar{v}}_i \widehat{\bar{v}}_j\right) = -\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\bar{v}}_i}{\partial x_j \partial x_j} - \frac{\partial T_{ij}}{\partial x_j}, \quad T_{ij} = \widehat{\overline{v_i v_j}} - \widehat{\bar{v}}_i \widehat{\bar{v}}_j \tag{37.1}$$

▶Second, we test filter the LES equations

$$\begin{aligned}
\frac{\partial \widehat{\bar{v}}_i}{\partial t} + \frac{\partial \widehat{\bar{v}_i \bar{v}_j}}{\partial x_j} &= -\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\bar{v}}_i}{\partial x_j \partial x_j} - \frac{\partial \widehat{\tau}_{ij}}{\partial x_j} \\
\text{Left side} &: \frac{\partial \widehat{\bar{v}_i \bar{v}_j}}{\partial x_j} - \frac{\partial \widehat{\bar{v}_i \bar{v}_j}}{\partial x_j} + \frac{\partial \widehat{\bar{v}}_i \widehat{\bar{v}}_j}{\partial x_j} = \frac{\partial \widehat{\bar{v}}_i \widehat{\bar{v}}_j}{\partial x_j} \\
\text{Right side} &: \underbrace{-\frac{\partial \widehat{\bar{v}_i \bar{v}_j}}{\partial x_j} + \frac{\partial \widehat{\bar{v}}_i \widehat{\bar{v}}_j}{\partial x_j}}_{-\partial \mathcal{L}_{ij}/\partial x_j}
\end{aligned}$$

▶We get

$$\frac{\partial \widehat{\bar{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\bar{v}}_i \widehat{\bar{v}}_j\right) = -\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\bar{v}}_i}{\partial x_j \partial x_j} - \frac{\partial \widehat{\tau}_{ij}}{\partial x_j} - \frac{\partial \mathcal{L}_{ij}}{\partial x_j} \tag{37.2}$$

Identification of Eqs. 37.1 and 37.2 gives

$$T_{ij} = \widehat{\overline{v_i v_j}} - \widehat{\bar{v}}_i \widehat{\bar{v}}_j + \widehat{\tau}_{ij} = \tag{37.3}$$

$$\frac{\partial \widehat{\bar{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\bar{v}}_i \widehat{\bar{v}}_j\right) = -\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\bar{v}}_i}{\partial x_j \partial x_j} - \frac{\partial T_{ij}}{\partial x_j}, \quad T_{ij} = \widehat{\overline{v_i v_j}} - \widehat{\bar{v}}_i \widehat{\bar{v}}_j \tag{37.1}$$

▶Second, we test filter the LES equations

$$\begin{aligned}
\frac{\partial \widehat{\bar{v}}_i}{\partial t} + \frac{\partial \widehat{\bar{v}_i \bar{v}_j}}{\partial x_j} &= -\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\bar{v}}_i}{\partial x_j \partial x_j} - \frac{\partial \widehat{\tau}_{ij}}{\partial x_j} \\
\text{Left side} \quad &: \quad \frac{\partial \widehat{\bar{v}_i \bar{v}_j}}{\partial x_j} - \frac{\partial \widehat{\bar{v}_i \bar{v}_j}}{\partial x_j} + \frac{\partial \widehat{\bar{v}}_i \widehat{\bar{v}}_j}{\partial x_j} = \frac{\partial \widehat{\bar{v}}_i \widehat{\bar{v}}_j}{\partial x_j} \\
\text{Right side} \quad &: \quad \underbrace{-\frac{\partial \widehat{\bar{v}_i \bar{v}_j}}{\partial x_j} + \frac{\partial \widehat{\bar{v}}_i \widehat{\bar{v}}_j}{\partial x_j}}_{-\partial \mathcal{L}_{ij}/\partial x_j}
\end{aligned}$$

▶We get

$$\frac{\partial \widehat{\bar{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\bar{v}}_i \widehat{\bar{v}}_j\right) = -\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\bar{v}}_i}{\partial x_j \partial x_j} - \frac{\partial \widehat{\tau}_{ij}}{\partial x_j} - \frac{\partial \mathcal{L}_{ij}}{\partial x_j} \tag{37.2}$$

Identification of Eqs. 37.1 and 37.2 gives

$$T_{ij} = \widehat{\bar{v}_i \bar{v}_j} - \widehat{\bar{v}}_i \widehat{\bar{v}}_j + \widehat{\tau}_{ij} = \mathcal{L}_{ij} + \widehat{\tau}_{ij} \tag{37.3}$$

$$\frac{\partial \widehat{\overline{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\overline{v}}_i \widehat{\overline{v}}_j\right) = -\frac{1}{\rho}\frac{\partial \widehat{\overline{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\overline{v}}_i}{\partial x_j \partial x_j} - \frac{\partial T_{ij}}{\partial x_j}, \quad T_{ij} = \widehat{\overline{v_i v_j}} - \widehat{\overline{v}}_i \widehat{\overline{v}}_j \tag{37.1}$$

► Second, we test filter the LES equations

$$\frac{\partial \widehat{\overline{v}}_i}{\partial t} + \frac{\partial \widehat{\overline{v}_i \overline{v}_j}}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \widehat{\overline{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\overline{v}}_i}{\partial x_j \partial x_j} - \frac{\partial \widehat{\tau}_{ij}}{\partial x_j}$$

$$\text{Left side} \quad : \quad \frac{\partial \widehat{\overline{v}_i \overline{v}_j}}{\partial x_j} - \frac{\partial \widehat{\overline{v}_i \overline{v}_j}}{\partial x_j} + \frac{\partial \widehat{\overline{v}}_i \widehat{\overline{v}}_j}{\partial x_j} = \frac{\partial \widehat{\overline{v}}_i \widehat{\overline{v}}_j}{\partial x_j}$$

$$\text{Right side} \quad : \quad \underbrace{-\frac{\partial \widehat{\overline{v}_i \overline{v}_j}}{\partial x_j} + \frac{\partial \widehat{\overline{v}}_i \widehat{\overline{v}}_j}{\partial x_j}}_{-\partial \mathcal{L}_{ij}/\partial x_j}$$

► We get

$$\frac{\partial \widehat{\overline{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\overline{v}}_i \widehat{\overline{v}}_j\right) = -\frac{1}{\rho}\frac{\partial \widehat{\overline{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\overline{v}}_i}{\partial x_j \partial x_j} - \frac{\partial \widehat{\tau}_{ij}}{\partial x_j} - \frac{\partial \mathcal{L}_{ij}}{\partial x_j} \tag{37.2}$$

Identification of Eqs. 37.1 and 37.2 gives

$$T_{ij} = \widehat{\overline{v_i v_j}} - \widehat{\overline{v}}_i \widehat{\overline{v}}_j + \widehat{\tau}_{ij} = \mathcal{L}_{ij} + \widehat{\tau}_{ij}, \quad \frac{1}{3}\delta_{ij} T_{kk} = \tag{37.3}$$

$$\frac{\partial \widehat{\bar{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\bar{v}}_i \widehat{\bar{v}}_j\right) = -\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\bar{v}}_i}{\partial x_j \partial x_j} - \frac{\partial T_{ij}}{\partial x_j}, \quad T_{ij} = \widehat{\overline{v_i v_j}} - \widehat{\bar{v}}_i \widehat{\bar{v}}_j \tag{37.1}$$

►Second, we test filter the LES equations

$$\frac{\partial \widehat{\bar{v}}_i}{\partial t} + \frac{\partial \widehat{\bar{v}_i \bar{v}_j}}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\bar{v}}_i}{\partial x_j \partial x_j} - \frac{\partial \widehat{\tau}_{ij}}{\partial x_j}$$

$$\text{Left side} : \frac{\partial \widehat{\bar{v}_i \bar{v}_j}}{\partial x_j} - \frac{\partial \widehat{\bar{v}_i \bar{v}_j}}{\partial x_j} + \frac{\partial \widehat{\bar{v}}_i \widehat{\bar{v}}_j}{\partial x_j} = \frac{\partial \widehat{\bar{v}}_i \widehat{\bar{v}}_j}{\partial x_j}$$

$$\text{Right side} : \underbrace{-\frac{\partial \widehat{\bar{v}_i \bar{v}_j}}{\partial x_j} + \frac{\partial \widehat{\bar{v}}_i \widehat{\bar{v}}_j}{\partial x_j}}_{-\partial \mathcal{L}_{ij}/\partial x_j}$$

►We get

$$\frac{\partial \widehat{\bar{v}}_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\widehat{\bar{v}}_i \widehat{\bar{v}}_j\right) = -\frac{1}{\rho}\frac{\partial \widehat{\bar{p}}}{\partial x_i} + \nu \frac{\partial^2 \widehat{\bar{v}}_i}{\partial x_j \partial x_j} - \frac{\partial \widehat{\tau}_{ij}}{\partial x_j} - \frac{\partial \mathcal{L}_{ij}}{\partial x_j} \tag{37.2}$$

Identification of Eqs. 37.1 and 37.2 gives

$$T_{ij} = \widehat{\bar{v}_i \bar{v}_j} - \widehat{\bar{v}}_i \widehat{\bar{v}}_j + \widehat{\tau}_{ij} = \mathcal{L}_{ij} + \widehat{\tau}_{ij}, \quad \frac{1}{3}\delta_{ij} T_{kk} = \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + \frac{1}{3}\delta_{ij}\widehat{\tau}_{kk} \tag{37.3}$$

$$T_{ij} - \frac{1}{3}\delta_{ij}T_{kk} + \widehat{\tau}_{ij} - \frac{1}{3}\delta_{ij}\widehat{\tau}_{kk} = \mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} \tag{37.4}$$

$$T_{ij} - \frac{1}{3}\delta_{ij}T_{kk} + \widehat{\tau}_{ij} - \frac{1}{3}\delta_{ij}\widehat{\tau}_{kk} = \mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} \quad (37.4)$$

►Smagorinsky model for both grid and test level SGS stresses:

(37.5)

(37.6)

$$T_{ij} - \frac{1}{3}\delta_{ij}T_{kk} + \widehat{\tau}_{ij} - \frac{1}{3}\delta_{ij}\widehat{\tau}_{kk} = \mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} \tag{37.4}$$

►Smagorinsky model for both grid and test level SGS stresses:

$$\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} = \tag{37.5}$$

$$\tag{37.6}$$

$$T_{ij} - \frac{1}{3}\delta_{ij}T_{kk} + \widehat{\tau}_{ij} - \frac{1}{3}\delta_{ij}\widehat{\tau}_{kk} = \mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} \quad (37.4)$$

►Smagorinsky model for both grid and test level SGS stresses:

$$\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} \; = \; -2C\Delta^2|\bar{s}|\bar{s}_{ij} \quad (37.5)$$

$$(37.6)$$

$$T_{ij} - \frac{1}{3}\delta_{ij}T_{kk} + \widehat{\tau}_{ij} - \frac{1}{3}\delta_{ij}\widehat{\tau}_{kk} = \mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} \tag{37.4}$$

►Smagorinsky model for both grid and test level SGS stresses:

$$\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} = -2C\Delta^2|\bar{s}|\bar{s}_{ij} \tag{37.5}$$

$$T_{ij} - \frac{1}{3}\delta_{ij}T_{kk} = \tag{37.6}$$

$$T_{ij} - \frac{1}{3}\delta_{ij}T_{kk} + \widehat{\tau}_{ij} - \frac{1}{3}\delta_{ij}\widehat{\tau}_{kk} = \mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} \quad (37.4)$$

►Smagorinsky model for both grid and test level SGS stresses:

$$\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} = -2C\Delta^2|\bar{s}|\bar{s}_{ij} \quad (37.5)$$

$$T_{ij} - \frac{1}{3}\delta_{ij}T_{kk} = -2C\widehat{\Delta}^2|\widehat{\bar{s}}|\widehat{\bar{s}}_{ij} \quad (37.6)$$

$$T_{ij} - \frac{1}{3}\delta_{ij}T_{kk} + \widehat{\tau}_{ij} - \frac{1}{3}\delta_{ij}\widehat{\tau}_{kk} = \mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} \tag{37.4}$$

►Smagorinsky model for both grid and test level SGS stresses:

$$\tau_{ij} - \frac{1}{3}\delta_{ij}\tau_{kk} = -2C\Delta^2|\bar{s}|\bar{s}_{ij} \tag{37.5}$$

$$T_{ij} - \frac{1}{3}\delta_{ij}T_{kk} = -2C\widehat{\Delta}^2|\widehat{\bar{s}}|\widehat{\bar{s}}_{ij} \tag{37.6}$$

where

$$\widehat{\bar{s}}_{ij} = \frac{1}{2}\left(\frac{\partial\widehat{\bar{v}}_i}{\partial x_j} + \frac{\partial\widehat{\bar{v}}_j}{\partial x_i}\right), \ |\widehat{\bar{s}}| = \left(2\widehat{\bar{s}}_{ij}\widehat{\bar{s}}_{ij}\right)^{1/2}$$

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►Three equations, three unknowns!

$$T_{ij} - \frac{1}{3}\delta_{ij}T_{kk} + \widehat{\tau}_{ij} - \frac{1}{3}\delta_{ij}\widehat{\tau}_{kk} = \mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} \tag{37.4}$$

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►Eqs. 37.5. 37.6 into Eq. 37.4 gives

$$\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} = -2\left(C\widehat{\Delta}^2|\widehat{\bar{s}}|\widehat{\bar{s}}_{ij} - \widehat{C\Delta^2|\bar{s}|\bar{s}_{ij}}\right)$$

$$T_{ij} - \frac{1}{3}\delta_{ij}T_{kk} + \widehat{\tau}_{ij} - \frac{1}{3}\delta_{ij}\widehat{\tau}_{kk} = \mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} \tag{37.4}$$

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►We need to yank C out of the test filter;

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$$\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} = -2C\left(\widehat{\Delta}^2|\widehat{\bar{s}}|\widehat{\bar{s}}_{ij} - \widehat{\Delta^2|\bar{s}|\bar{s}_{ij}}\right)$$

$$\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + 2C \underbrace{\left(\widehat{\Delta}^2 |\widehat{\bar{s}}| \widehat{\bar{s}}_{ij} - \Delta^2 \widehat{|\bar{s}|\bar{s}_{ij}} \right)}_{M_{ij}} = 0$$

$$\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + 2C\underbrace{\left(\widehat{\Delta}^2 |\widehat{\bar{s}}|\widehat{\bar{s}}_{ij} - \Delta^2 \widehat{|\bar{s}|\bar{s}_{ij}}\right)}_{M_{ij}} = 0$$

► Now we get

$$\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + 2C \underbrace{\left(\widehat{\Delta}^2 |\widehat{\bar{s}}| \widehat{\bar{s}}_{ij} - \Delta^2 \widehat{|\bar{s}|\bar{s}_{ij}} \right)}_{M_{ij}} = 0$$

▶Now we get

$$\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + 2CM_{ij} = 0 \tag{37.7}$$

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$$\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + 2CM_{ij} = 0 \tag{37.7}$$

▶This cannot be satisfied for all i, j ▶Least-square problem:

$$Q = \left(\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + 2CM_{ij}\right)^2$$

$$\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + 2C \underbrace{\left(\widehat{\Delta}^2 |\widehat{\bar{s}}| \widehat{\bar{s}}_{ij} - \Delta^2 \widehat{|\bar{s}|\bar{s}_{ij}} \right)}_{M_{ij}} = 0$$

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►Find a minimum of Q which best satisfies Eq. 37.7 for all i, j

(37.8)

$$\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + 2C\underbrace{\left(\widehat{\Delta}^2 |\widehat{\bar{s}}|\widehat{\bar{s}}_{ij} - \Delta^2 \widehat{|\bar{s}|\bar{s}_{ij}}\right)}_{M_{ij}} = 0$$

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$$\frac{\partial Q}{\partial C} = \tag{37.8}$$

$$\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + 2C\underbrace{\left(\widehat{\Delta}^2|\widehat{\bar{s}}|\widehat{\bar{s}}_{ij} - \Delta^2\widehat{|\bar{s}|\bar{s}_{ij}}\right)}_{M_{ij}} = 0$$

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$$\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + 2CM_{ij} = 0 \qquad (37.7)$$

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►Find a minimum of Q which best satisfies Eq. 37.7 for all i, j

$$\frac{\partial Q}{\partial C} = 4M_{ij}\left(\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + 2CM_{ij}\right) = \qquad (37.8)$$

$$\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + 2C \underbrace{\left(\widehat{\Delta}^2 |\widehat{\bar{s}}| \widehat{\bar{s}}_{ij} - \Delta^2 \widehat{|\bar{s}| \bar{s}_{ij}} \right)}_{M_{ij}} = 0$$

►Now we get

$$\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + 2CM_{ij} = 0 \tag{37.7}$$

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$$\frac{\partial Q}{\partial C} = 4M_{ij} \left(\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + 2CM_{ij} \right) = 4M_{ij} \left(\mathcal{L}_{ij} + 2CM_{ij} \right) = 0 \tag{37.8}$$

$$\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + 2C \underbrace{\left(\widehat{\Delta}^2 |\widehat{\bar{s}}|\widehat{\bar{s}}_{ij} - \Delta^2 \widehat{|\bar{s}|\bar{s}_{ij}} \right)}_{M_{ij}} = 0$$

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since $\frac{1}{3}\delta_{ij}\mathcal{L}_{kk}M_{ij} = \frac{1}{3}\mathcal{L}_{kk}M_{ii} = 0$ since $\widehat{\bar{s}}_{ii} = \bar{s}_{ii} = 0$ thanks to continuity.

$$\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + 2C \underbrace{\left(\widehat{\Delta}^2 |\widehat{\bar{s}}| \widehat{\bar{s}}_{ij} - \Delta^2 \widehat{|\bar{s}|\bar{s}_{ij}} \right)}_{M_{ij}} = 0$$

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Eq. 37.8: Minimum or maximunm?

$$\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + 2C \underbrace{\left(\widehat{\Delta}^2 |\widehat{\bar{s}}|\widehat{\bar{s}}_{ij} - \Delta^2 \widehat{|\bar{s}|\bar{s}_{ij}} \right)}_{M_{ij}} = 0$$

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Eq. 37.8: Minimum or maximunm?

►$\partial^2 Q/\partial C^2 = 8M_{ij}M_{ij} > 0$

$$\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + 2C \underbrace{\left(\widehat{\Delta}^2 |\widehat{\bar{s}}| \widehat{\bar{s}}_{ij} - \Delta^2 \widehat{|\bar{s}|\bar{s}_{ij}} \right)}_{M_{ij}} = 0$$

►Now we get

$$\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + 2CM_{ij} = 0 \tag{37.7}$$

►This cannot be satisfied for all i, j ►Least-square problem:

$$Q = \left(\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + 2CM_{ij} \right)^2$$

►Find a minimum of Q which best satisfies Eq. 37.7 for all i, j

$$\frac{\partial Q}{\partial C} = 4M_{ij} \left(\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + 2CM_{ij} \right) = 4M_{ij} \left(\mathcal{L}_{ij} + 2CM_{ij} \right) = 0 \tag{37.8}$$

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Eq. 37.8: Minimum or maximunm?

►$\partial^2 Q / \partial C^2 = 8M_{ij}M_{ij} > 0$ ►Hence, minimum

$$\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + 2C\underbrace{\left(\widehat{\Delta}^2|\widehat{\bar{s}}|\widehat{\bar{s}}_{ij} - \Delta^2\widehat{|\bar{s}|\bar{s}_{ij}}\right)}_{M_{ij}} = 0$$

►Now we get

$$\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + 2CM_{ij} = 0 \tag{37.7}$$

►This cannot be satisfied for all i, j ►Least-square problem:

$$Q = \left(\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + 2CM_{ij}\right)^2$$

►Find a minimum of Q which best satisfies Eq. 37.7 for all i, j

$$\frac{\partial Q}{\partial C} = 4M_{ij}\left(\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} + 2CM_{ij}\right) = 4M_{ij}\left(\mathcal{L}_{ij} + 2CM_{ij}\right) = 0 \tag{37.8}$$

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Eq. 37.8: Minimum or maximunm?

►$\partial^2 Q/\partial C^2 = 8M_{ij}M_{ij} > 0$ ►Hence, minimum (fortunately)

$$\frac{\partial Q}{\partial C} = 4M_{ij}\left(\mathcal{L}_{ij} + 2CM_{ij}\right) = 0$$

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▶ We get

$$\frac{\partial Q}{\partial C} = 4M_{ij}\left(\mathcal{L}_{ij} + 2CM_{ij}\right) = 0$$

- We get

$$C = -\frac{\mathcal{L}_{ij}M_{ij}}{2M_{ij}M_{ij}}$$

$$\frac{\partial Q}{\partial C} = 4M_{ij}\left(\mathcal{L}_{ij} + 2CM_{ij}\right) = 0$$

- We get

$$C = -\frac{\mathcal{L}_{ij}M_{ij}}{2M_{ij}M_{ij}}, \quad \text{stability problems: needs smoothing}$$

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¶ See Section 18.12, The test filter

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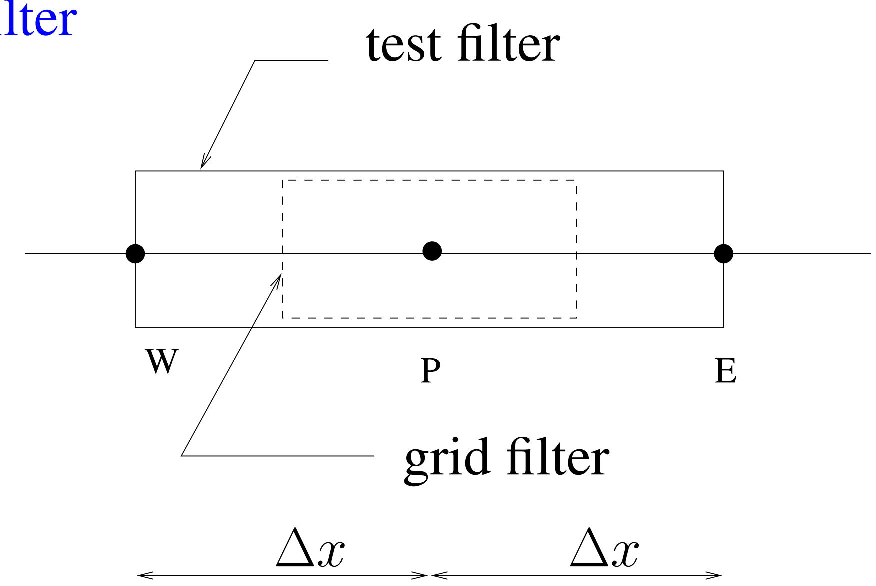
¶See Section 18.12, The test filter



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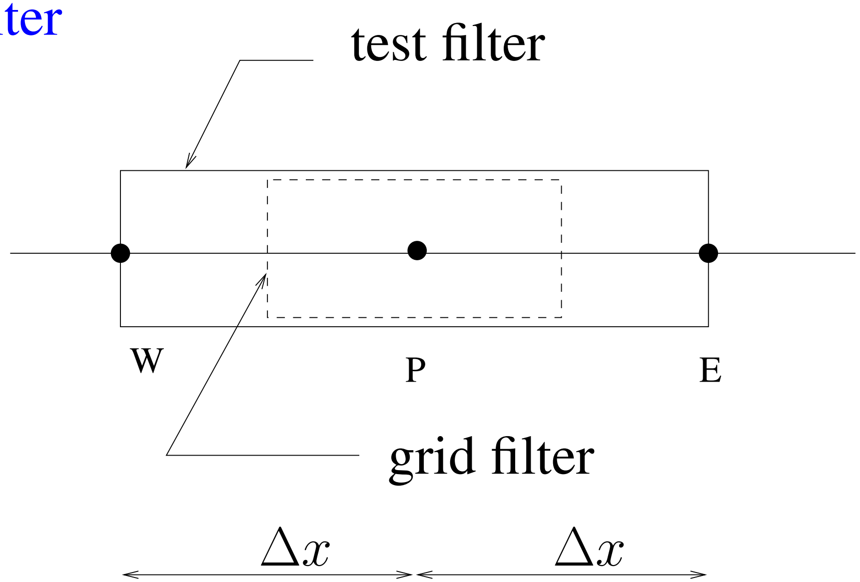

$\widehat{\bar{v}}_P$ is computed as ($\widehat{\Delta x} = 2\Delta x$)

P

$$\frac{\partial Q}{\partial C} = 4M_{ij}\left(\mathcal{L}_{ij} + 2CM_{ij}\right) = 0$$

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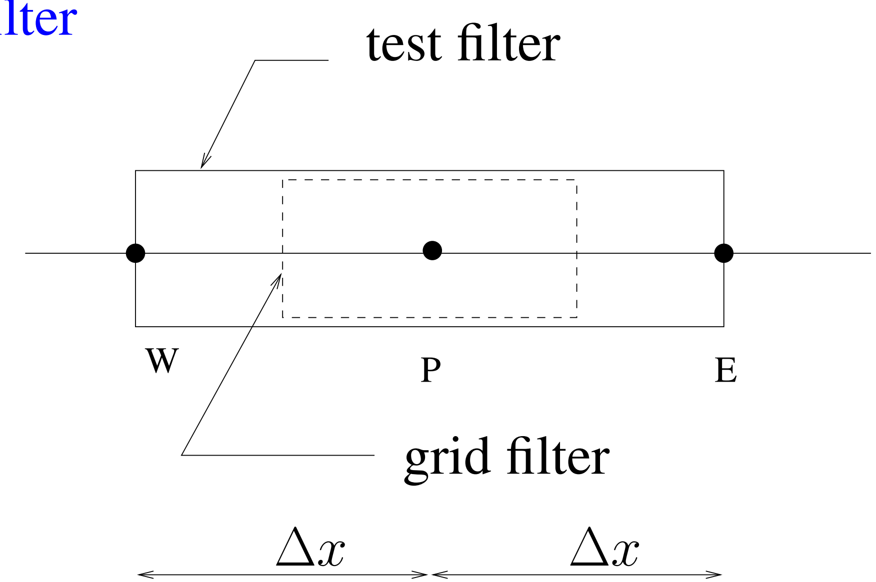
¶See Section 18.12, The test filter



$\widehat{\overline{v}}_P$ is computed as ($\widehat{\Delta x} = 2\Delta x$)

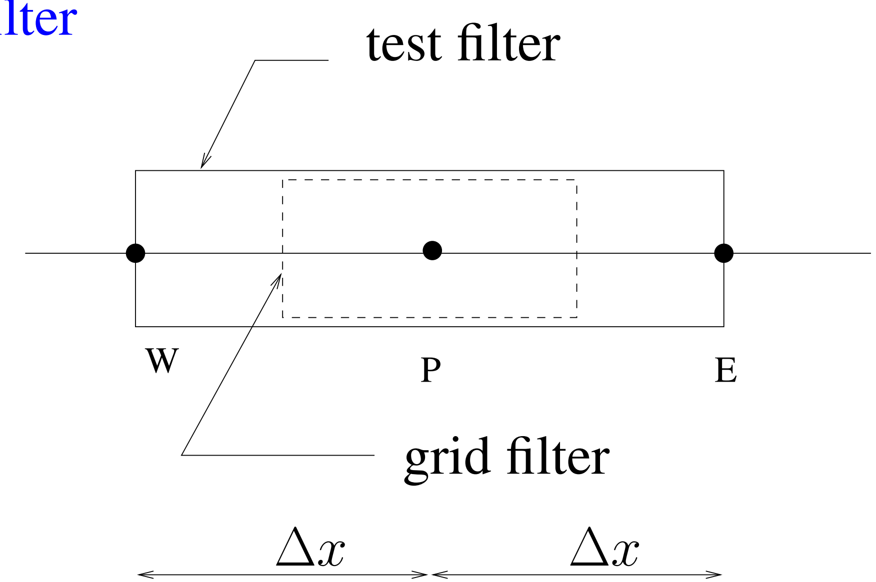
$$\widehat{\overline{v}}_P =$$

$$\frac{\partial Q}{\partial C} = 4M_{ij}\left(\mathcal{L}_{ij} + 2CM_{ij}\right) = 0$$

▶We get

$$C = -\frac{\mathcal{L}_{ij}M_{ij}}{2M_{ij}M_{ij}}, \quad \text{stability problems: needs smoothing}$$

¶See Section 18.12, The test filter



$\widehat{\overline{v}}_P$ is computed as ($\widehat{\Delta x} = 2\Delta x$)

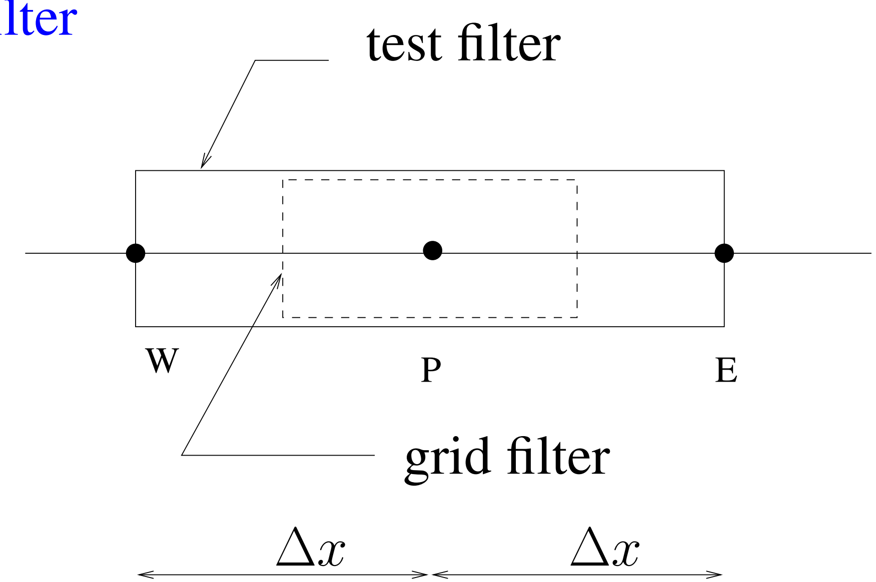
$$\widehat{\overline{v}}_P = \frac{1}{2\Delta x}\int_W^E \bar{v}dx =$$

$$\frac{\partial Q}{\partial C} = 4M_{ij}\left(\mathcal{L}_{ij} + 2CM_{ij}\right) = 0$$

▶ We get

$$C = -\frac{\mathcal{L}_{ij}M_{ij}}{2M_{ij}M_{ij}}, \quad \text{stability problems: needs smoothing}$$

¶ See Section 18.12, The test filter



$\widehat{\bar{v}}_P$ is computed as ($\widehat{\Delta x} = 2\Delta x$)

$$\widehat{\bar{v}}_P = \frac{1}{2\Delta x}\int_W^E \bar{v}dx = \frac{1}{2\Delta x}\left(\int_W^P \bar{v}dx + \int_P^E \bar{v}dx\right)$$

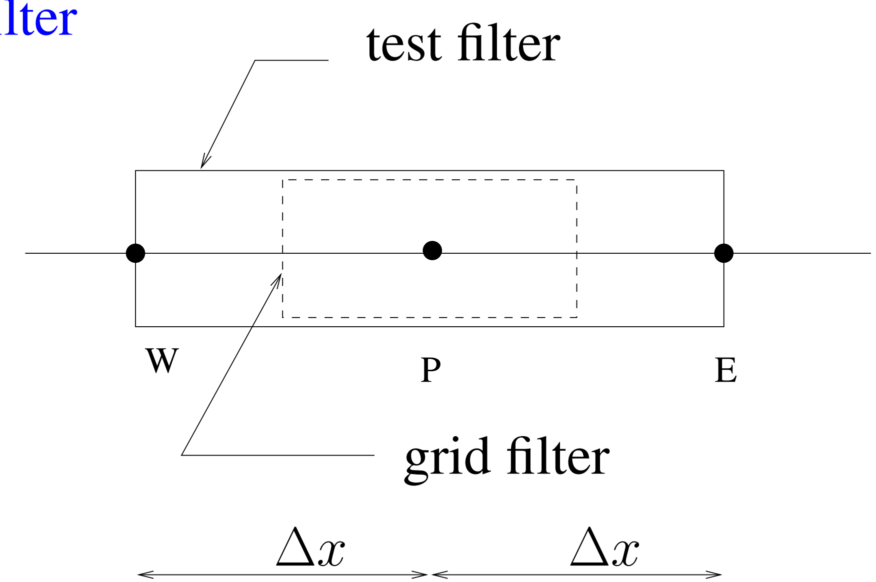
$$=$$

$$\frac{\partial Q}{\partial C} = 4M_{ij}\left(\mathcal{L}_{ij} + 2CM_{ij}\right) = 0$$

▶We get

$$C = -\frac{\mathcal{L}_{ij}M_{ij}}{2M_{ij}M_{ij}}, \quad \text{stability problems: needs smoothing}$$

¶See Section 18.12, The test filter



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$$\begin{aligned}\widehat{\bar{v}}_P &= \frac{1}{2\Delta x}\int_W^E \bar{v}dx = \frac{1}{2\Delta x}\left(\int_W^P \bar{v}dx + \int_P^E \bar{v}dx\right)\\ &= \frac{1}{2\Delta x}\left(\bar{v}_w\Delta x + \bar{v}_e\Delta x\right) =\end{aligned}$$

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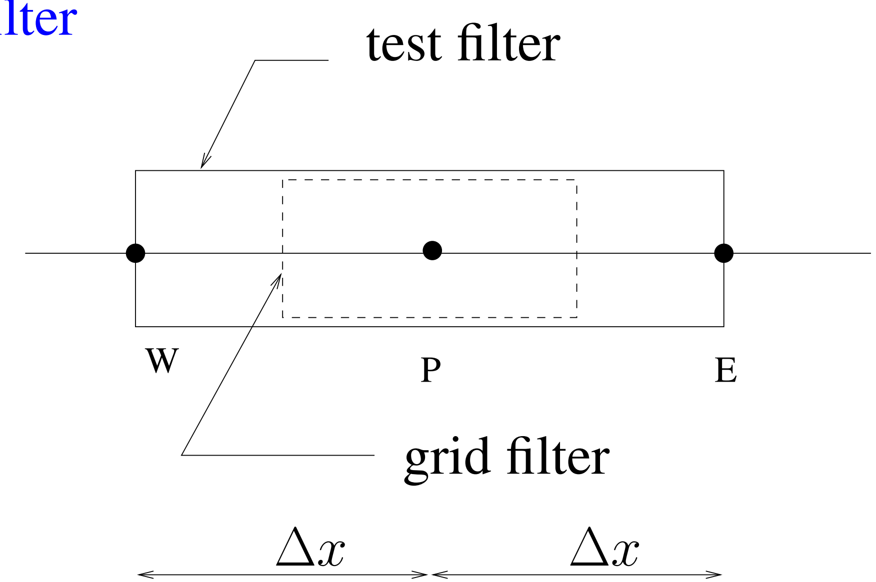

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&= \frac{1}{2\Delta x}\left(\bar{v}_w\Delta x + \bar{v}_e\Delta x\right) = \frac{1}{2}\left(\frac{\bar{v}_W + \bar{v}_P}{2} + \frac{\bar{v}_P + \bar{v}_E}{2}\right) =
\end{aligned}$$

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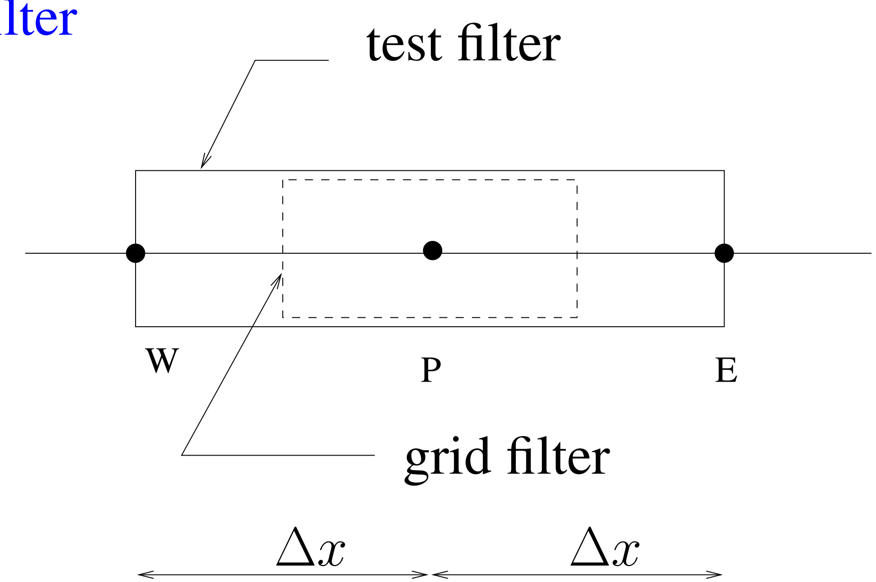

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\end{aligned}$$

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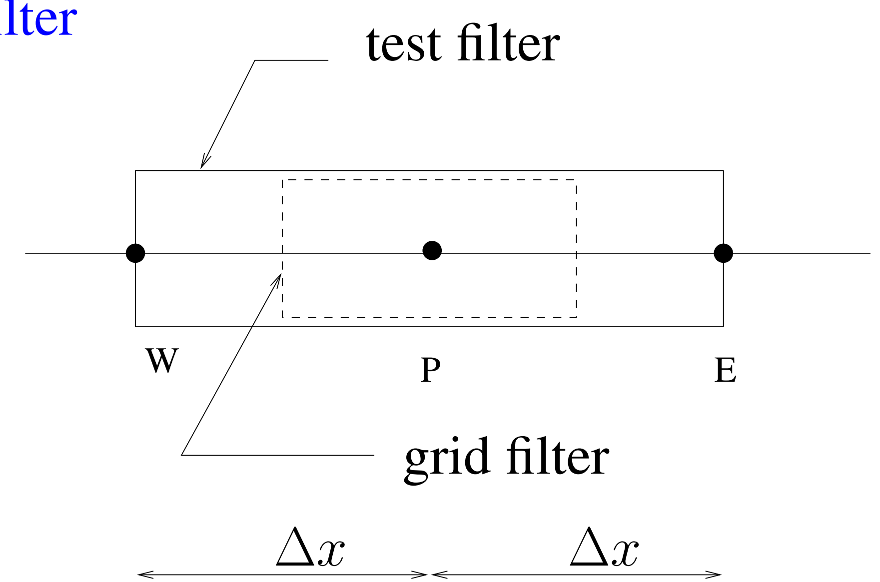

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\end{aligned}$$

$$C = -\frac{\mathcal{L}_{ij} M_{ij}}{2 M_{ij} M_{ij}}, \quad \nu_{sgs} = C \Delta^2 |\bar{s}|$$

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▶ Is C positive?

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▶Is C positive? ▶Do we want it to stay positive?

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$$\nu_{tot} =$$

$$C = -\frac{\mathcal{L}_{ij} M_{ij}}{2 M_{ij} M_{ij}}, \qquad \nu_{sgs} = C \Delta^2 |\bar{s}|$$

▶Is C positive? ▶Do we want it to stay positive? ▶Limits on C?

$$\nu_{tot} = \nu + \nu_{sgs} =$$

$$C = -\frac{\mathcal{L}_{ij} M_{ij}}{2 M_{ij} M_{ij}}, \qquad \nu_{sgs} = C \Delta^2 |\bar{s}|$$

▶Is C positive? ▶Do we want it to stay positive? ▶Limits on C?

$$\nu_{tot} = \nu + \nu_{sgs} = \nu + C \Delta^2 |\bar{\bar{s}}| > 0$$

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$$\nu_{tot} = \nu + \nu_{sgs} = \nu + C \Delta^2 |\bar{\bar{s}}| > 0 \quad \Rightarrow \nu_{sgs}$$

$$C = -\frac{\mathcal{L}_{ij} M_{ij}}{2 M_{ij} M_{ij}}, \qquad \nu_{sgs} = C \Delta^2 |\bar{s}|$$

▶Is C positive? ▶Do we want it to stay positive? ▶Limits on C?

$$\nu_{tot} = \nu + \nu_{sgs} = \nu + C\Delta^2 |\bar{\bar{s}}| > 0 \quad \Rightarrow \nu_{sgs} > -\nu$$

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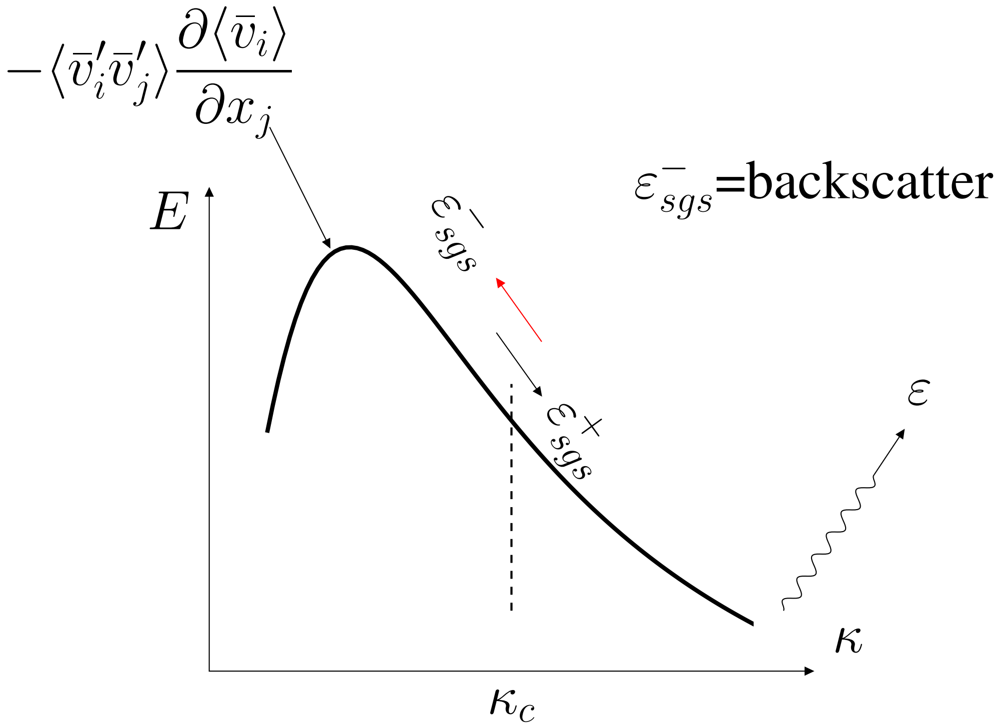

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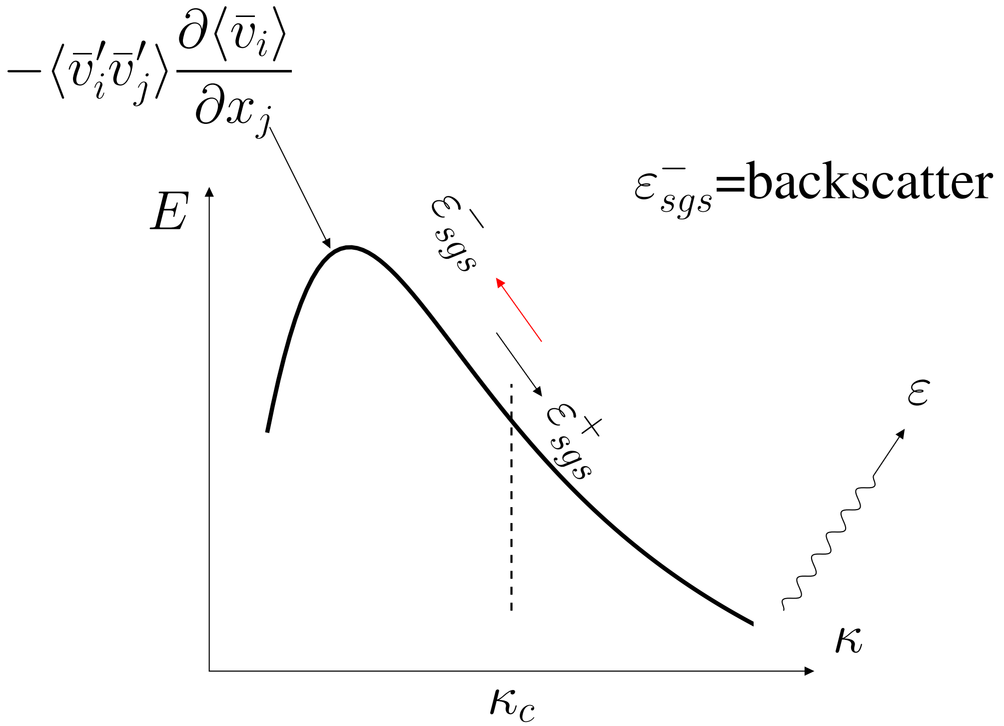

$$\varepsilon_{sgs} = 2\nu_{sgs} \bar{s}_{ij} \bar{s}_{ij} = \varepsilon_{sgs}^{+} + \varepsilon_{sgs}^{-}$$

¶See Section 18.13, Stresses on grid, test and intermediate level

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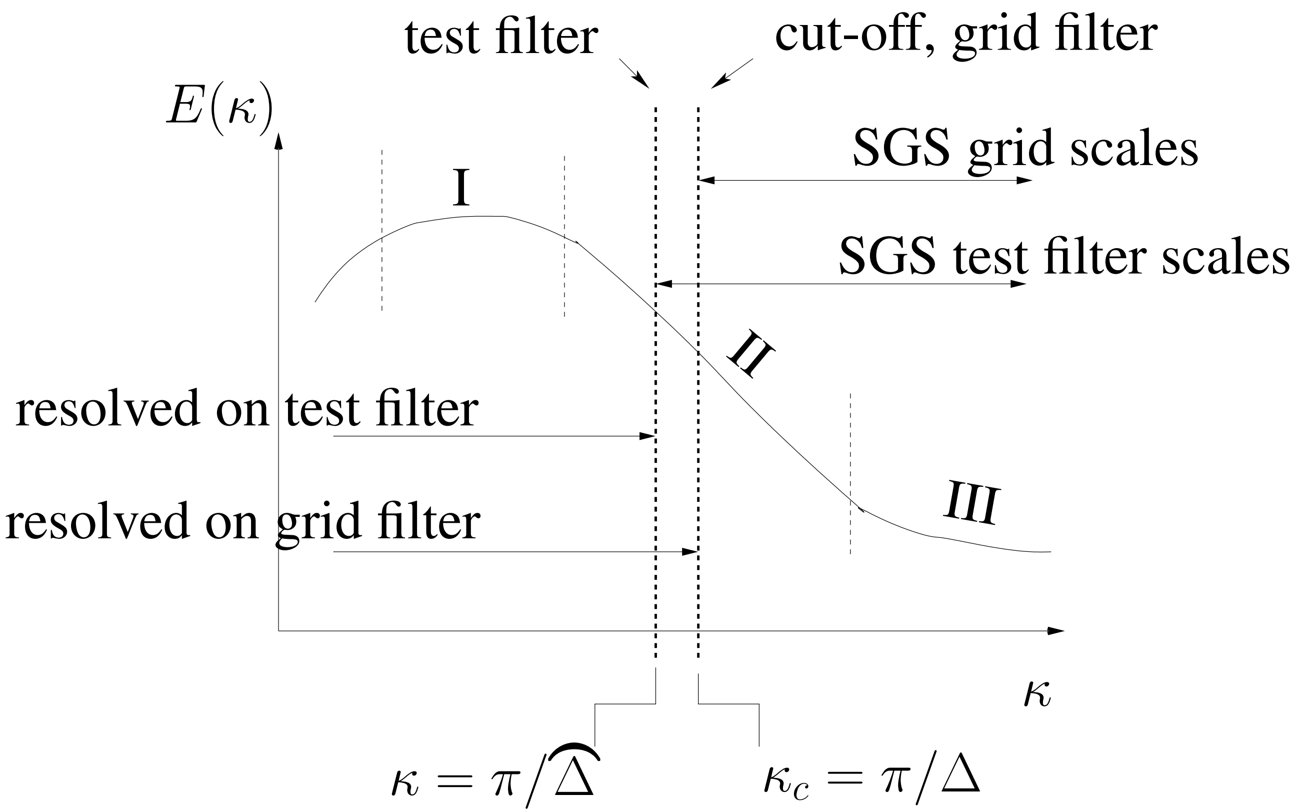

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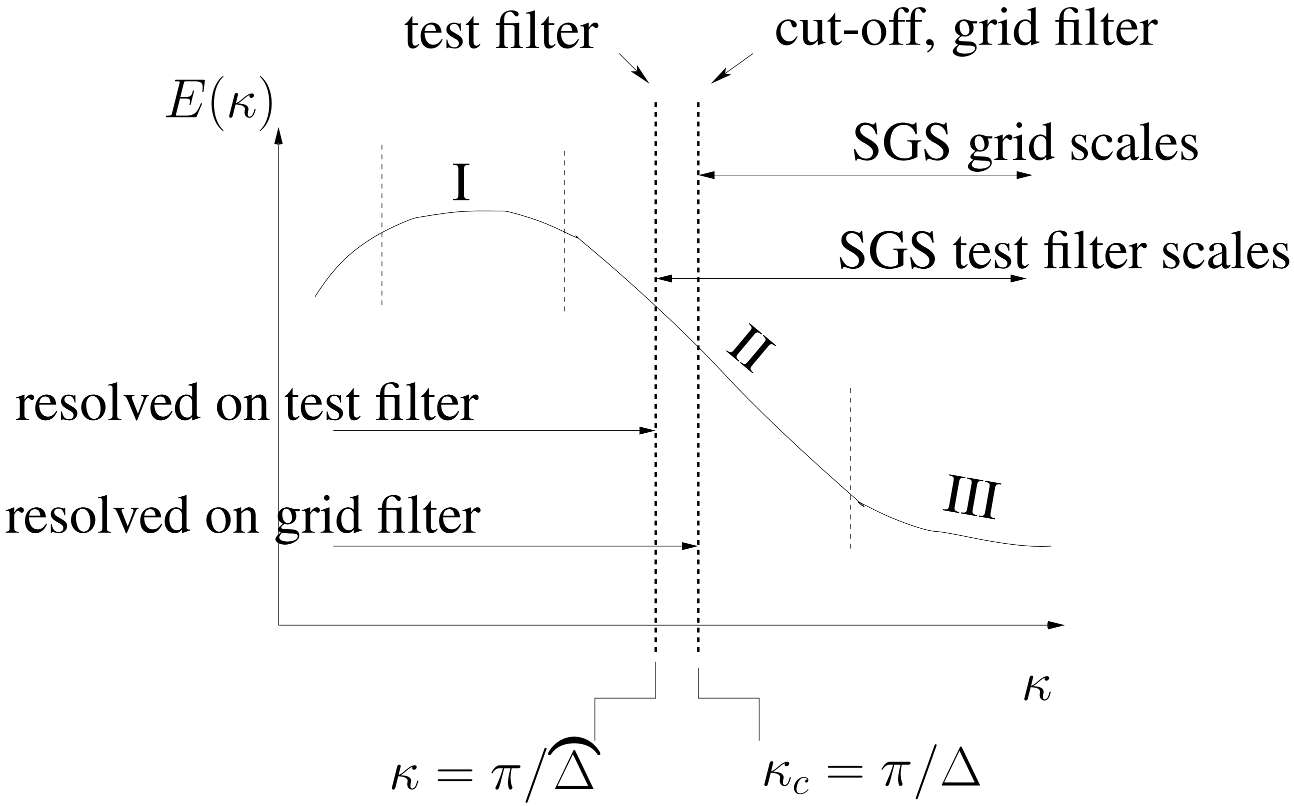

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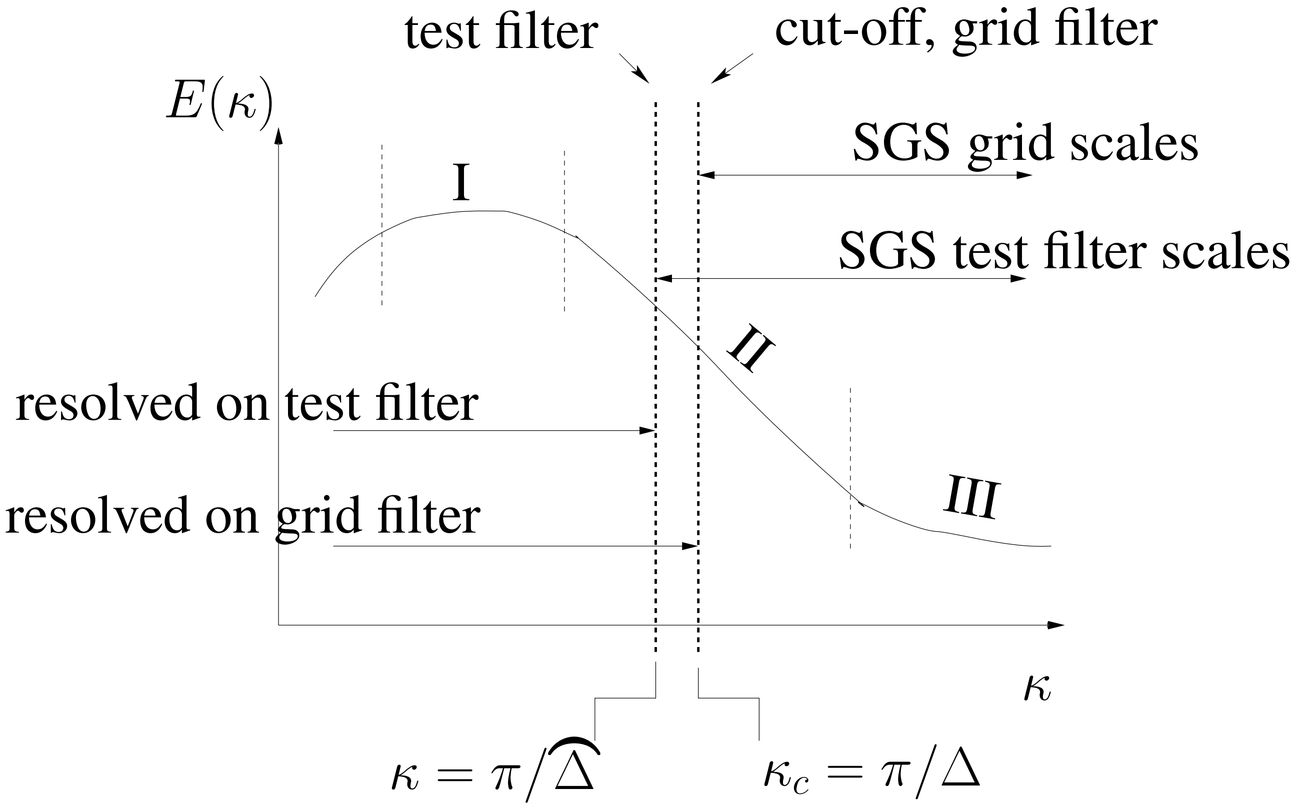
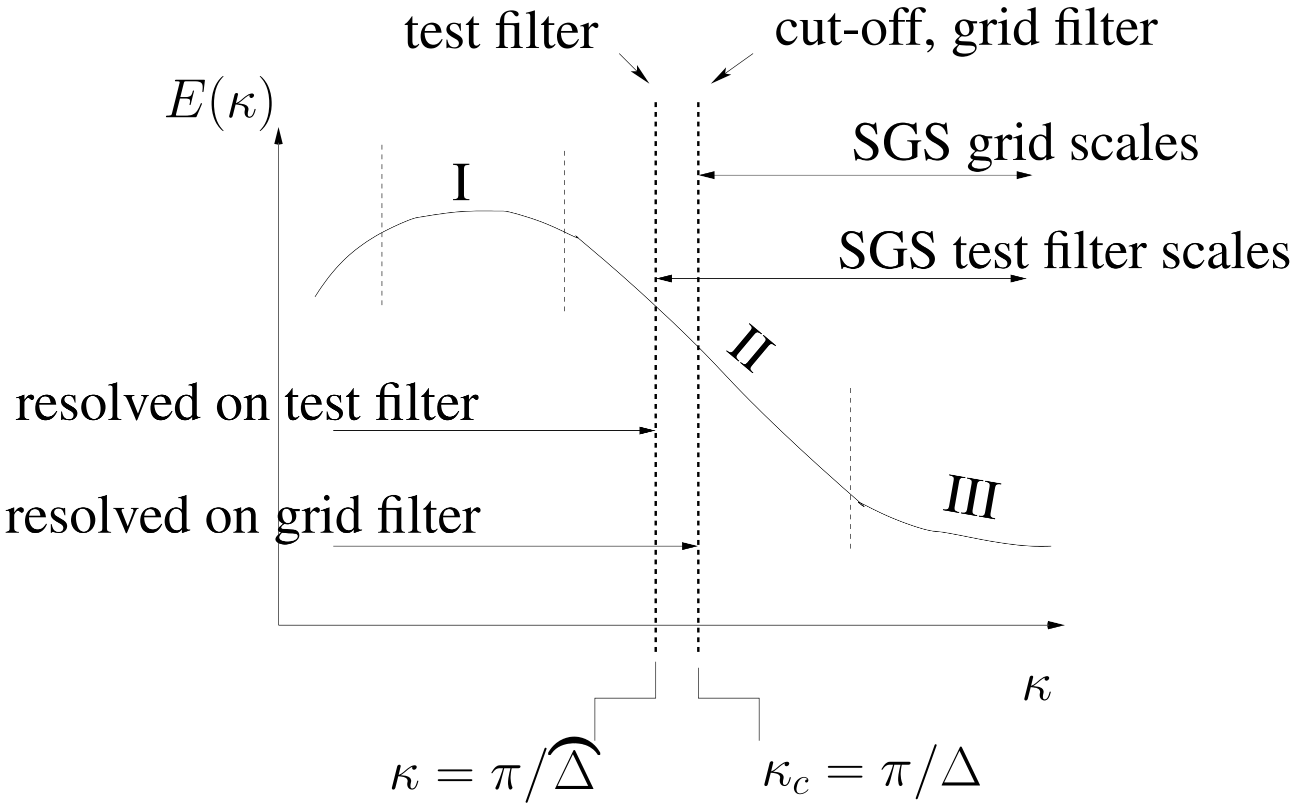


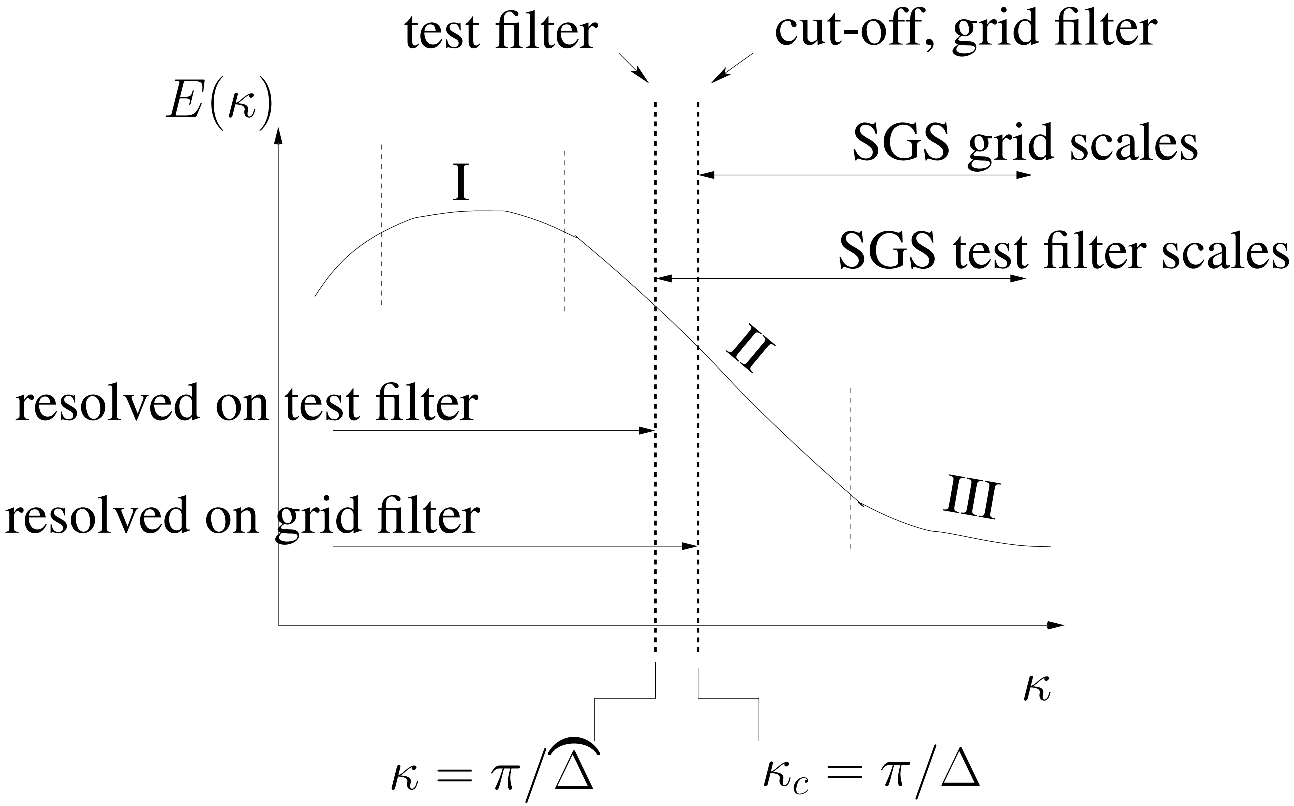
$$\tau_{ij} = \overline{v_i v_j} - \bar{v}_i \bar{v}_j$$

¶See Section 18.13, Stresses on grid, test and intermediate level



$\tau_{ij} = \overline{v_i v_j} - \bar{v}_i \bar{v}_j$ **stresses with $\ell < \Delta$**

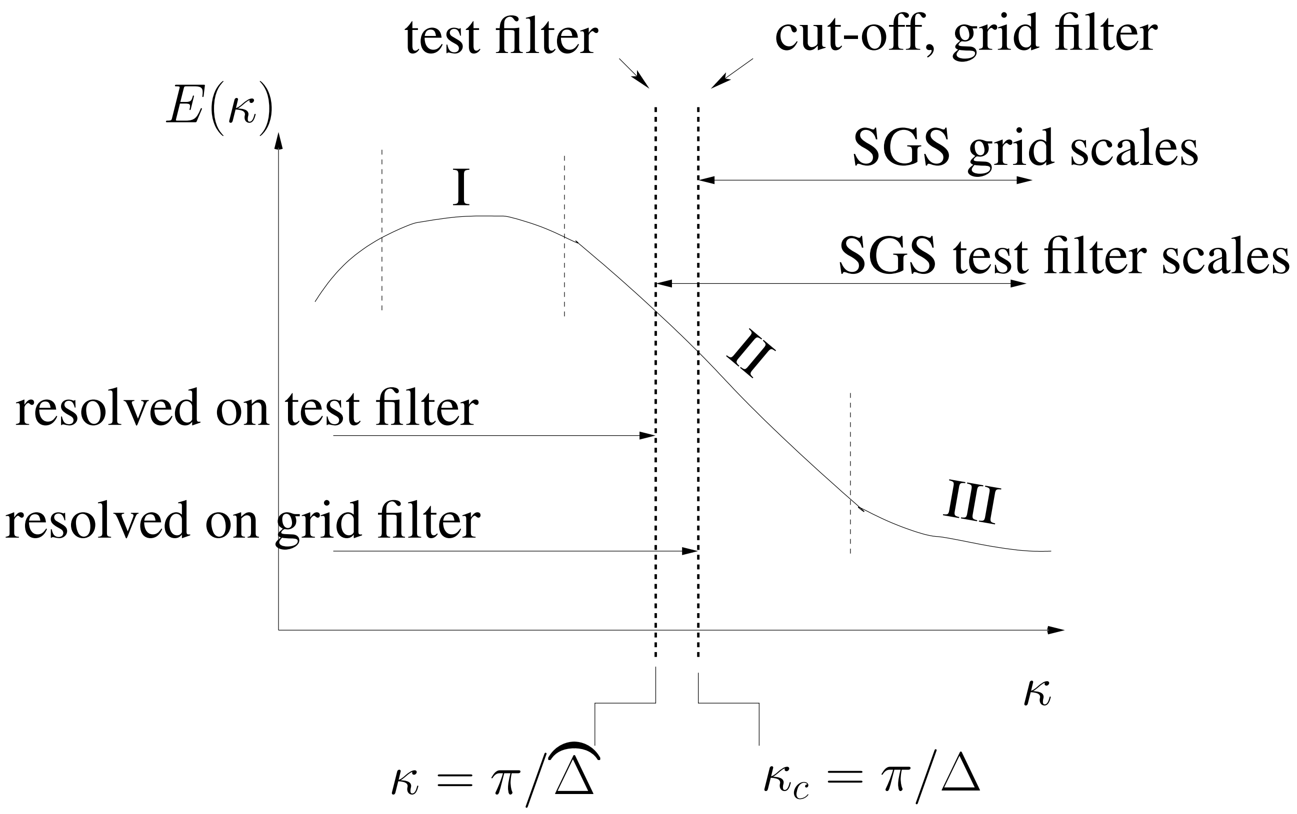
¶See Section 18.13, Stresses on grid, test and intermediate level



$$\tau_{ij} = \overline{v_i v_j} - \bar{v}_i \bar{v}_j \qquad \textbf{stresses with } \ell < \Delta$$

$$T_{ij} = \widehat{\overline{v_i v_j}} - \widehat{\bar{v}}_i \widehat{\bar{v}}_j$$

¶See Section 18.13, Stresses on grid, test and intermediate level



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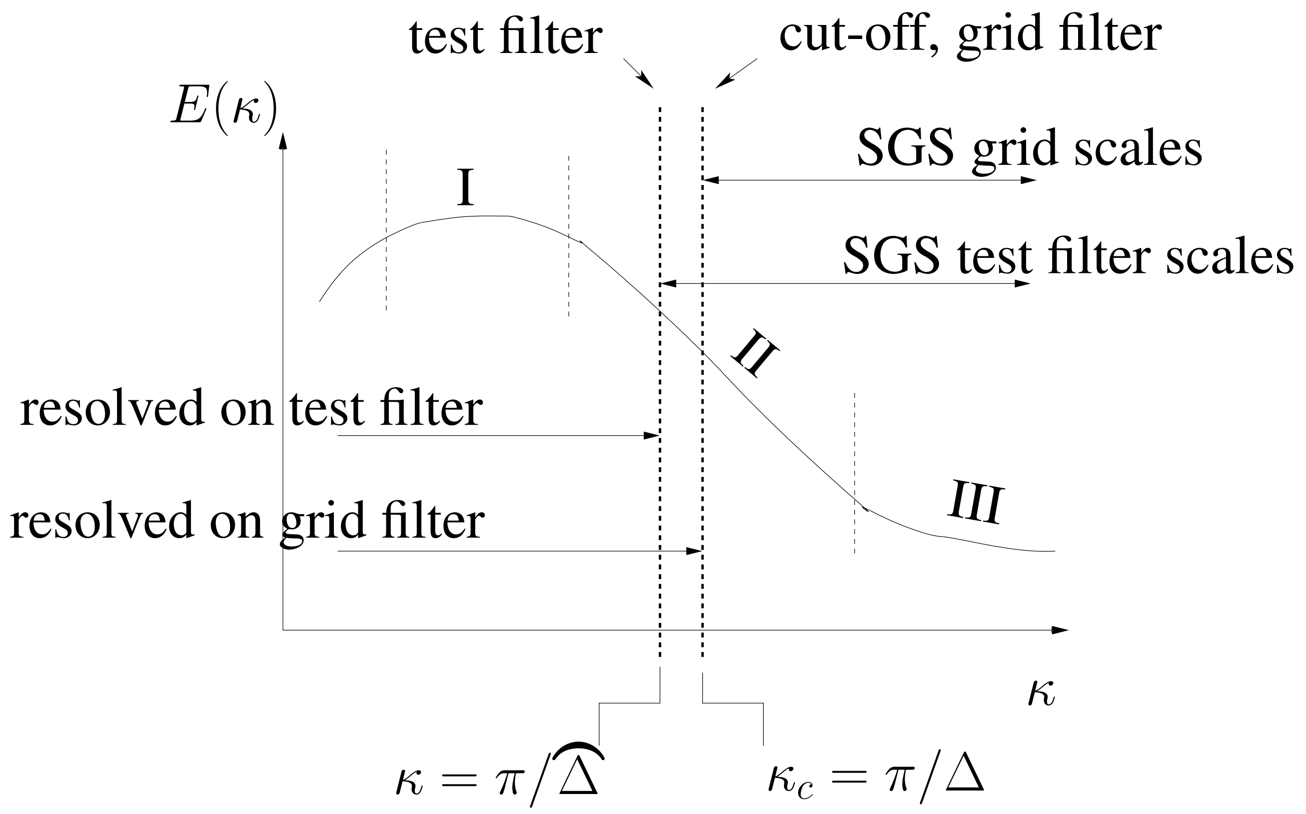
¶See Section 18.13, Stresses on grid, test and intermediate level



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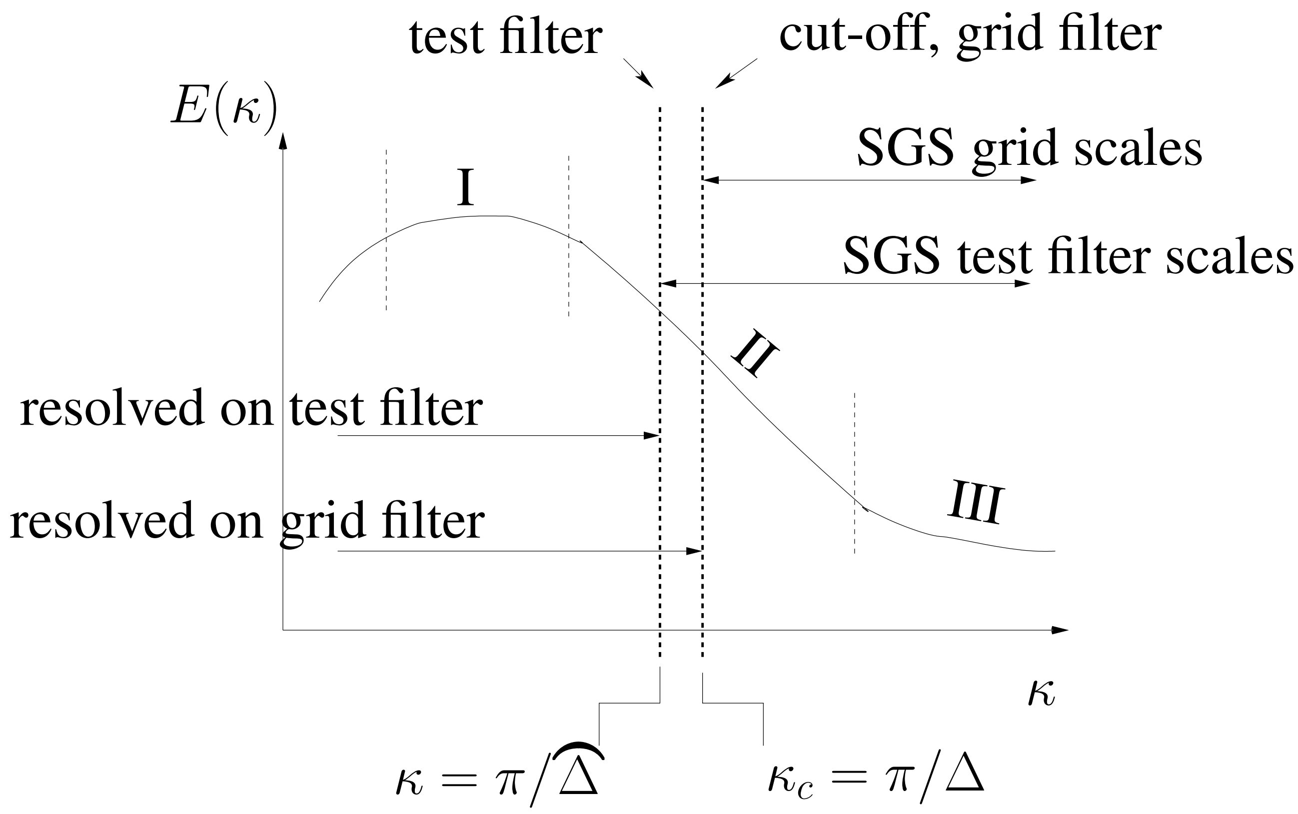
$$T_{ij} = \widehat{\overline{v_i v_j}} - \widehat{\bar{v}}_i \widehat{\bar{v}}_j \qquad \textbf{stresses with } \ell < \widehat{\Delta}$$

$$\mathcal{L}_{ij} = T_{ij} - \widehat{\tau}_{ij}$$

¶See Section 18.13, Stresses on grid, test and intermediate level



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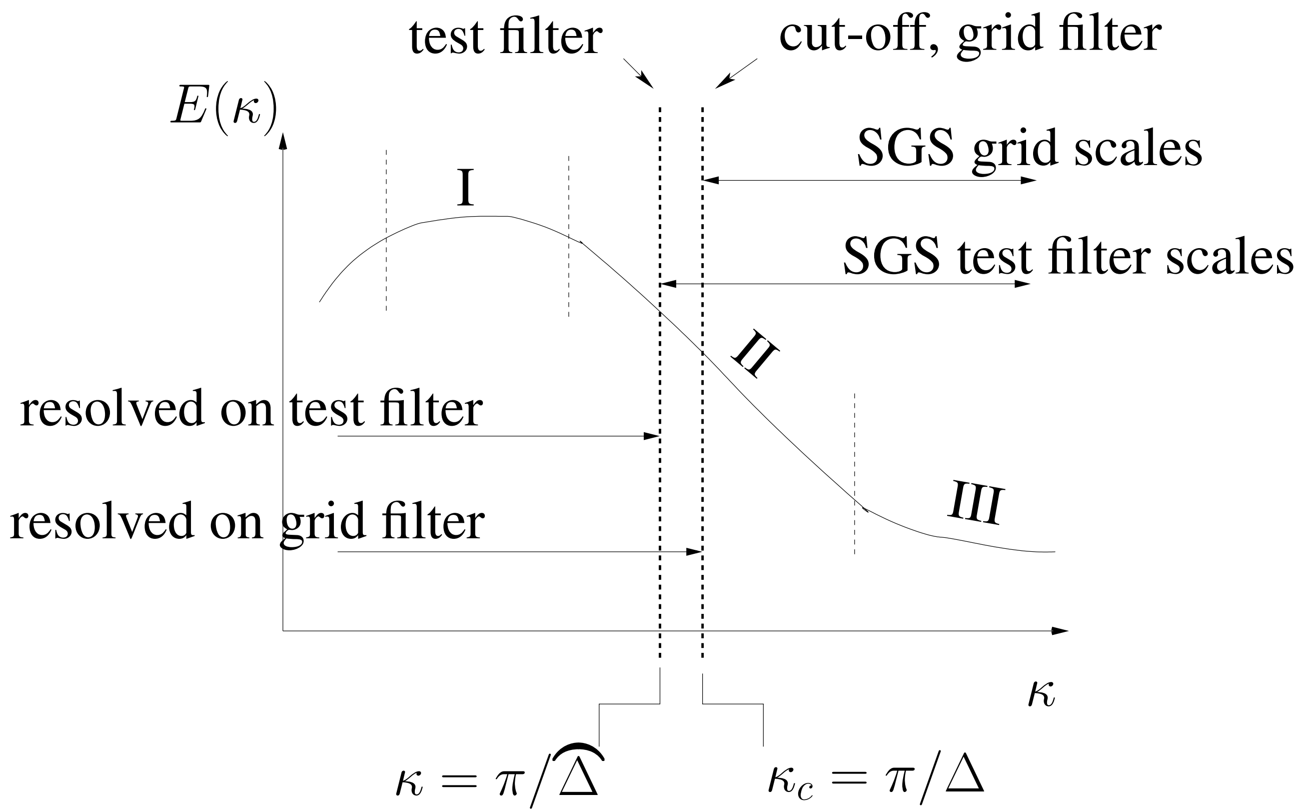
¶See Section 18.13, Stresses on grid, test and intermediate level



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¶See Section 18.20, Numerical method

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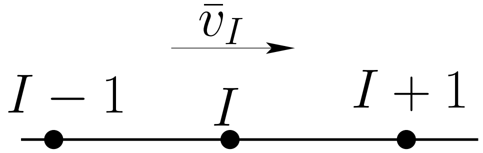
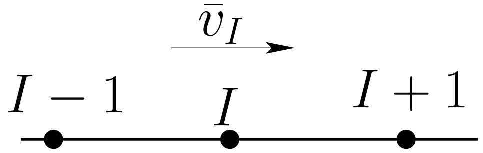


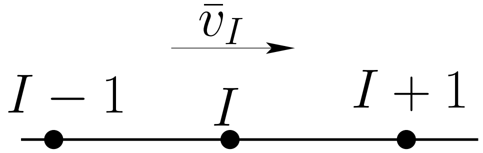
(37.9)

¶See Section 18.20, Numerical method



$$\bar{v}_I \left(\frac{\partial \bar{v}}{\partial x}\right)_{exact} = \tag{37.9}$$

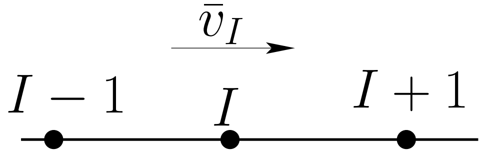
¶See Section 18.20, Numerical method



$$\bar{v}_I \left(\frac{\partial \bar{v}}{\partial x}\right)_{exact} = \bar{v}_I \left(\frac{\bar{v}_I - \bar{v}_{I-1}}{\Delta x} + \mathcal{O}\left(\Delta x\right)\right) \tag{37.9}$$

(37.10)

¶See Section 18.20, Numerical method



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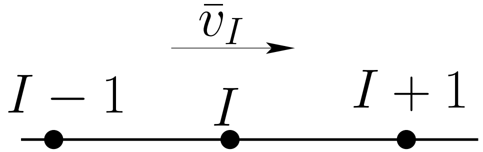
$$\bar{v}_{I-1} = \tag{37.10}$$

¶See Section 18.20, Numerical method



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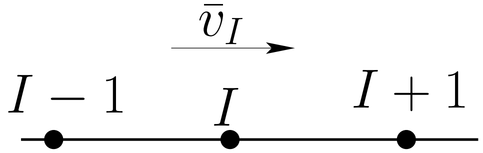
$$\bar{v}_{I-1} = \bar{v}_I - \Delta x \left(\frac{\partial \bar{v}}{\partial x}\right)_I + \tag{37.10}$$

¶See Section 18.20, Numerical method



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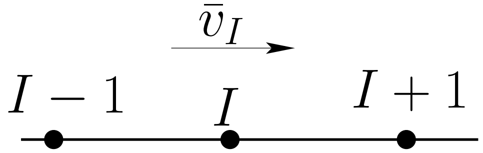
$$\bar{v}_{I-1} = \bar{v}_I - \Delta x \left(\frac{\partial \bar{v}}{\partial x}\right)_I + \frac{1}{2}(\Delta x)^2 \left(\frac{\partial^2 \bar{v}}{\partial x^2}\right)_I + \tag{37.10}$$

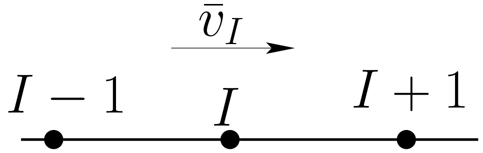
¶See Section 18.20, Numerical method



$$\bar{v}_I \left(\frac{\partial \bar{v}}{\partial x} \right)_{exact} = \bar{v}_I \left(\frac{\bar{v}_I - \bar{v}_{I-1}}{\Delta x} + \mathcal{O}\left(\Delta x\right) \right) \tag{37.9}$$

$$\bar{v}_{I-1} = \bar{v}_I - \Delta x \left(\frac{\partial \bar{v}}{\partial x} \right)_I + \frac{1}{2} (\Delta x)^2 \left(\frac{\partial^2 \bar{v}}{\partial x^2} \right)_I + \mathcal{O}\left((\Delta x)^3 \right) \tag{37.10}$$

¶See Section 18.20, Numerical method



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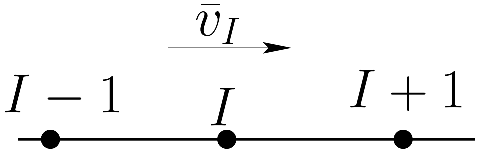
►Insert Eq. 37.9 into Eq. 37.9

$$\bar{v}\left(\frac{\partial \bar{v}}{\partial x}\right)_{exact} = \bar{v}\frac{\partial \bar{v}}{\partial x} - \underbrace{\frac{1}{2}\Delta x \bar{v}\frac{\partial^2 \bar{v}}{\partial x^2}}_{\mathcal{O}(\Delta x)} + \bar{v}\mathcal{O}\left((\Delta x)^2\right)$$

¶See Section 18.20, Numerical method



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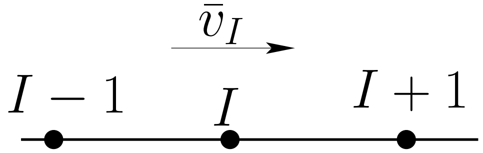
►$\Delta x \bar{v}/2$ acts as an additional numerical viscosity

¶See Section 18.20, Numerical method



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►Insert Eq. 37.9 into Eq. 37.9

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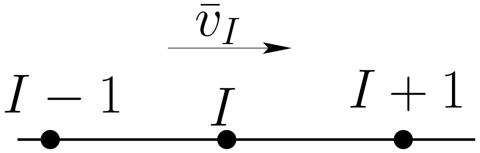
►$\Delta x \bar{v}/2$ acts as an additional numerical viscosity

►The total diffusion now consists of

¶See Section 18.20, Numerical method



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►Insert Eq. 37.9 into Eq. 37.9

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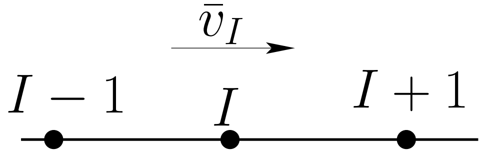
►$\Delta x \bar{v}/2$ acts as an additional numerical viscosity

►The total diffusion now consists of

$$\text{diffusion term} = \frac{\partial}{\partial x}\left\{(\nu + \nu_{sgs} + \nu_{num})\frac{\partial \bar{v}}{\partial x}\right\}$$

¶See Section 18.20, Numerical method



$$\bar{v}_I\left(\frac{\partial \bar{v}}{\partial x}\right)_{exact} = \bar{v}_I\left(\frac{\bar{v}_I-\bar{v}_{I-1}}{\Delta x}+\mathcal{O}\left(\Delta x\right)\right) \tag{37.9}$$

$$\bar{v}_{I-1} = \bar{v}_I-\Delta x\left(\frac{\partial \bar{v}}{\partial x}\right)_I+\frac{1}{2}(\Delta x)^2\left(\frac{\partial^2 \bar{v}}{\partial x^2}\right)_I+\mathcal{O}\left((\Delta x)^3\right) \tag{37.10}$$

▶Insert Eq. 37.9 into Eq. 37.9

$$\bar{v}\left(\frac{\partial \bar{v}}{\partial x}\right)_{exact} = \bar{v}\frac{\partial \bar{v}}{\partial x}-\underbrace{\frac{1}{2}\Delta x\bar{v}\frac{\partial^2 \bar{v}}{\partial x^2}}_{\mathcal{O}(\Delta x)}+\bar{v}\mathcal{O}\left((\Delta x)^2\right)$$

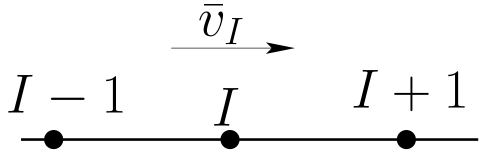
▶$\Delta x\bar{v}/2$ acts as an additional numerical viscosity

▶The total diffusion now consists of

$$\text{diffusion term} = \frac{\partial}{\partial x}\left\{(\nu+\nu_{sgs}+\nu_{num})\frac{\partial \bar{v}}{\partial x}\right\}$$

▶And the total dissipation

¶See Section 18.20, Numerical method



$$\bar{v}_I\left(\frac{\partial \bar{v}}{\partial x}\right)_{exact} = \bar{v}_I\left(\frac{\bar{v}_I-\bar{v}_{I-1}}{\Delta x}+\mathcal{O}\left(\Delta x\right)\right) \tag{37.9}$$

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►Insert Eq. 37.9 into Eq. 37.9

$$\bar{v}\left(\frac{\partial \bar{v}}{\partial x}\right)_{exact} = \bar{v}\frac{\partial \bar{v}}{\partial x} - \underbrace{\frac{1}{2}\Delta x \bar{v}\frac{\partial^2 \bar{v}}{\partial x^2}}_{\mathcal{O}(\Delta x)} + \bar{v}\mathcal{O}\left((\Delta x)^2\right)$$

►$\Delta x \bar{v}/2$ acts as an additional numerical viscosity

►The total diffusion now consists of

$$\text{diffusion term} = \frac{\partial}{\partial x}\left\{(\nu+\nu_{sgs}+\nu_{num})\frac{\partial \bar{v}}{\partial x}\right\}$$

►And the total dissipation

$$\varepsilon_{tot} = 2(\nu+\nu_{sgs}+\nu_{num})\bar{s}_{ij}\bar{s}_{ij}$$

On-line Lecture 9

¶See Section 18.15, Scale-similarity Models

On-line Lecture 9

¶See Section 18.15, Scale-similarity Models

$$\tau_{ij} =$$

On-line Lecture 9

¶See Section 18.15, Scale-similarity Models

$$\tau_{ij} = \overline{v_i v_j} - \bar{v}_i \bar{v}_j =$$

On-line Lecture 9

¶See Section 18.15, Scale-similarity Models

$$\tau_{ij} = \overline{v_i v_j} - \bar{v}_i \bar{v}_j = \overline{(\bar{v}_i + v''_i)(\bar{v}_j + v''_j)} -$$

On-line Lecture 9

¶See Section 18.15, Scale-similarity Models

$$\tau_{ij} = \overline{v_i v_j} - \bar{v}_i \bar{v}_j = \overline{(\bar{v}_i + v_i'')(\bar{v}_j + v_j'')} - \bar{v}_i \bar{v}_j$$

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¶See Section 18.15, Scale-similarity Models

$$\begin{aligned}\tau_{ij} &= \overline{v_i v_j} - \bar{v}_i \bar{v}_j = \overline{(\bar{v}_i + v''_i)(\bar{v}_j + v''_j)} - \bar{v}_i \bar{v}_j \\ &= \end{aligned}$$

On-line Lecture 9

¶See Section 18.15, Scale-similarity Models

$$\begin{aligned}\tau_{ij} &= \overline{v_i v_j} - \bar{v}_i \bar{v}_j = \overline{(\bar{v}_i + v_i'')(\bar{v}_j + v_j'')} - \bar{v}_i \bar{v}_j \\ &= \underbrace{\left(\overline{\bar{v}_i \bar{v}_j} - \bar{v}_i \bar{v}_j\right)}_{L_{ij}} +\end{aligned}$$

On-line Lecture 9

¶See Section 18.15, Scale-similarity Models

$$
\begin{aligned}
\tau_{ij} &= \overline{v_i v_j} - \bar{v}_i \bar{v}_j = \overline{(\bar{v}_i + v_i'')(\bar{v}_j + v_j'')} - \bar{v}_i \bar{v}_j \\
&= \underbrace{\left(\overline{\bar{v}_i \bar{v}_j} - \bar{v}_i \bar{v}_j\right)}_{L_{ij}} + \underbrace{\left[\overline{\bar{v}_i v_j'' + \bar{v}_j v_i''}\right]}_{C_{ij}} +
\end{aligned}
$$

On-line Lecture 9

¶See Section 18.15, Scale-similarity Models

$$\begin{aligned}\tau_{ij} &= \overline{v_i v_j} - \bar{v}_i \bar{v}_j = \overline{(\bar{v}_i + v''_i)(\bar{v}_j + v''_j)} - \bar{v}_i \bar{v}_j \\ &= \underbrace{\left(\overline{\bar{v}_i \bar{v}_j} - \bar{v}_i \bar{v}_j\right)}_{L_{ij}} + \underbrace{\left[\overline{\bar{v}_i v''_j + \bar{v}_j v''_i}\right]}_{C_{ij}} + \underbrace{\overline{v''_i v''_j}}_{R_{ij}}\end{aligned}$$

On-line Lecture 9

¶See Section 18.15, Scale-similarity Models

$$
\begin{aligned}
\tau_{ij} &= \overline{v_i v_j} - \bar{v}_i \bar{v}_j = \overline{(\bar{v}_i + v_i'')(\bar{v}_j + v_j'')} - \bar{v}_i \bar{v}_j \\
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\end{aligned}
$$

►C_{ij} denotes scale-similar stresses

On-line Lecture 9

¶See Section 18.15, Scale-similarity Models

$$
\begin{aligned}
\tau_{ij} &= \overline{v_i v_j} - \bar{v}_i \bar{v}_j = \overline{(\bar{v}_i + v_i'')(\bar{v}_j + v_j'')} - \bar{v}_i \bar{v}_j \\
&= \underbrace{(\overline{\bar{v}_i \bar{v}_j} - \bar{v}_i \bar{v}_j)}_{L_{ij}} + \underbrace{\left[\overline{\bar{v}_i v_j'' + \bar{v}_j v_i''}\right]}_{C_{ij}} + \underbrace{\overline{v_i'' v_j''}}_{R_{ij}}
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$$v_i =$$

On-line Lecture 9

¶See Section 18.15, Scale-similarity Models

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$$
v_i = \bar{v}_i + v_j''
$$

On-line Lecture 9

¶See Section 18.15, Scale-similarity Models

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On-line Lecture 9

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► Scale-similarity model

On-line Lecture 9

¶See Section 18.15, Scale-similarity Models

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$$

► Scale-similarity model

$$
C_{ij}^M = c_r(\bar{v}_i \bar{v}_j - \bar{\bar{v}}_i \bar{\bar{v}}_j), \quad R_{ij} = 0 \tag{38.1}
$$

On-line Lecture 9

¶See Section 18.15, Scale-similarity Models

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$$C_{ij}^M = c_r(\bar{v}_i \bar{v}_j - \bar{\bar{v}}_i \bar{\bar{v}}_j), \quad R_{ij} = 0 \tag{38.1}$$

►The $C_{ij} = \overline{\bar{v}_i v_j''} + \overline{\bar{v}_j v_i''}$ and $L_{ij} = \overline{\bar{v}_i \bar{v}_j} - \bar{v}_i \bar{v}_j$ stresses are not Galilean invariant (but $C_{ij} + L_{ij}$ is).

On-line Lecture 9

¶See Section 18.15, Scale-similarity Models

$$\begin{aligned}\tau_{ij} &= \overline{\bar{v}_i\bar{v}_j} - \bar{v}_i\bar{v}_j = \overline{(\bar{v}_i + v''_i)(\bar{v}_j + v''_j)} - \bar{v}_i\bar{v}_j \\ &= \underbrace{(\overline{\bar{v}_i\bar{v}_j} - \bar{v}_i\bar{v}_j)}_{L_{ij}} + \underbrace{\left[\overline{\bar{v}_i v''_j + \bar{v}_j v''_i}\right]}_{C_{ij}} + \underbrace{\overline{v''_i v''_j}}_{R_{ij}}\end{aligned}$$

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On-line Lecture 9

¶See Section 18.15, Scale-similarity Models

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&= \underbrace{(\overline{\bar{\bar{v}}_i \bar{\bar{v}}_j} - \bar{\bar{v}}_i \bar{\bar{v}}_j)}_{L_{ij}} + \underbrace{\left[\overline{\bar{\bar{v}}_i v''_j + \bar{\bar{v}}_j v''_i}\right]}_{C_{ij}} + \underbrace{\overline{v''_i v''_j}}_{R_{ij}}
\end{aligned}
$$

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¶See Section 18.16, The Bardina Model

►The Bardina model reads (since C_{ij}^M was not sufficiently dissipative, a Smagorinsky model is added)

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¶See Section 18.16, The Bardina Model

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$$\tau^m_{ij} = \tau_{ij} = C^m_{ij} + L^m_{ij} + R^m_{ij}$$

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$$\begin{aligned} \tau_{ij}^m &= \tau_{ij} = C_{ij}^m + L_{ij}^m + R_{ij}^m \\ L_{ij}^m &= c_r\left(\overline{\bar{v}_i\bar{v}_j} - \bar{\bar{v}}_i\bar{\bar{v}}_j\right) \end{aligned}$$

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$$\begin{aligned}
\tau_{ij}^m &= \tau_{ij} = C_{ij}^m + L_{ij}^m + R_{ij}^m \\
L_{ij}^m &= c_r\left(\overline{\bar{v}_i\bar{v}_j} - \bar{\bar{v}}_i\bar{\bar{v}}_j\right) \\
C_{ij}^m &=
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\tau_{ij}^m &= \tau_{ij} = C_{ij}^m + L_{ij}^m + R_{ij}^m \\
L_{ij}^m &= c_r\left(\overline{\bar{v}_i\bar{v}_j} - \bar{\bar{v}}_i\bar{\bar{v}}_j\right) \\
C_{ij}^m &= 0
\end{aligned}$$

¶See Section 18.16, The Bardina Model

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C_{ij}^m &= 0 \\
R_{ij}^m &= R_{ij} =
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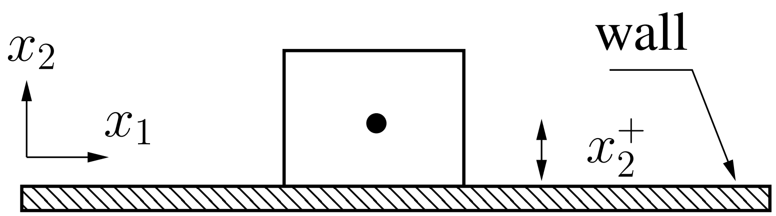

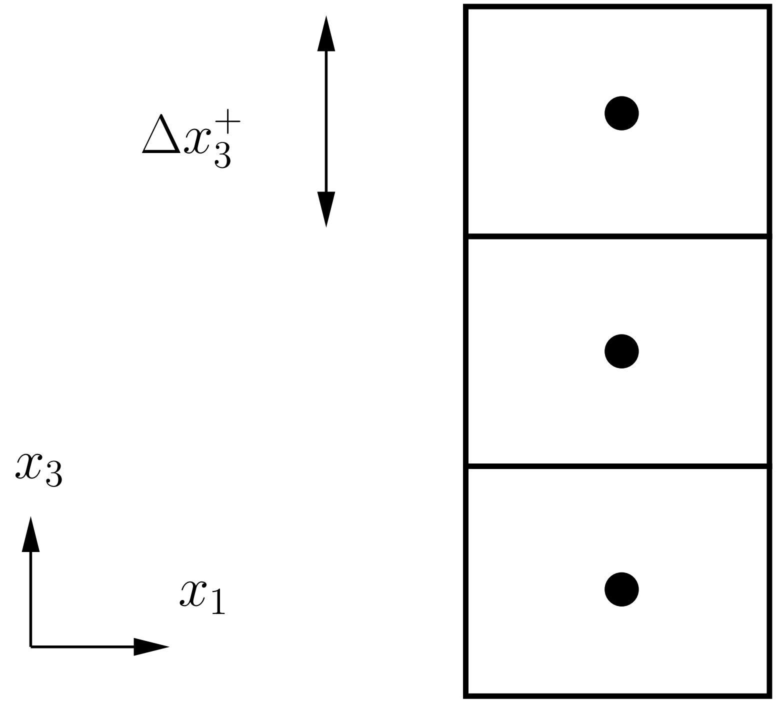

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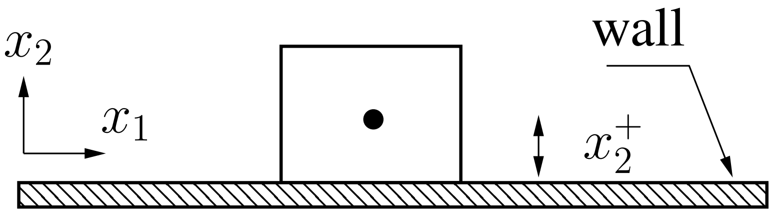

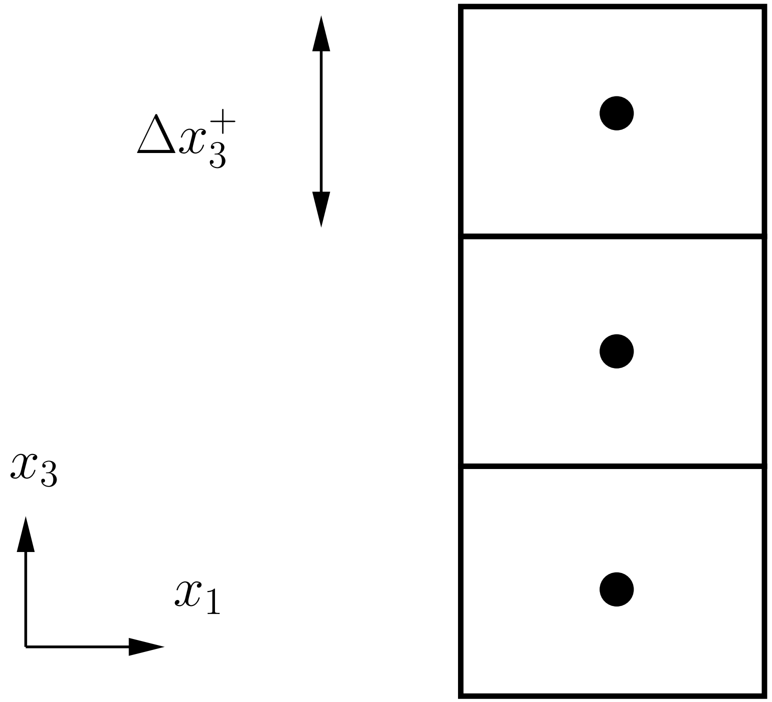

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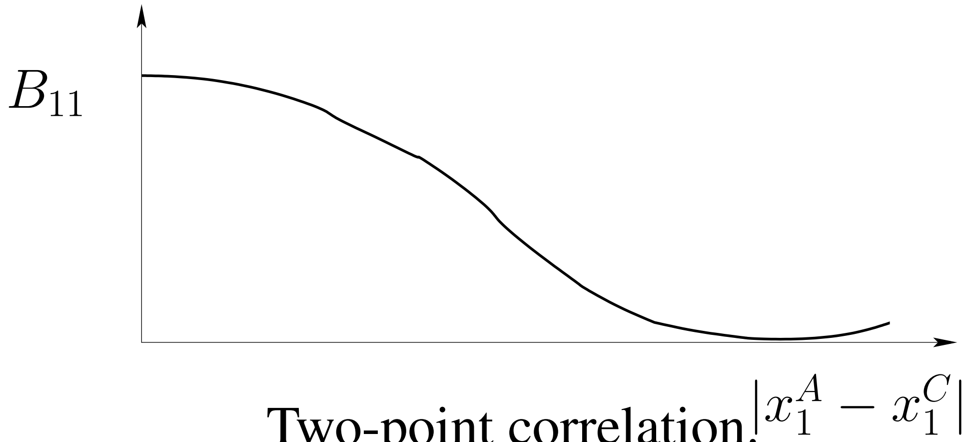

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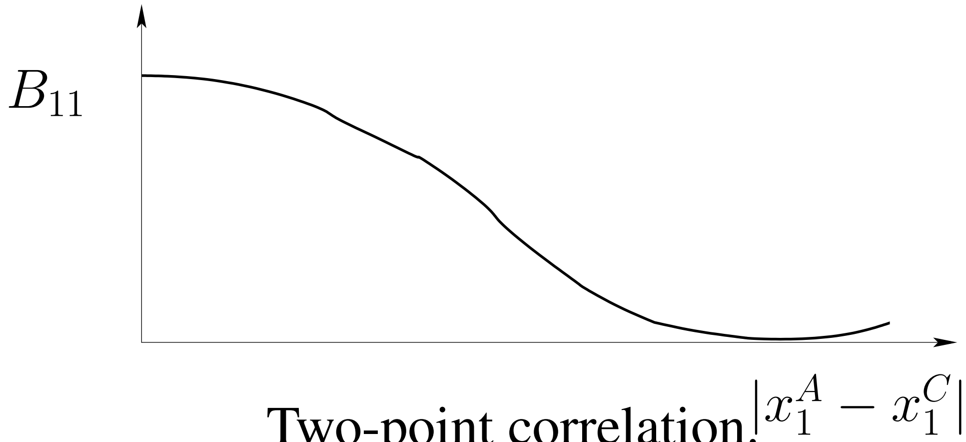

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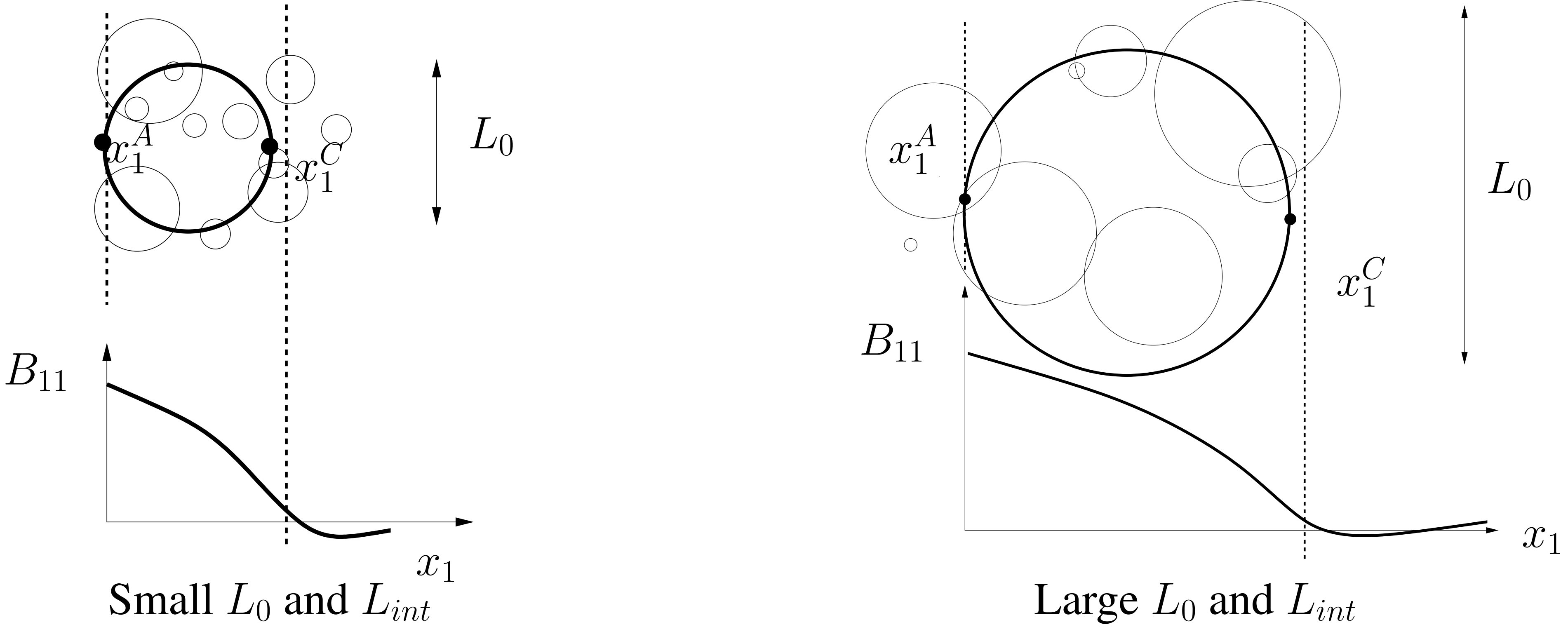

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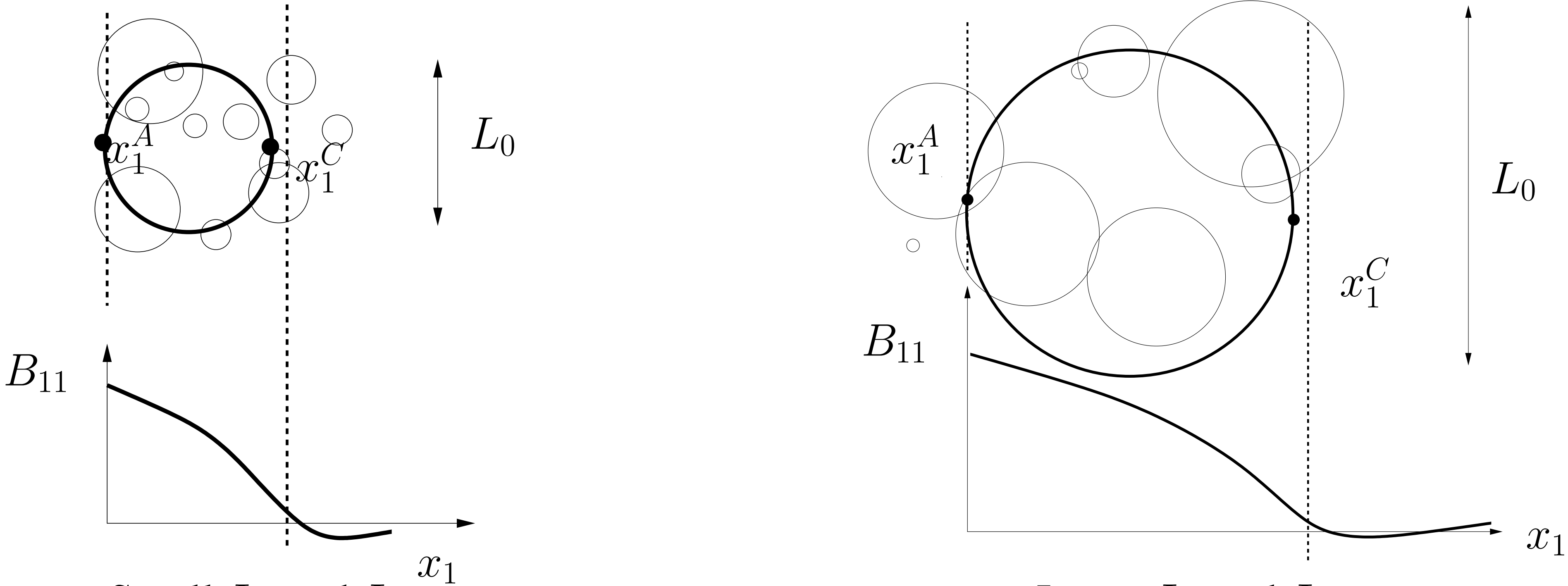

Small L_0 and L_{int} Large L_0 and L_{int}

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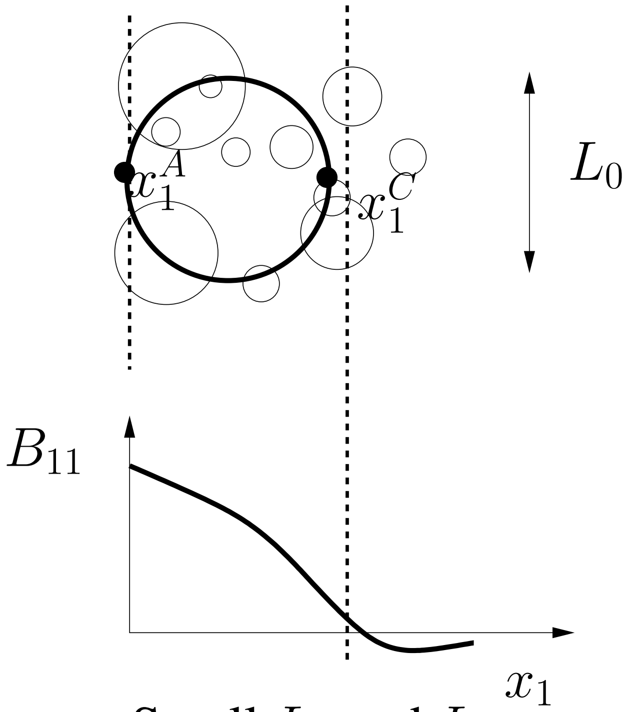

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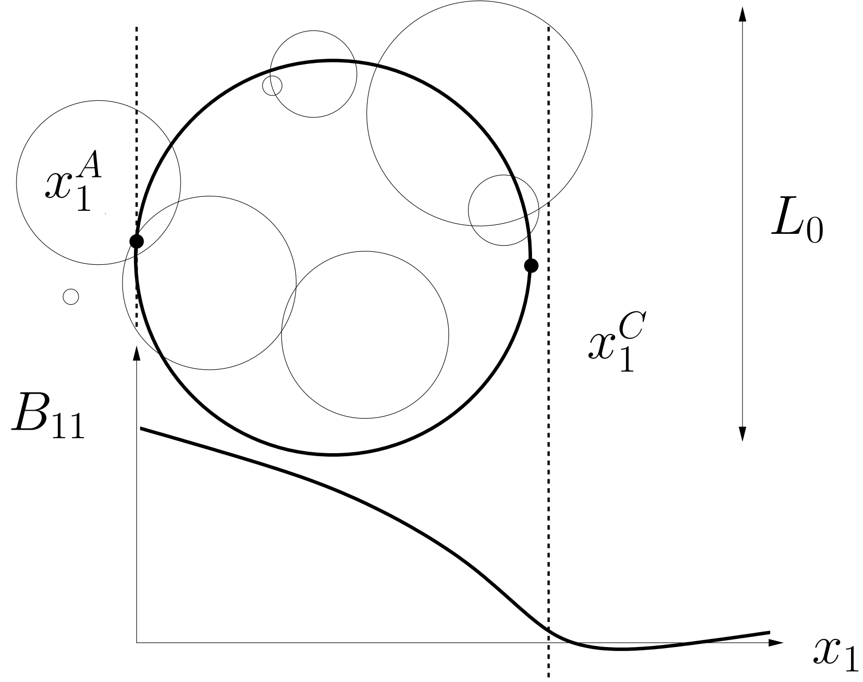

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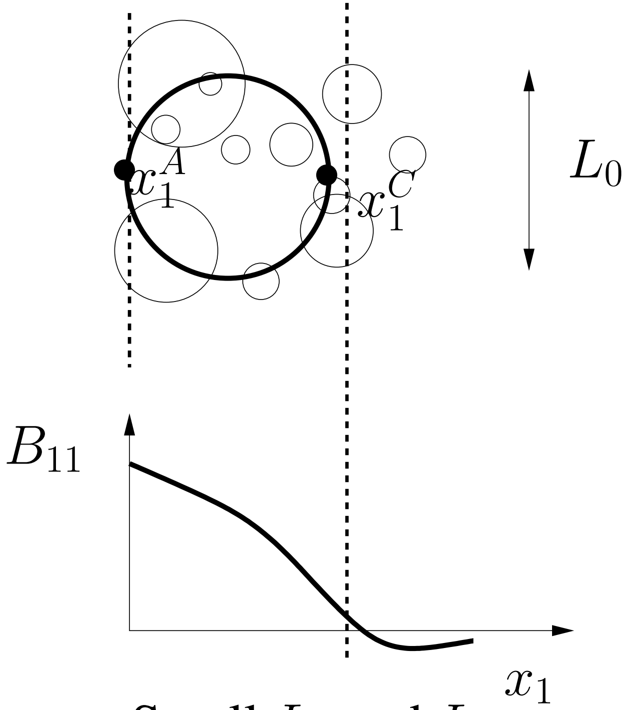

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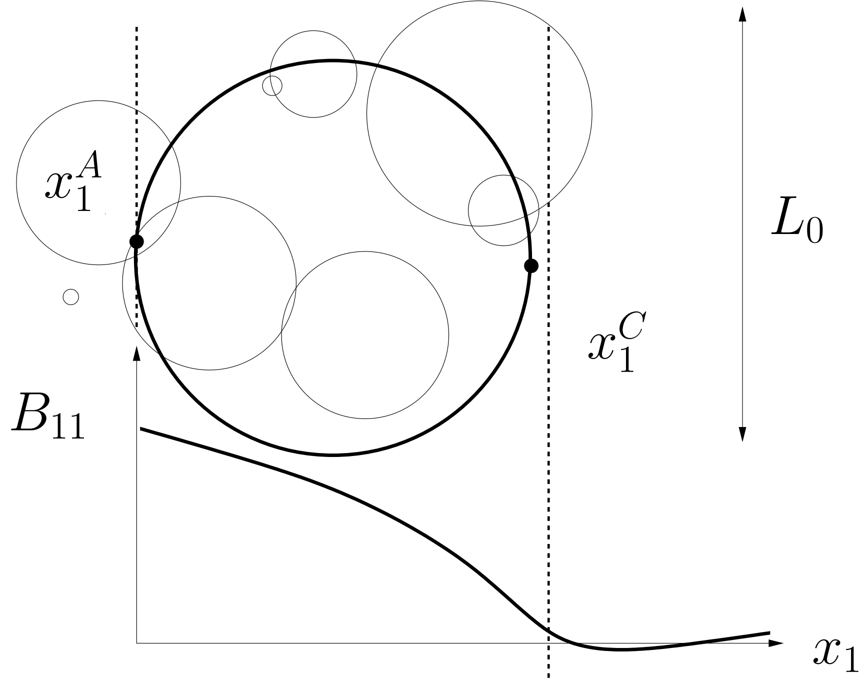

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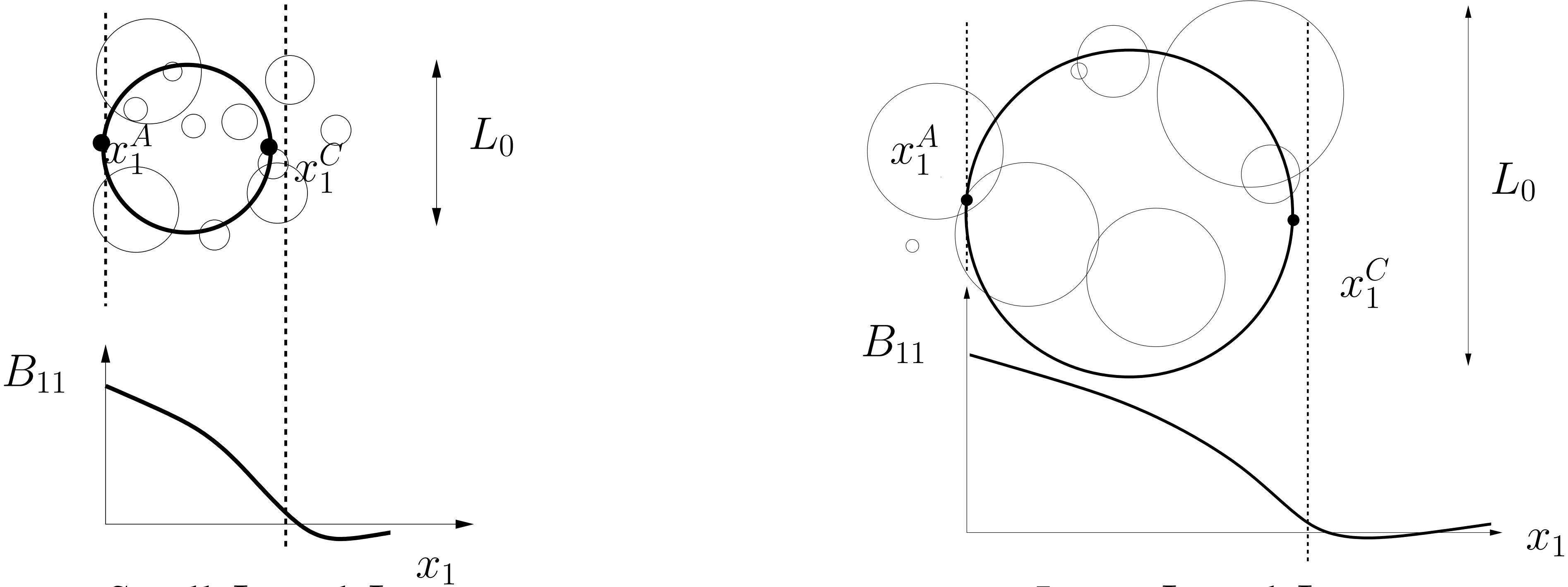

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- When we move point A and C closer to each other, B_{11} increases; when A=C, then $B_{11} = \overline{v'^2}(x_1^A)$
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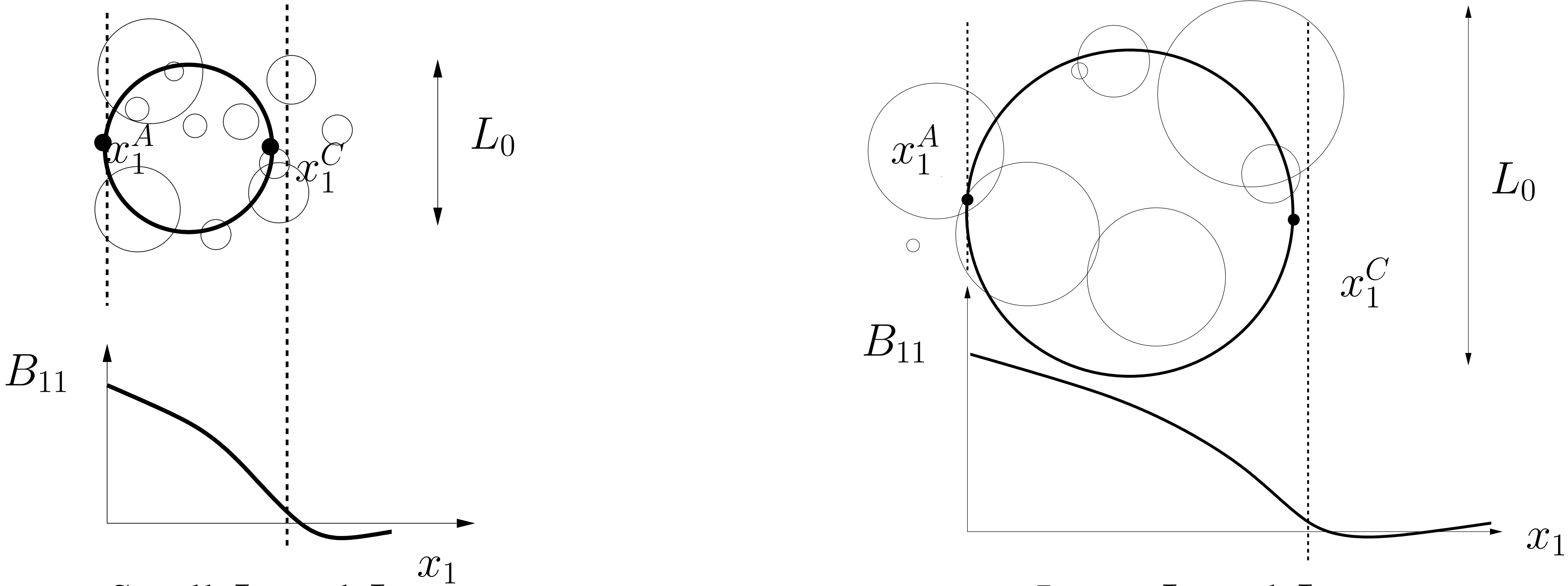

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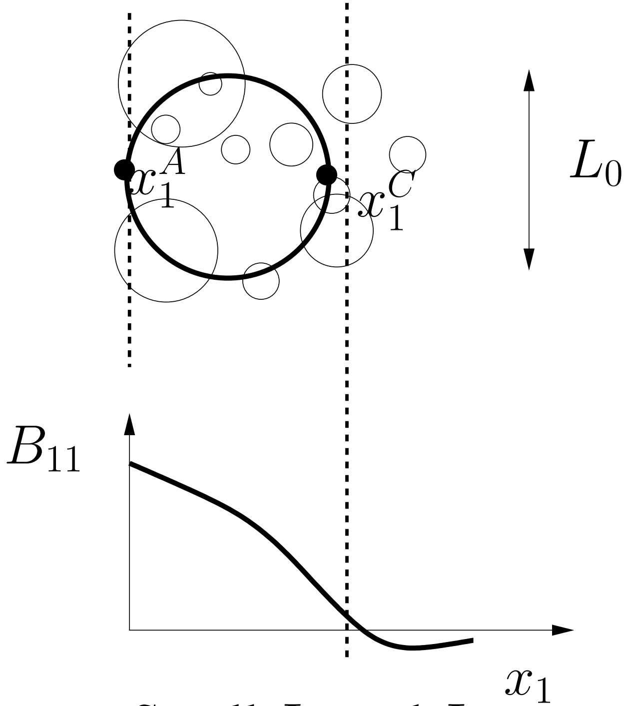

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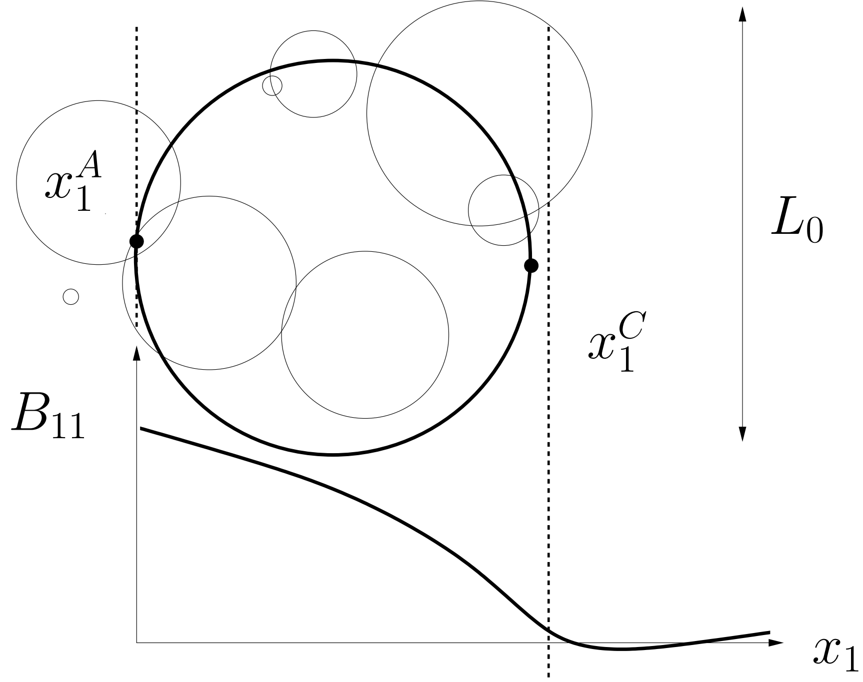

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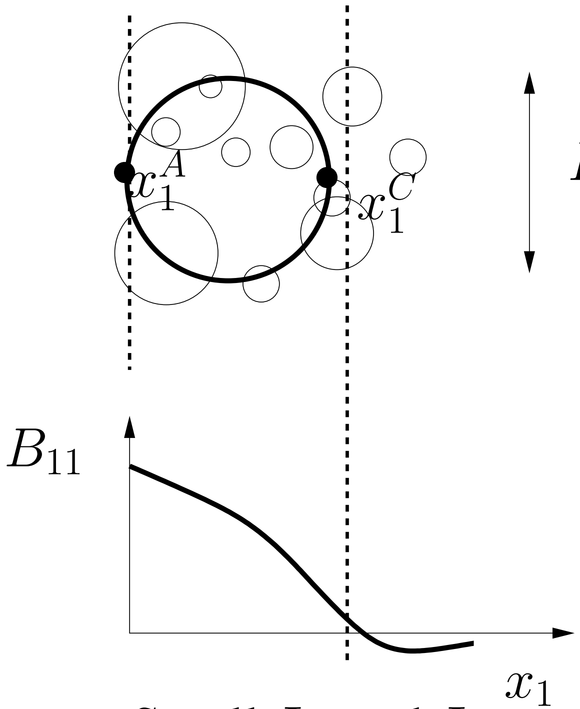

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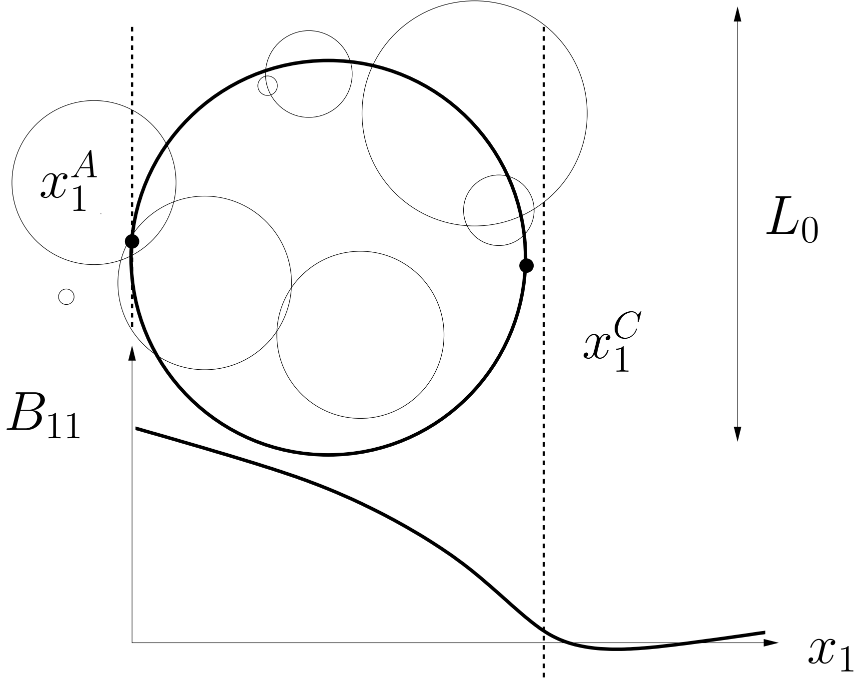

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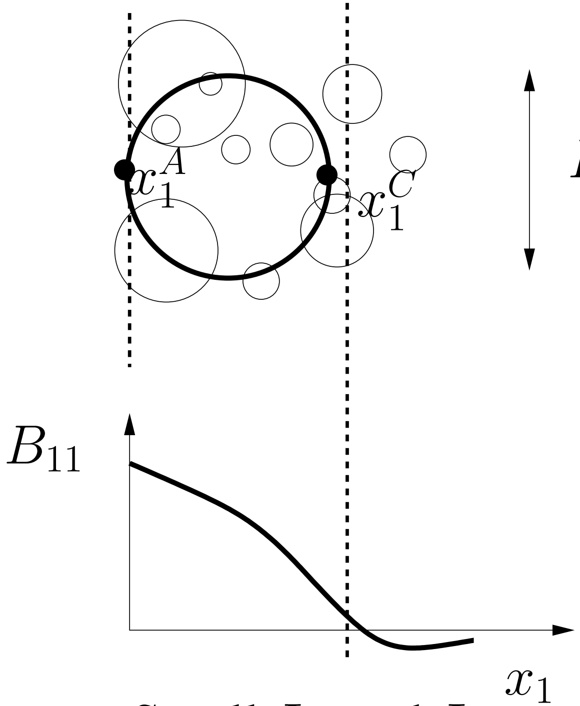

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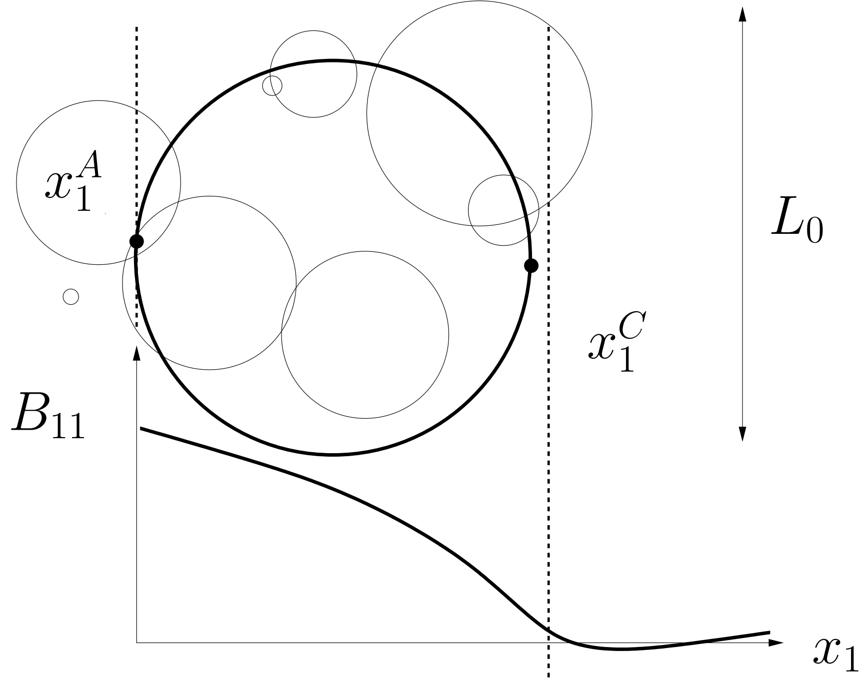

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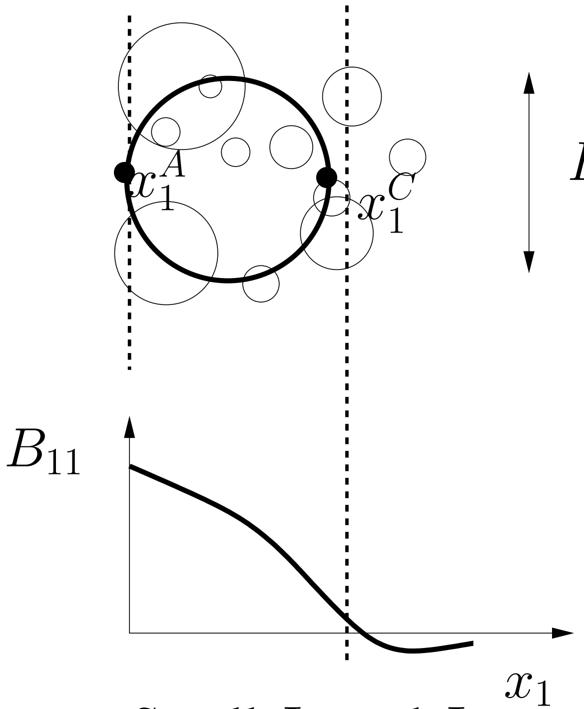

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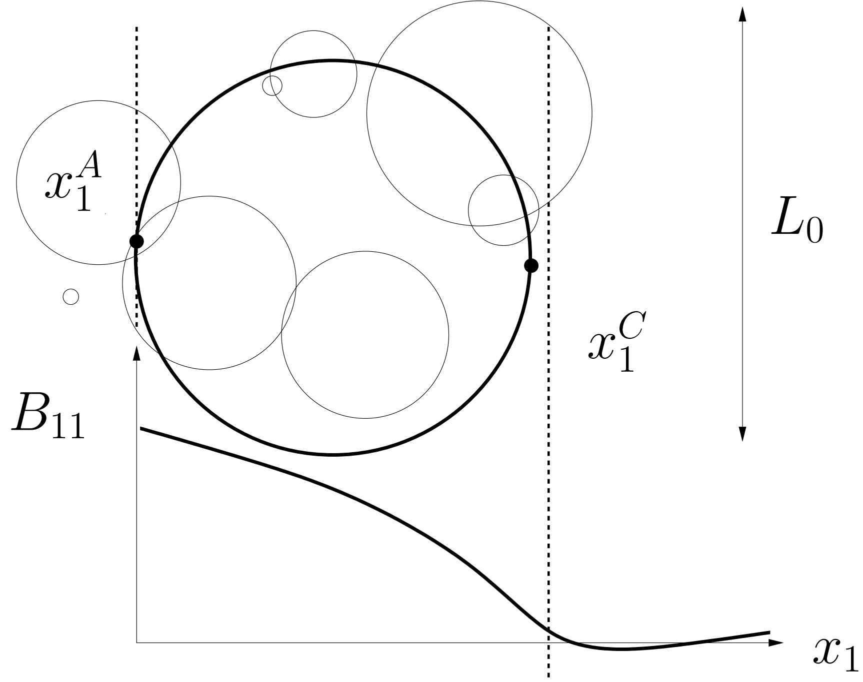

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¶See Section 10.2, Auto correlation

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►Auto correlation is a “two-point correlation in time” which reads

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¶See Section 10.2, Auto correlation

►Auto correlation is a "two-point correlation in time" which reads

$$B_{11}(t^A, \hat{t}\,) = \overline{v_1'(t^A)v_1'(t^A + \hat{t}\,)}$$

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►In analogy to L_{int}, the *integral time scale*, T_{int}, is defined

$$T_{int} = \int_0^\infty B_{11}^{norm}(\hat{t})d\hat{t}$$

►Integral timescale is used an Assignment 2a for finding time samples that are *independent* (i.e. the time between the samples is at least one integral timescale).

¶See Section 18.20.1, RANS vs. LES

¶See Section 18.20.1, RANS vs. LES

▶Numerical method: RANS vs. LES

¶See Section 18.20.1, RANS vs. LES

►Numerical method: RANS vs. LES

	RANS	

¶See Section 18.20.1, RANS vs. LES

►Numerical method: RANS vs. LES

	RANS	**LES**

¶See Section 18.20.1, RANS vs. LES

►Numerical method: RANS vs. LES

	RANS	**LES**
Domain		

¶See Section 18.20.1, RANS vs. LES

►Numerical method: RANS vs. LES

	RANS	**LES**
Domain	2D or 3D	

¶See Section 18.20.1, RANS vs. LES

►Numerical method: RANS vs. LES

	RANS	**LES**
Domain	2D or 3D	always 3D

¶See Section 18.20.1, RANS vs. LES

►Numerical method: RANS vs. LES

	RANS	**LES**
Domain	2D or 3D	always 3D
Time domain		

¶See Section 18.20.1, RANS vs. LES

►Numerical method: RANS vs. LES

	RANS	**LES**
Domain	2D or 3D	always 3D
Time domain	steady or unsteady	

¶See Section 18.20.1, RANS vs. LES

▶Numerical method: RANS vs. LES

	RANS	**LES**
Domain	2D or 3D	always 3D
Time domain	steady or unsteady	always unsteady

¶See Section 18.20.1, RANS vs. LES

▶Numerical method: RANS vs. LES

	RANS	**LES**
Domain	2D or 3D	always 3D
Time domain	steady or unsteady	always unsteady
Space discretization		

¶See Section 18.20.1, RANS vs. LES

▶Numerical method: RANS vs. LES

	RANS	**LES**
Domain	2D or 3D	always 3D
Time domain	steady or unsteady	always unsteady
Space discretization	2nd order upwind	

¶See Section 18.20.1, RANS vs. LES

▶Numerical method: RANS vs. LES

	RANS	**LES**
Domain	2D or 3D	always 3D
Time domain	steady or unsteady	always unsteady
Space discretization	2nd order upwind	central differencing

¶See Section 18.20.1, RANS vs. LES

►Numerical method: RANS vs. LES

	RANS	**LES**
Domain	2D or 3D	always 3D
Time domain	steady or unsteady	always unsteady
Space discretization	2nd order upwind	central differencing
Time discretization		

¶See Section 18.20.1, RANS vs. LES

►Numerical method: RANS vs. LES

	RANS	**LES**
Domain	2D or 3D	always 3D
Time domain	steady or unsteady	always unsteady
Space discretization	2nd order upwind	central differencing
Time discretization	1st order	

¶See Section 18.20.1, RANS vs. LES

►Numerical method: RANS vs. LES

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¶See Section 18.20.1, RANS vs. LES

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Turbulence model		

¶See Section 18.20.1, RANS vs. LES

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¶See Section 18.20.1, RANS vs. LES

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¶See Section 18.20.1, RANS vs. LES

►Numerical method: RANS vs. LES

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►**Start** and **end** time averaging.

¶See Section 18.20.1, RANS vs. LES

▶Numerical method: RANS vs. LES

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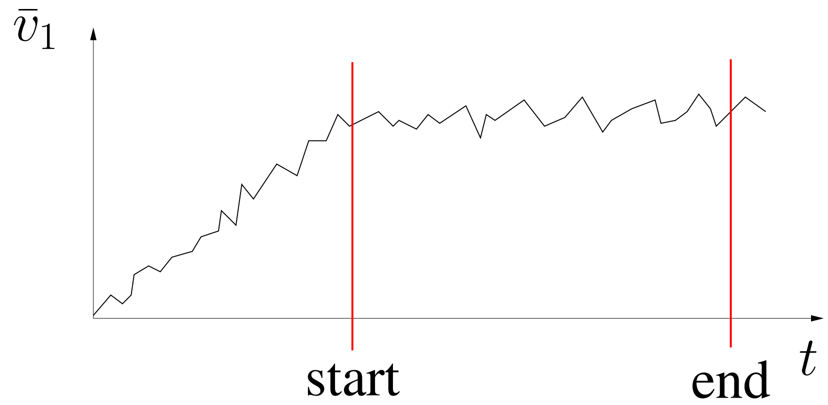

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¶See Section 18.20.1, RANS vs. LES

▶Numerical method: RANS vs. LES

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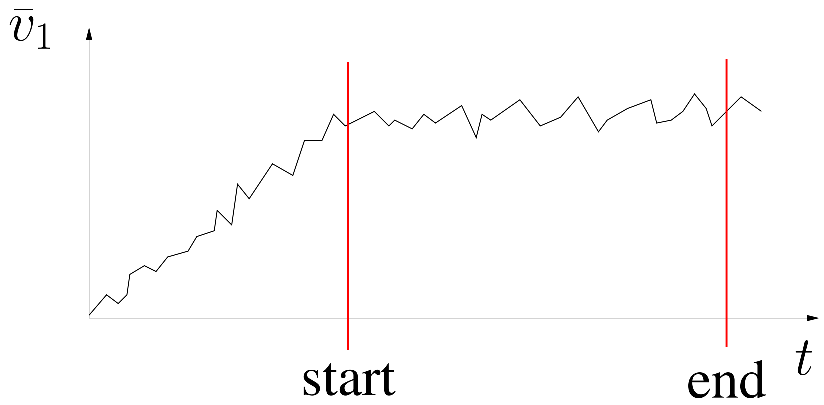

- Say that we want to store 3D inst. fields

¶See Section 18.20.1, RANS vs. LES

▶Numerical method: RANS vs. LES

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Domain	2D or 3D	always 3D
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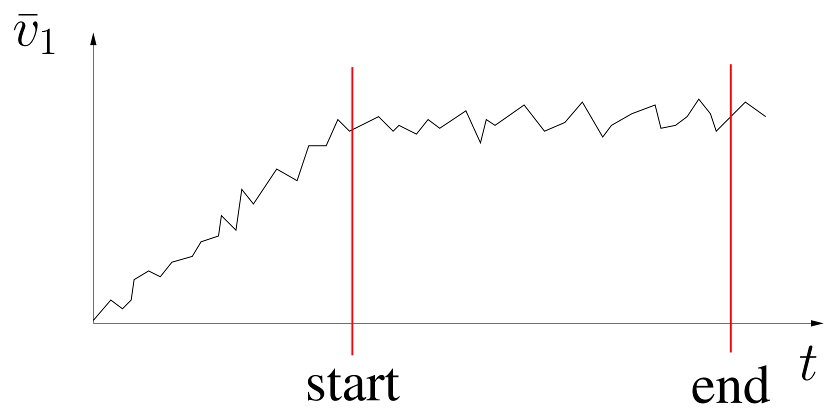

- Say that we want to store 3D inst. fields $\Rightarrow$ we can post-proc, e.g., two-point corr anywhere

¶See Section 18.20.1, RANS vs. LES

▶Numerical method: RANS vs. LES

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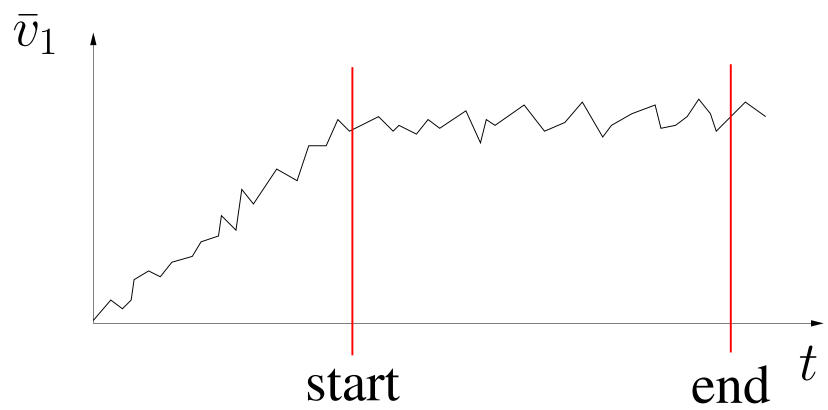

- Say that we want to store 3D inst. fields $\Rightarrow$ we can post-proc, e.g., two-point corr anywhere
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¶See Section 18.20.1, RANS vs. LES

▶Numerical method: RANS vs. LES

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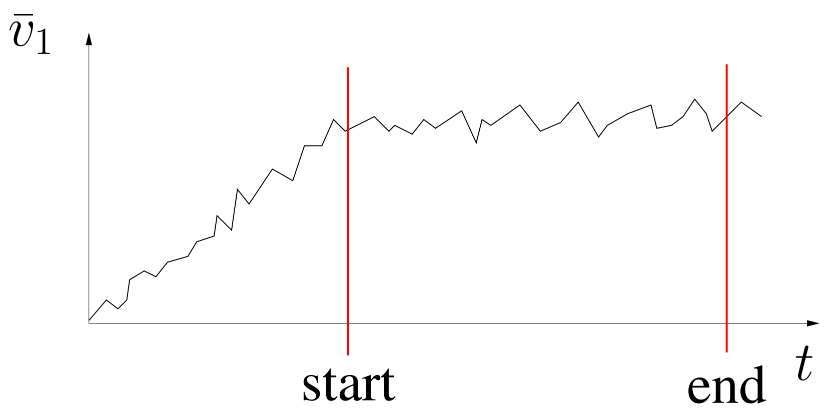

- Say that we want to store 3D inst. fields $\Rightarrow$ we can post-proc, e.g., two-point corr anywhere
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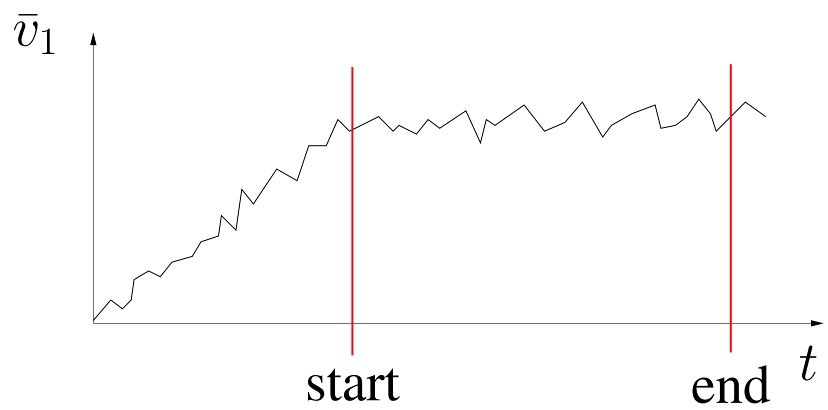

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- Answer: store only every T_{int} second:

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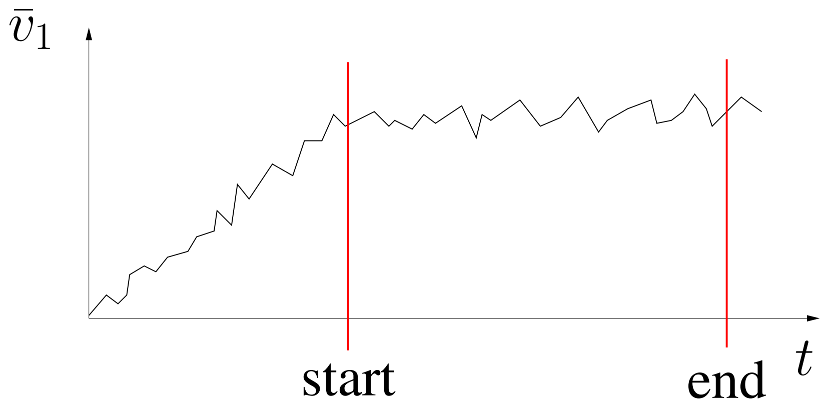

- Say that we want to store 3D inst. fields $\Rightarrow$ we can post-proc, e.g., two-point corr anywhere
- Then we want to store as few 3D fields as possible (otherwise our disk space will quickly be saturated)
- Answer: store only every T_{int} second: 100 indepedent samples gives a statistical error 0.01

¶See Section 19, URANS: Unsteady RANS

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▶The usual Reynolds decomposition is employed

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¶See Section 19, URANS: Unsteady RANS

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$$\bar{v}(t) = \frac{1}{2T}\int_{t-T}^{t+T} v(t)dt$$

¶See Section 19, URANS: Unsteady RANS

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¶See Section 19, URANS: Unsteady RANS

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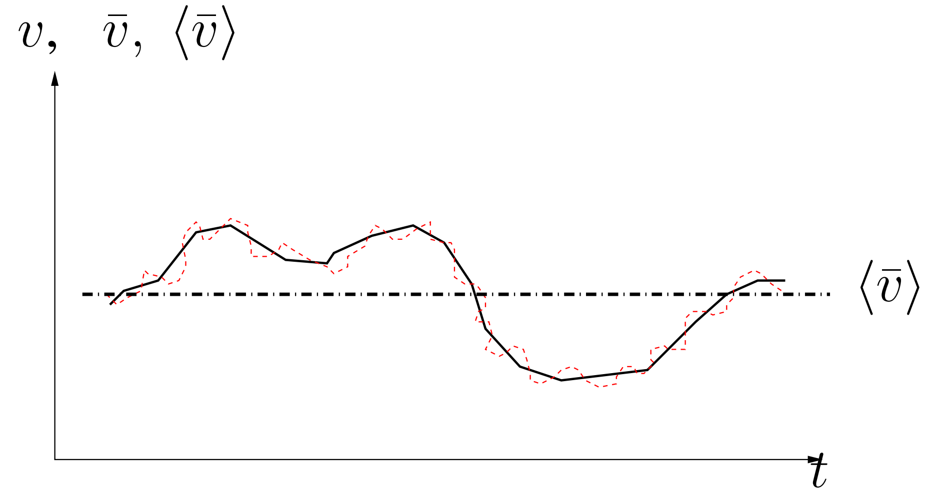

Decomposion in URANS. —: $\bar{v}$; - -: v; -·-$\langle\bar{v}\rangle$.

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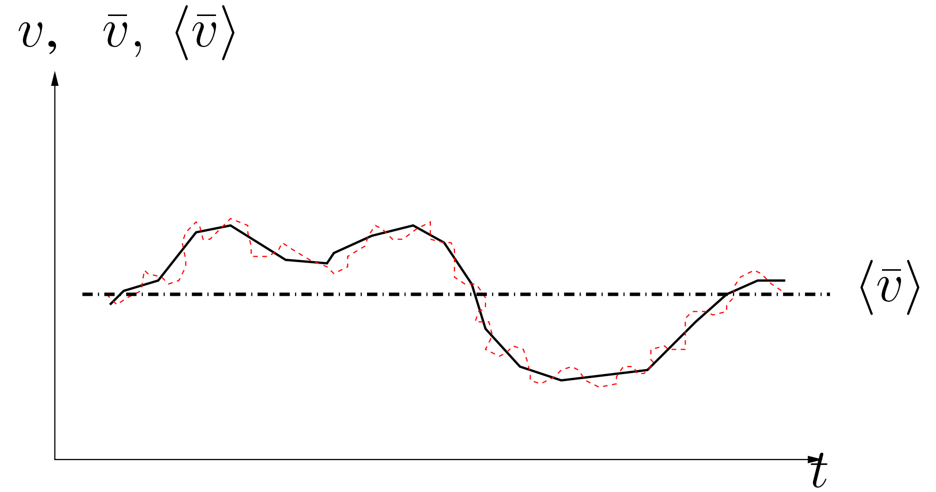

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- Decomposition of velocities:

¶See Section 19, URANS: Unsteady RANS

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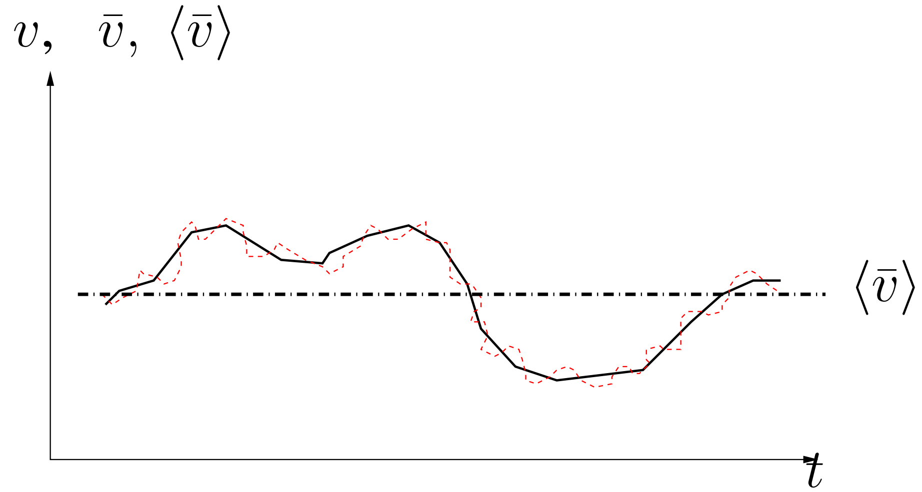

Decomposion in URANS. —: $\bar{v}$; - -: v; -·- $\langle \bar{v} \rangle$.

- Decomposition of velocities: $v = \bar{v} + v'' =$

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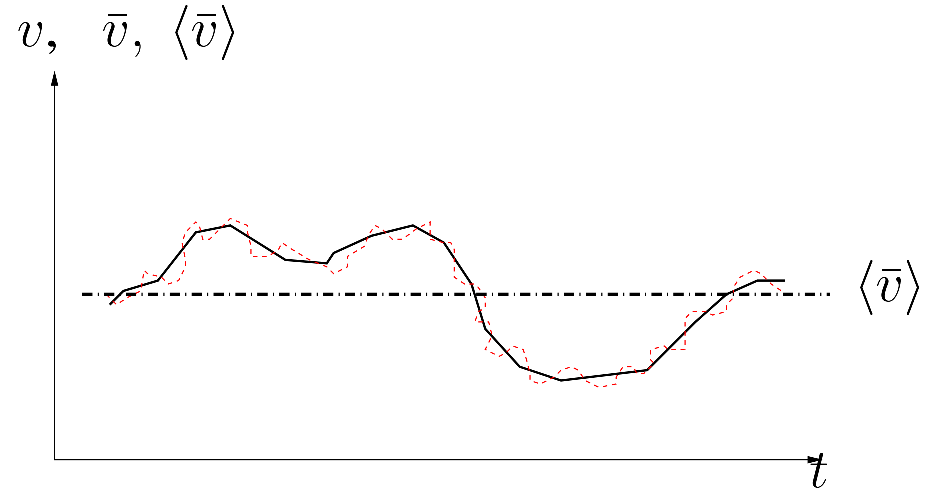

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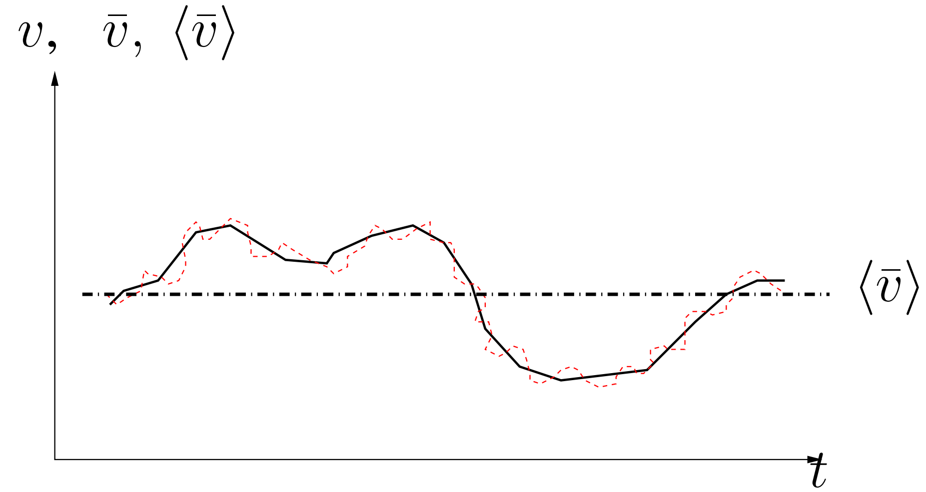

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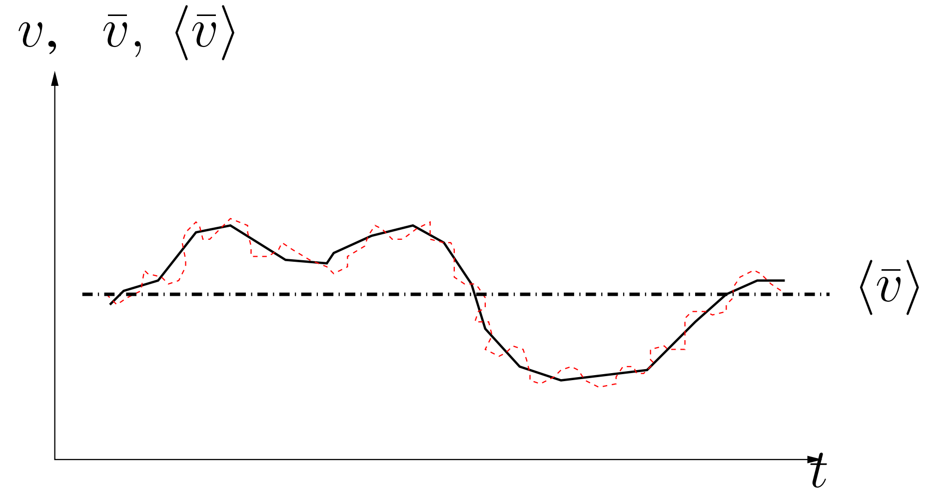

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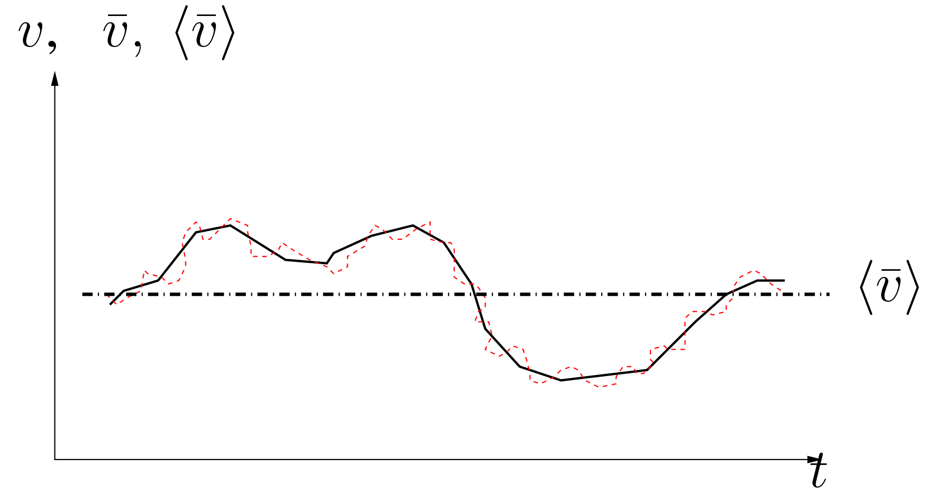

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$$C^k = D^k + P^k - \varepsilon \quad \Rightarrow \quad C^k = D^k + P^k - \varepsilon_T, \quad \varepsilon_T = \max\left(\varepsilon, C_\varepsilon \frac{k^{3/2}}{\Delta}\right), \quad \nu_t = C_\mu \frac{k^2}{\varepsilon}$$

▶ $\varepsilon_T \uparrow$ in LES region ▶ $\Rightarrow k \downarrow$ in LES region ▶ $\Rightarrow \nu_t \downarrow$ in LES region

▶ $k-\varepsilon$ DES II (modify ν_T and ε_T)

$$C^k = D^k + P^k - \varepsilon_T, \quad C^\varepsilon = D^\varepsilon + P^\varepsilon - \Psi^\varepsilon, \quad \ell_t = \min\left(C_\mu \frac{k^{3/2}}{\varepsilon}, C_{DES}\Delta\right), \quad \nu_T = k^{1/2}\ell_t$$

▶ $\varepsilon_T \uparrow$ in LES region ▶ $\Rightarrow k \downarrow$ in LES region

▶ $\ell_t \downarrow$ in LES region ▶ $\Rightarrow \nu_t \downarrow$ in LES region

On-line Lecture 10

¶See Section 20.2, DES based on the $k-\omega$ SST model

On-line Lecture 10

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$k-\omega$ SST DES (modify $\beta^* k\omega$)

On-line Lecture 10

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$$C^k = D^k + P^k - F_{DES}\beta^* k\omega$$

On-line Lecture 10

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$k-\omega$ SST DES (modify $\beta^* k\omega$)

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- $L_t = \dfrac{k^{3/2}}{\varepsilon}$

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- $L_t = \dfrac{k^{3/2}}{\varepsilon}$
- $\omega = \dfrac{\varepsilon}{\beta^* k}$

On-line Lecture 10

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On-line Lecture 10

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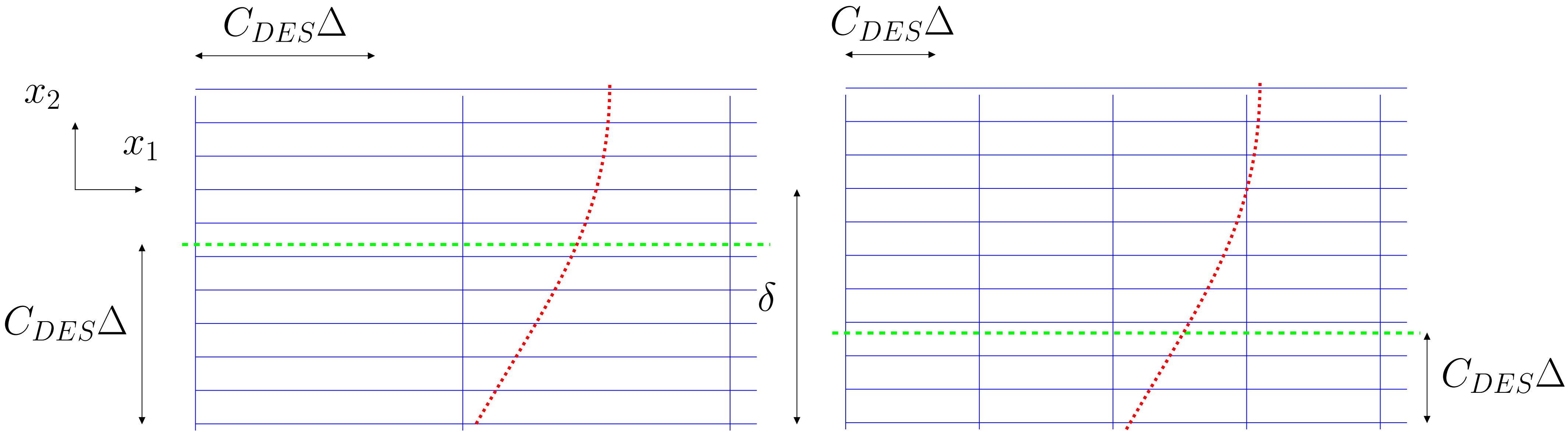
¶See Section 20.3, DDES



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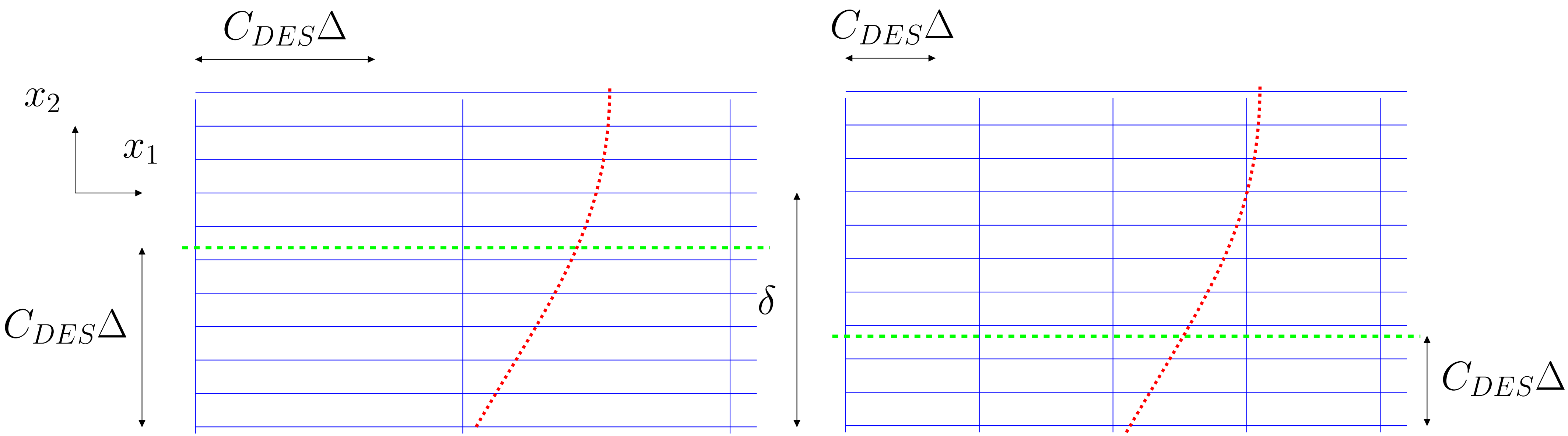

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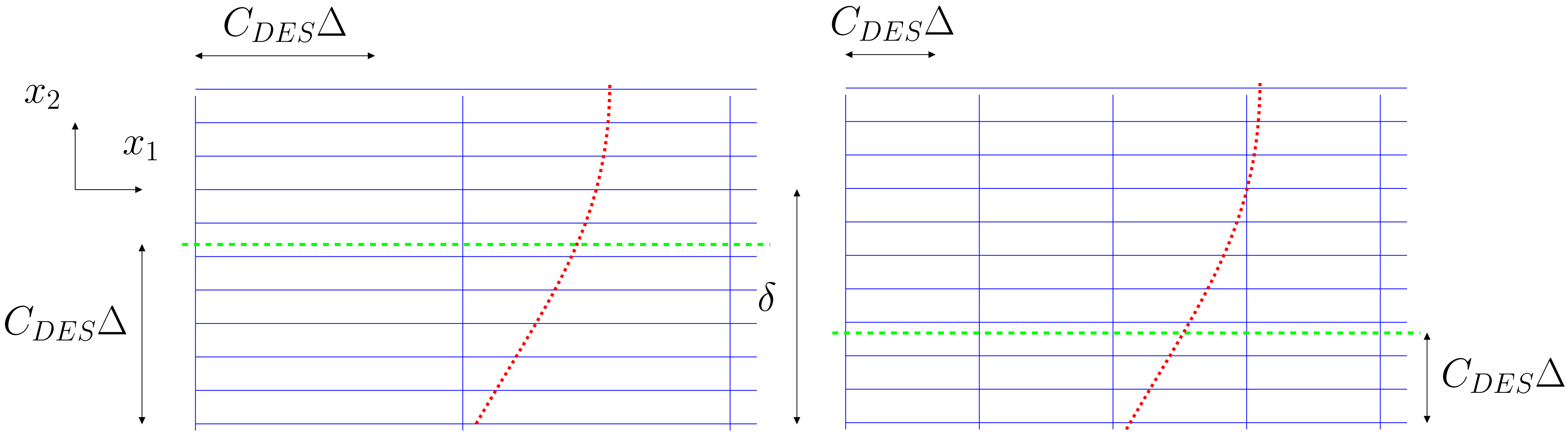
¶See Section 20.3, DDES



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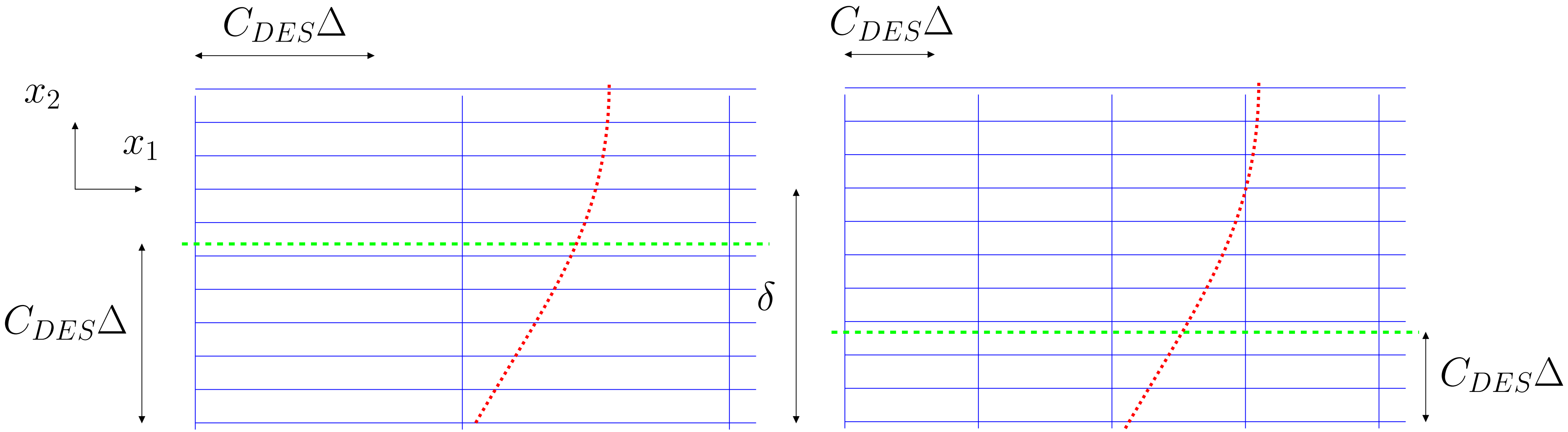

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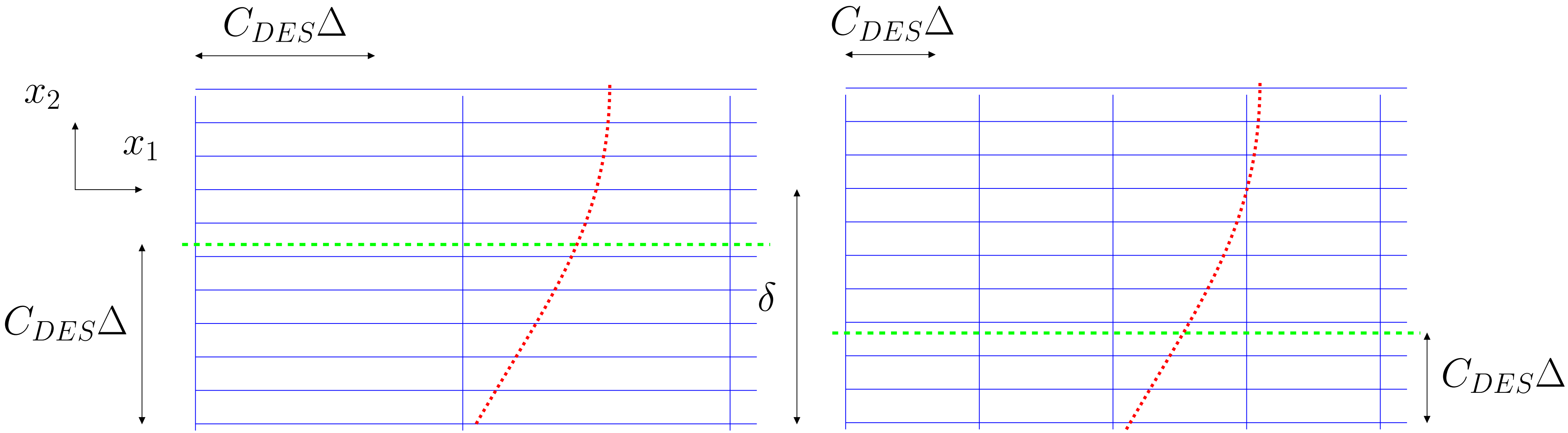

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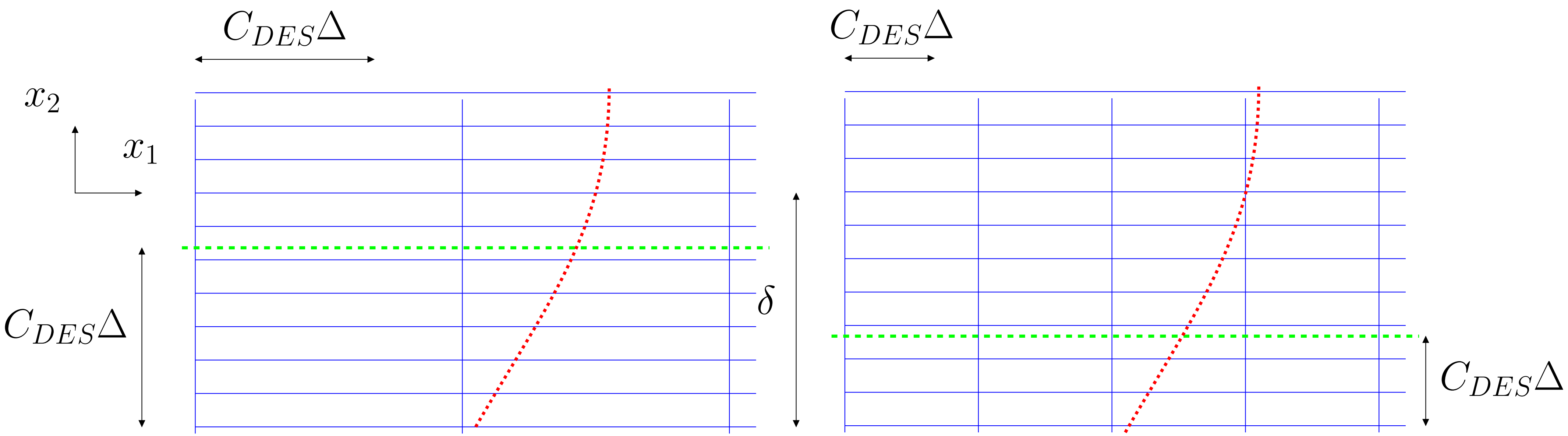

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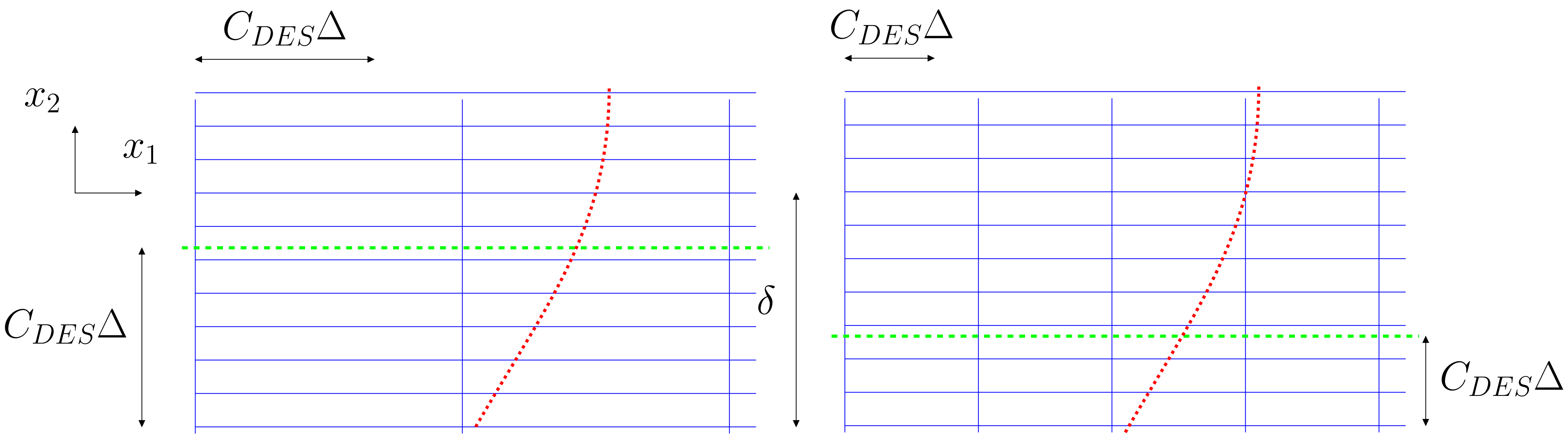

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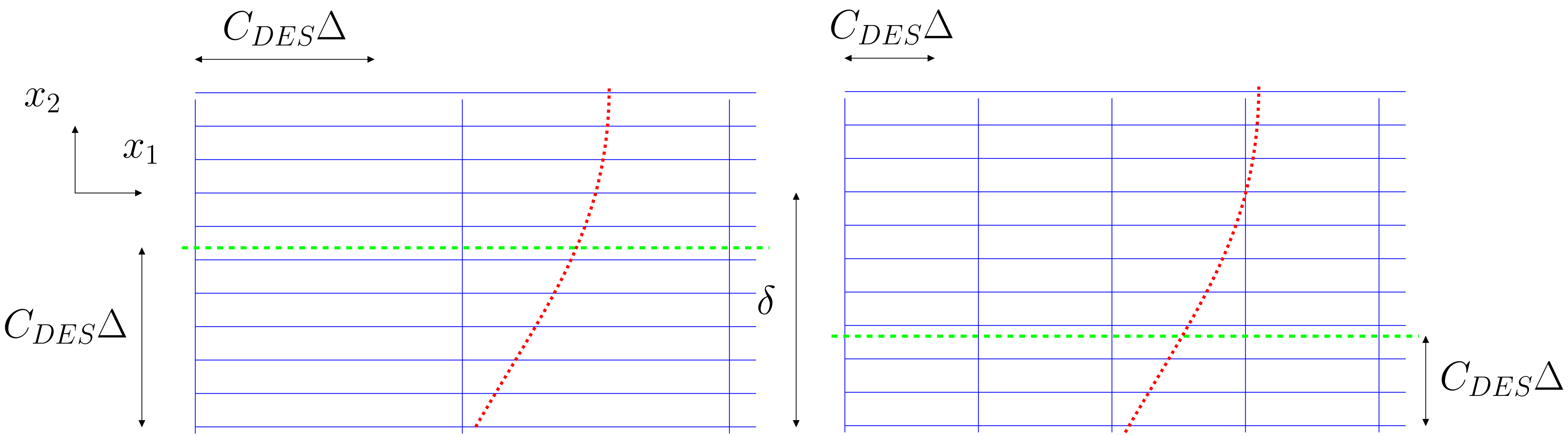

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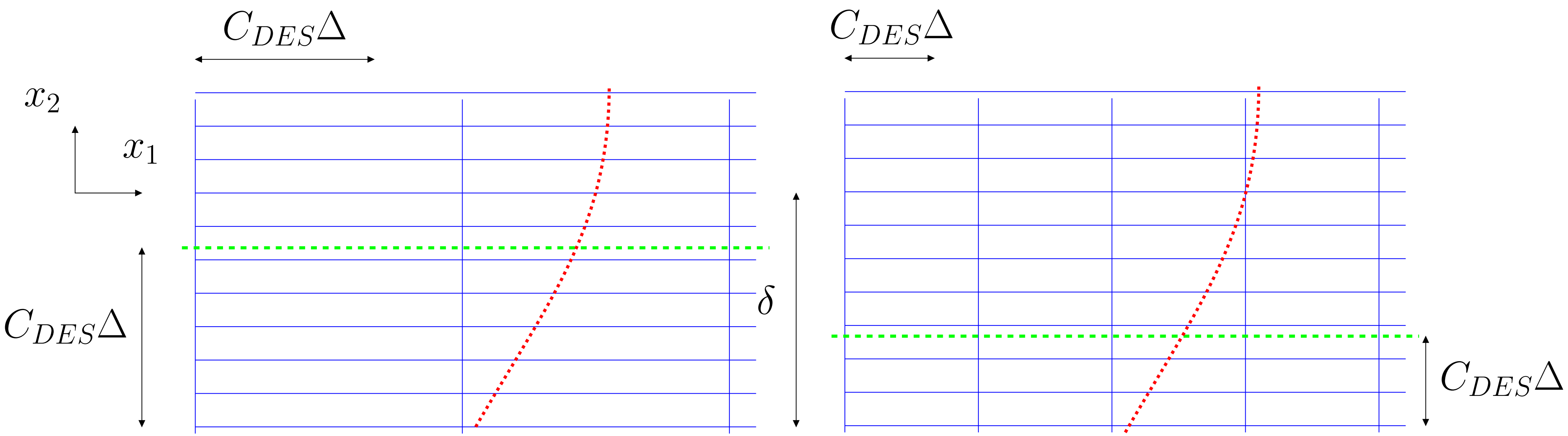

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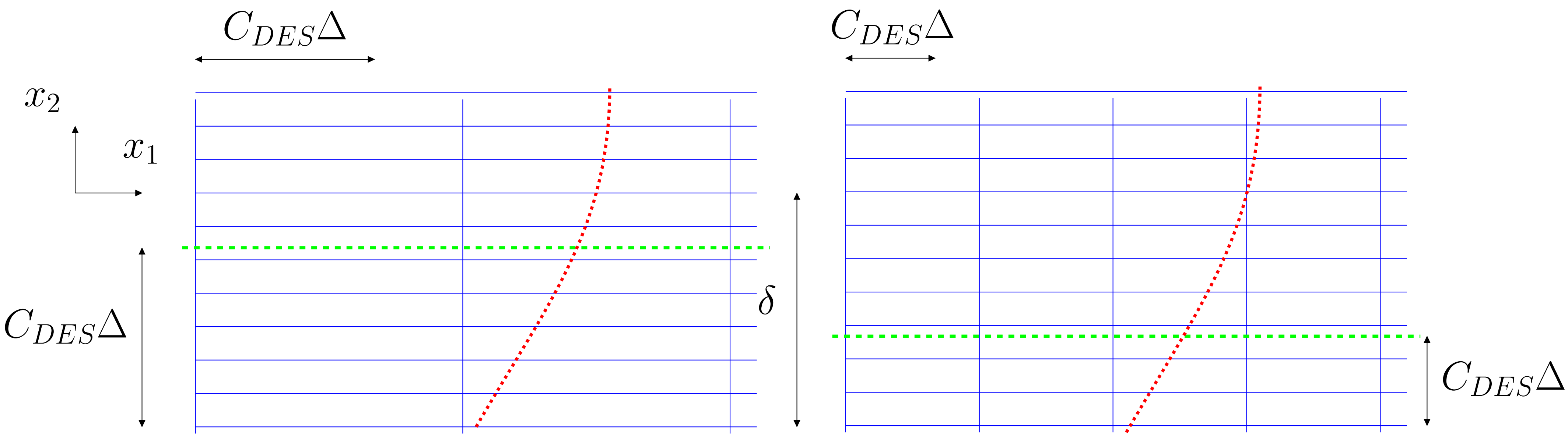
¶See Section 20.3, DDES



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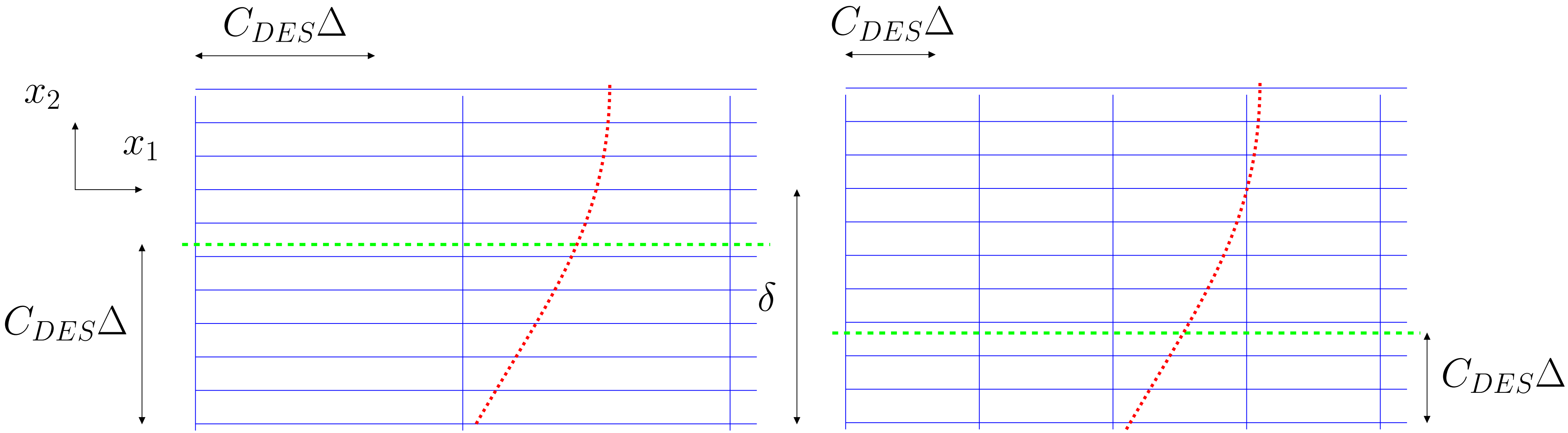

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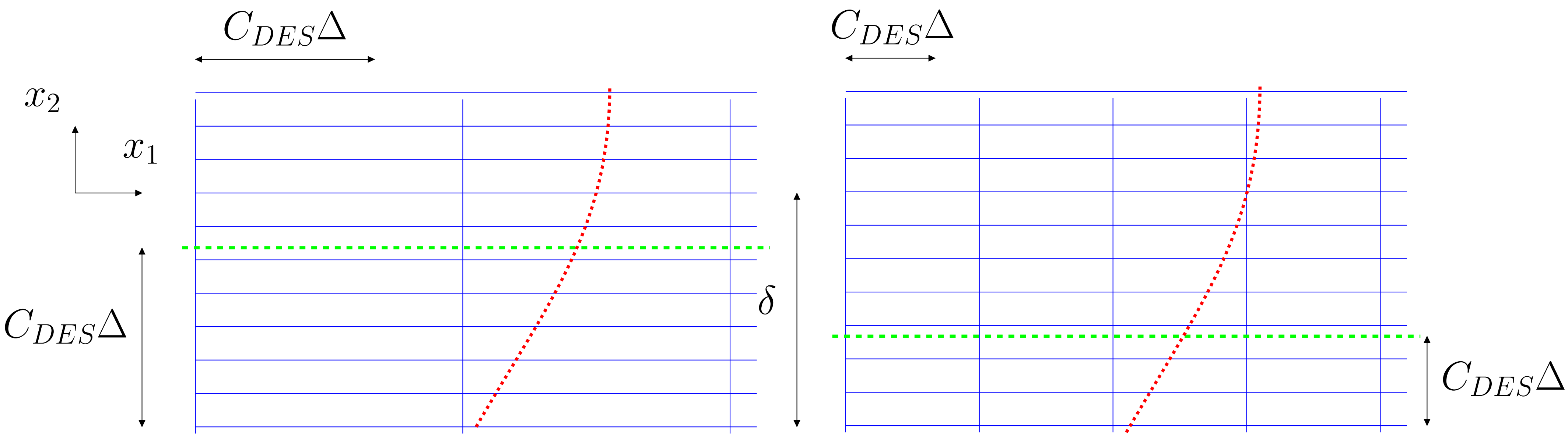

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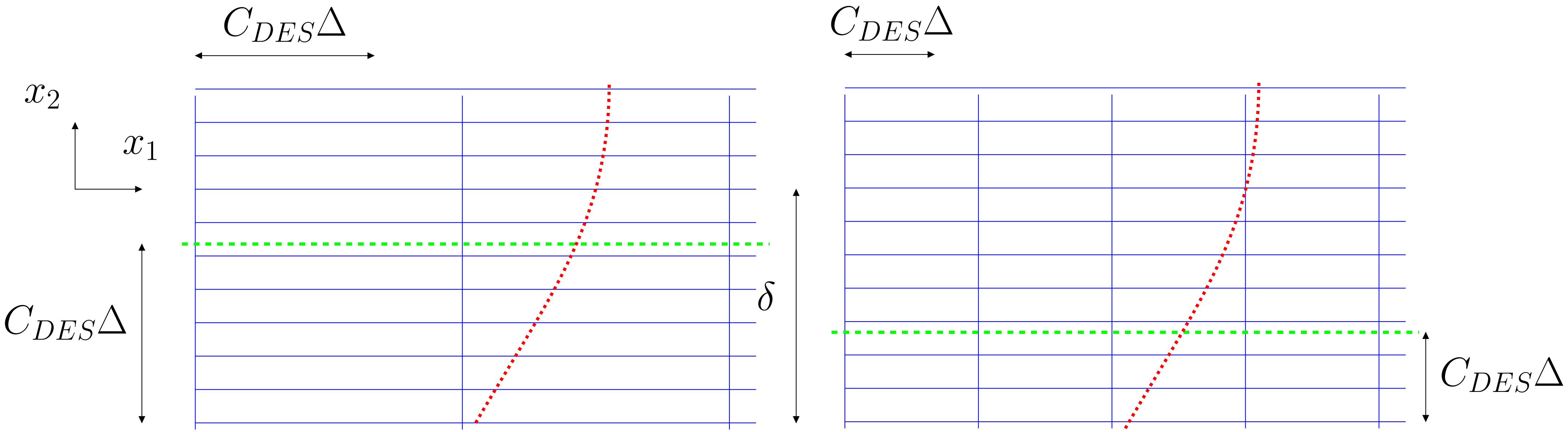

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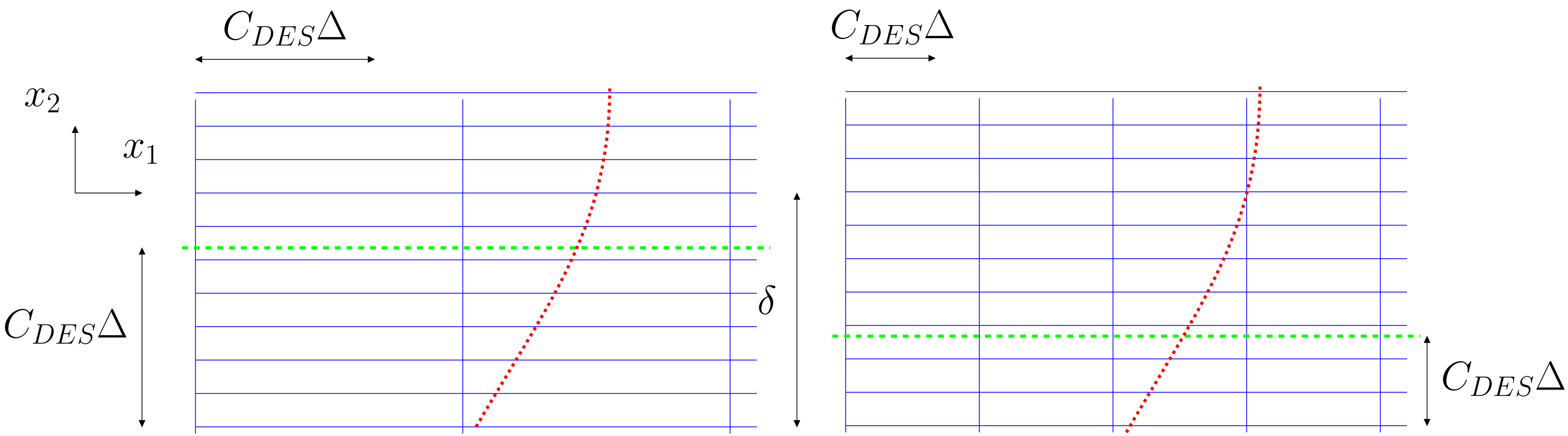
¶See Section 20.3, DDES



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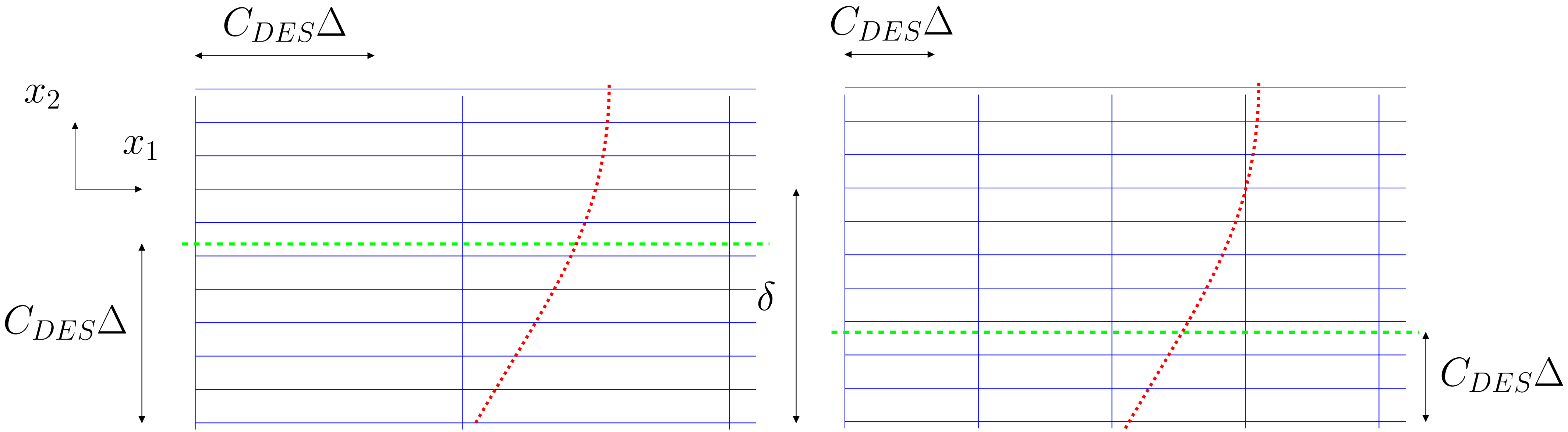
¶See Section 20.3, DDES



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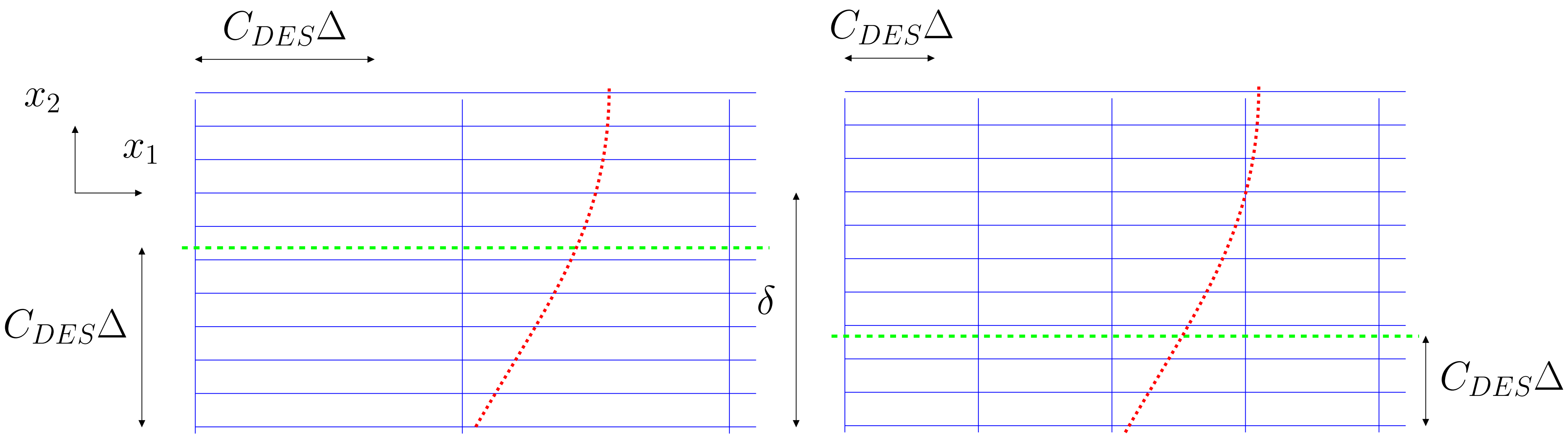

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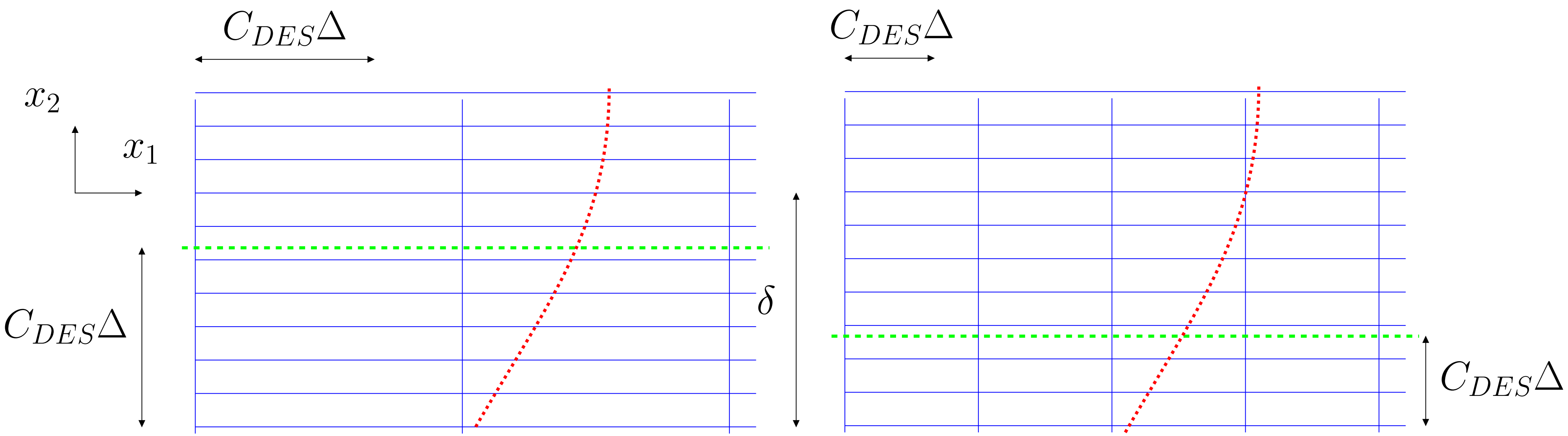
¶See Section 20.3, DDES



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► F_S is called the shielding function: ► it protects the boundary layer from LES

¶See Section 21, Hybrid LES-RANS

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►DES:

¶See Section 21, Hybrid LES-RANS

►DES: The entire boundary layer is modelled with URANS

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Hybrid LES-RANS: Only the inner part of the log region is modelled with URANS.

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Hybrid LES-RANS: Only the inner part of the log region is modelled with URANS.

▶Hybrid LES-RANS is also called WM-LES (WM=Wall-Modelled)

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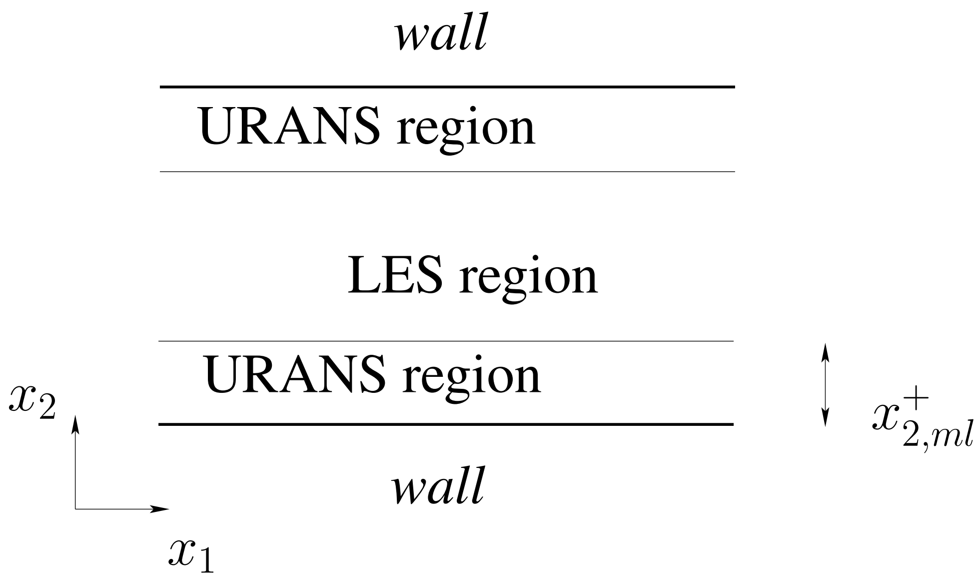

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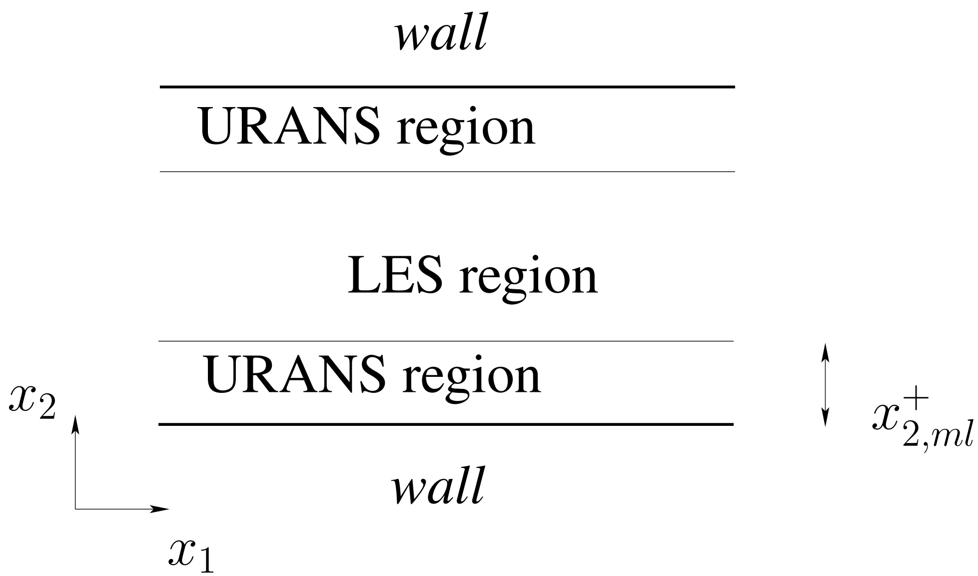

▶One-equation model in both URANS and LES region

¶See Section 21, Hybrid LES-RANS

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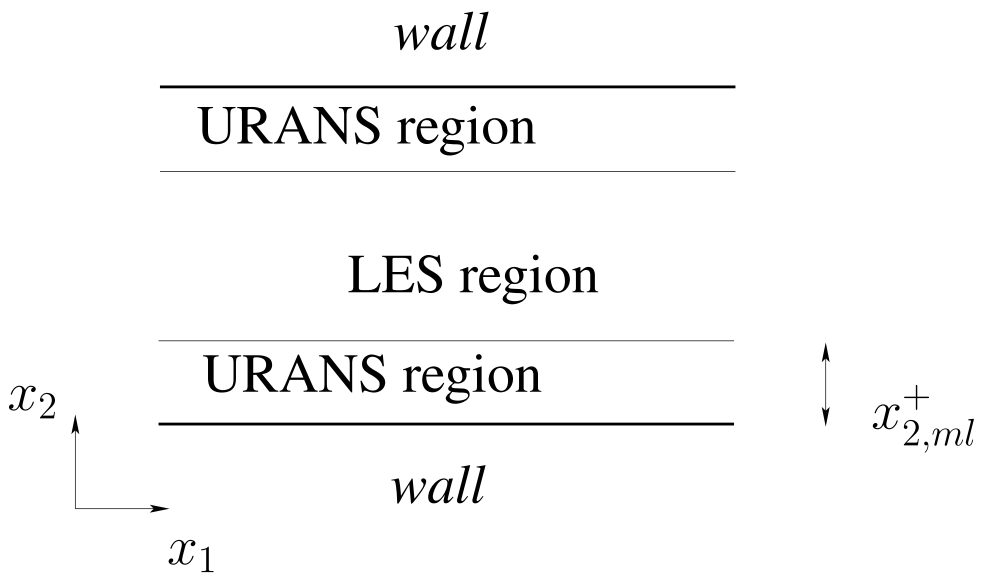

►One-equation model in both URANS and LES region

$$\frac{\partial k_T}{\partial t} + \frac{\partial}{\partial x_j}(\bar{v}_j k_T) =$$

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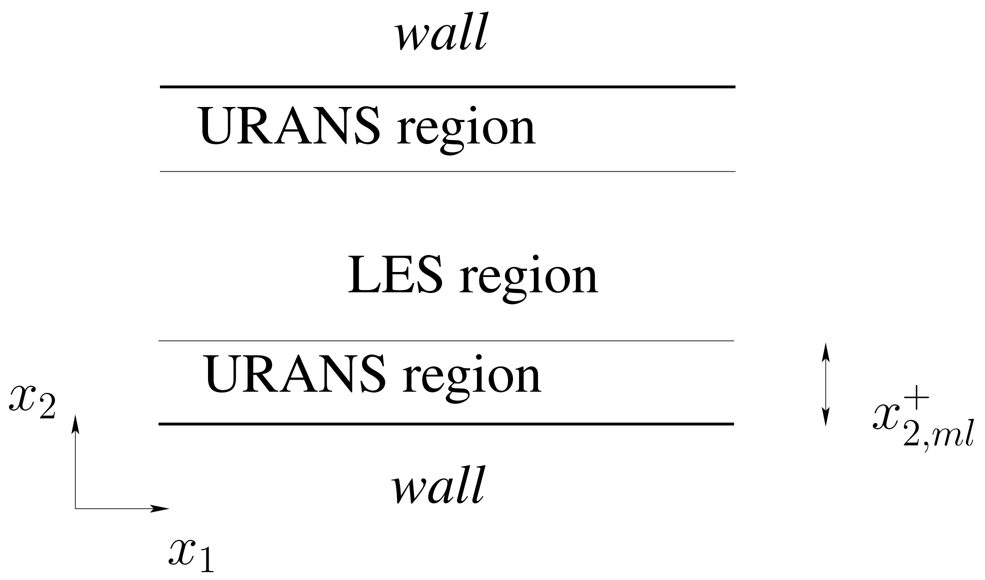

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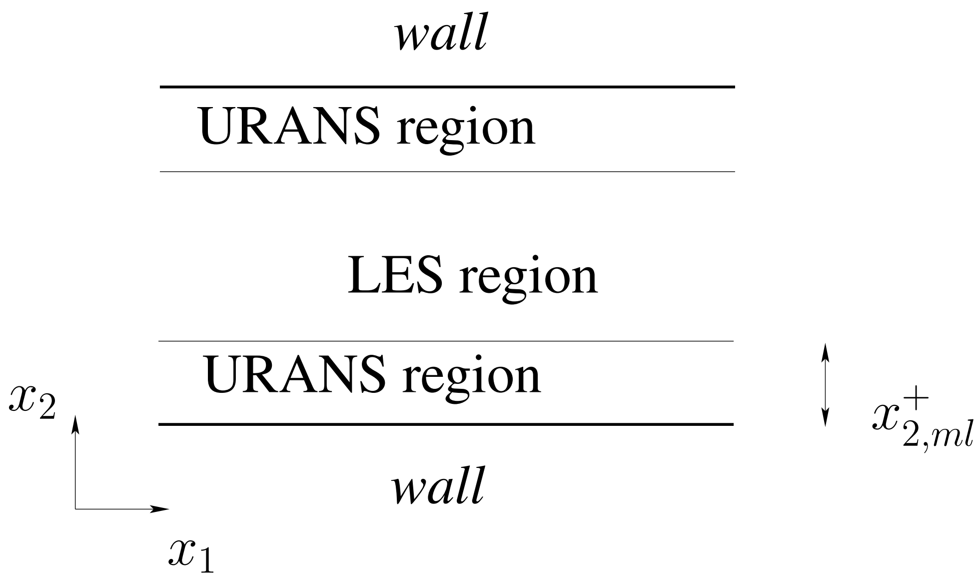

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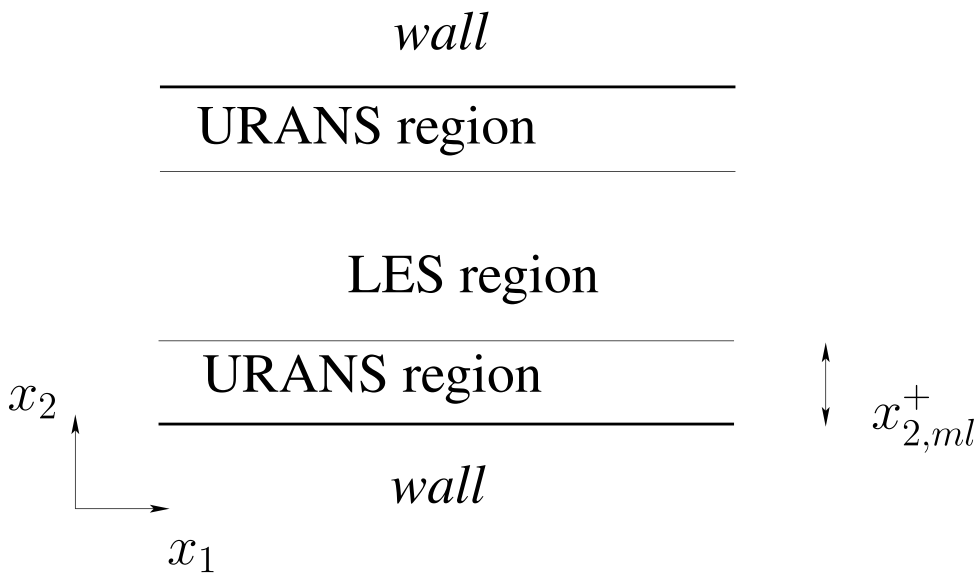

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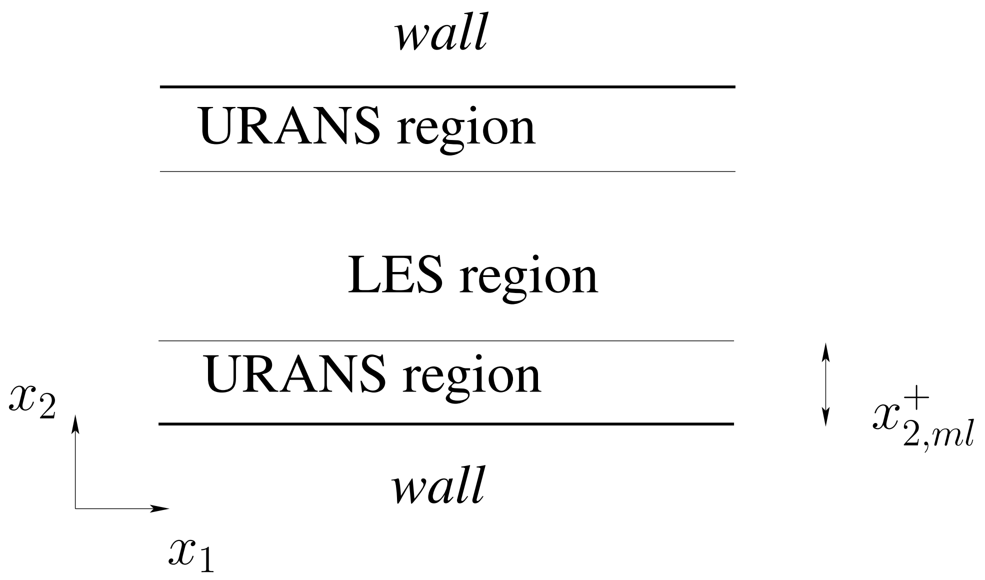

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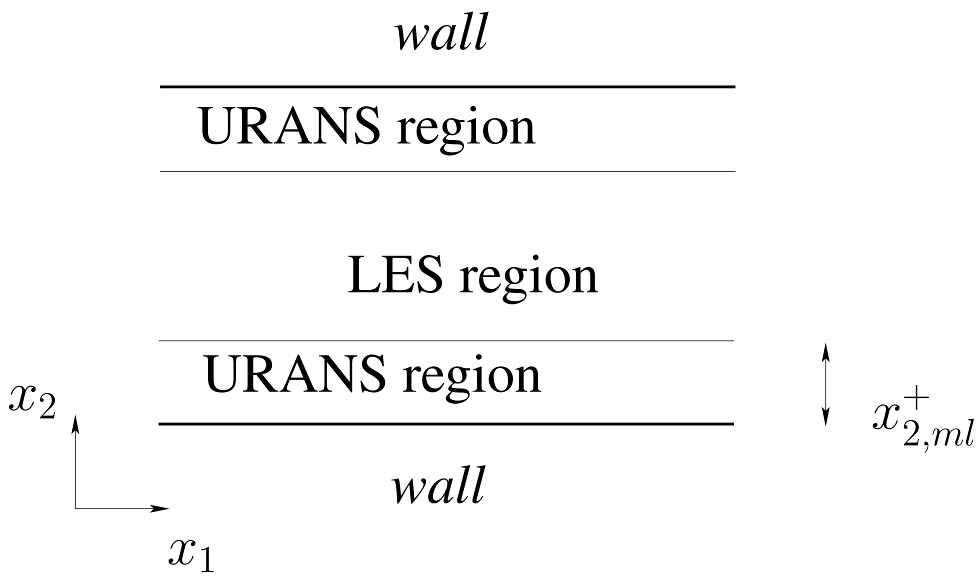

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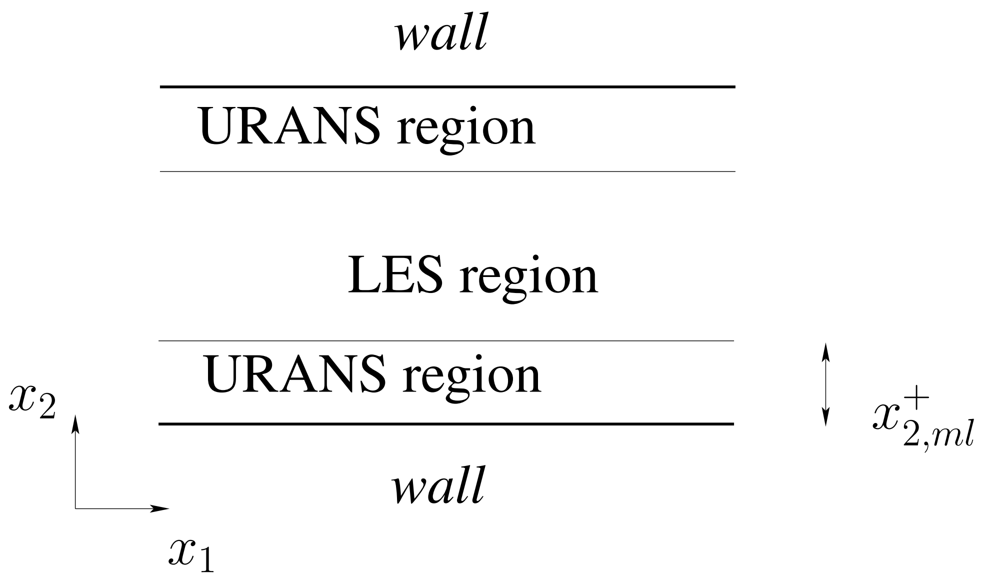

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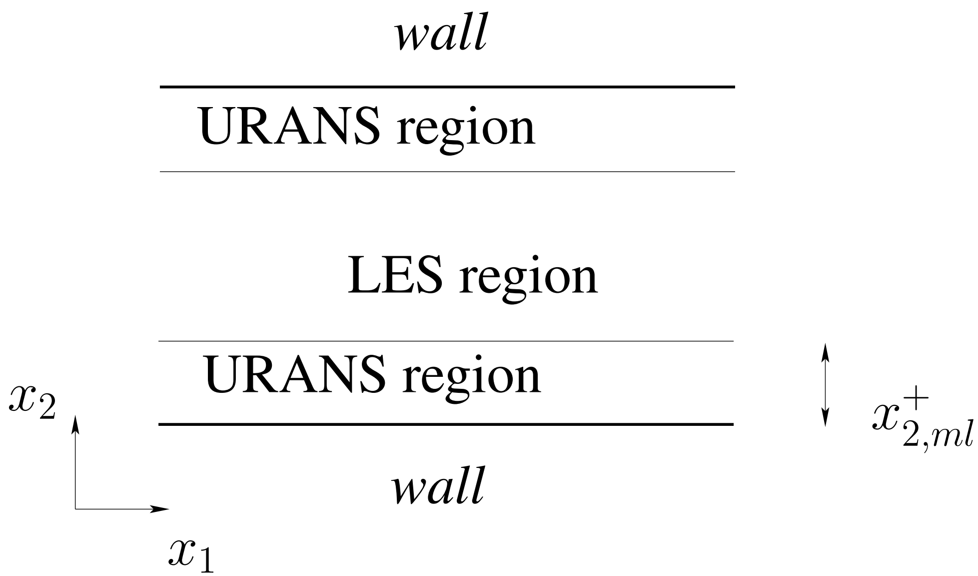

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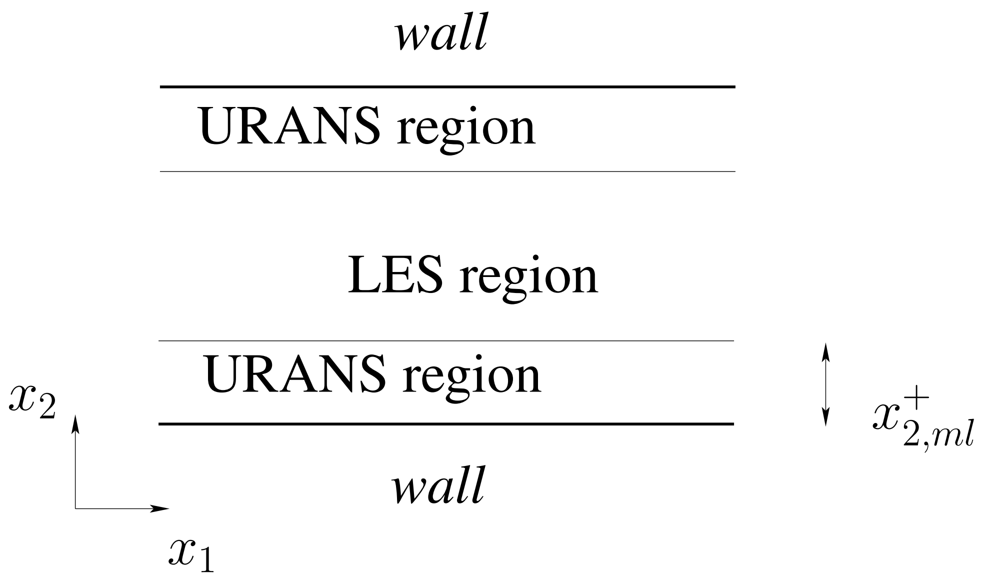

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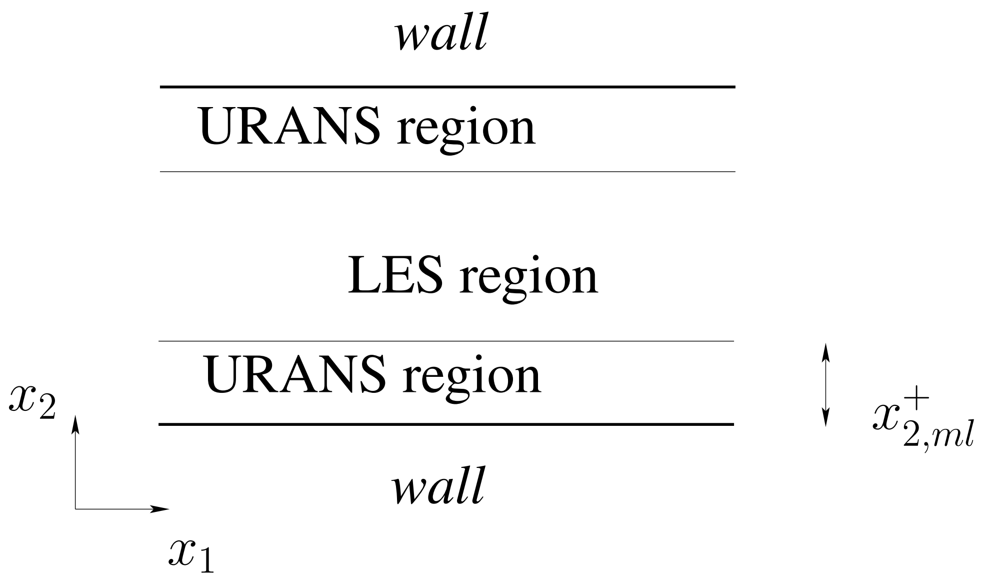

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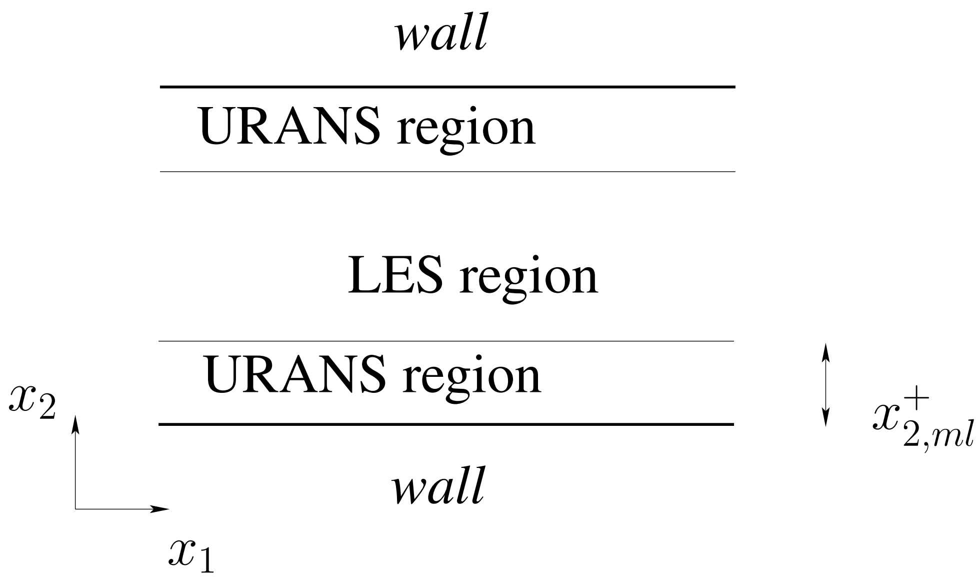

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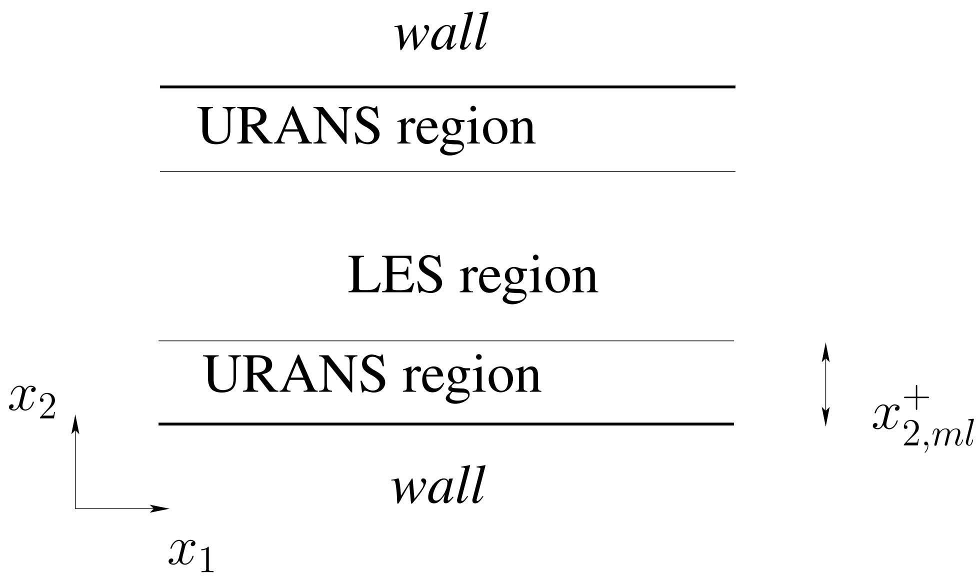

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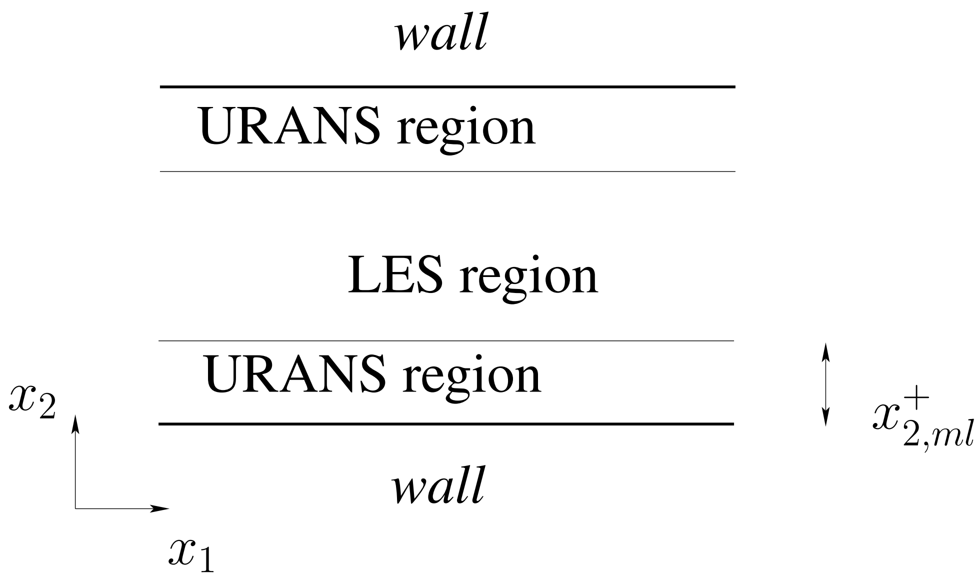

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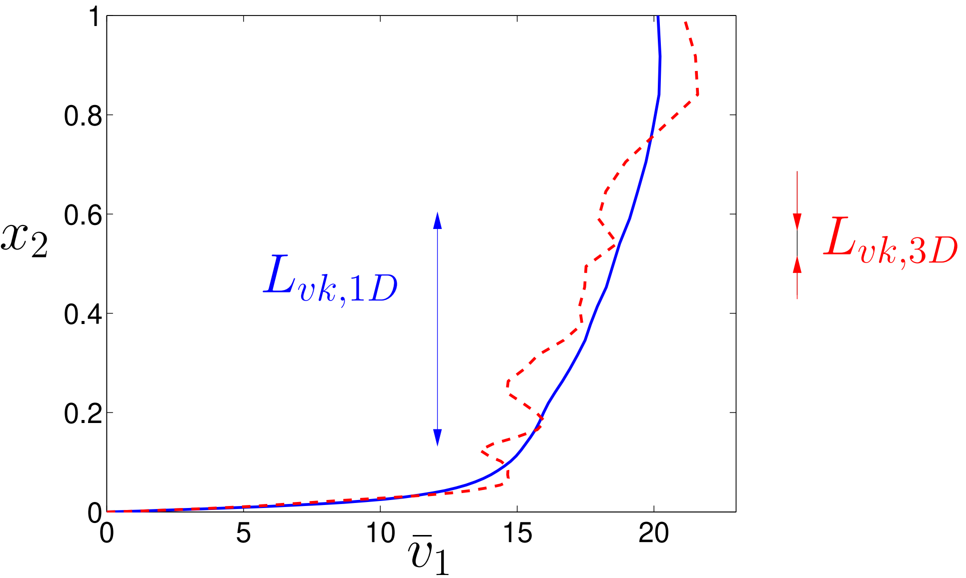

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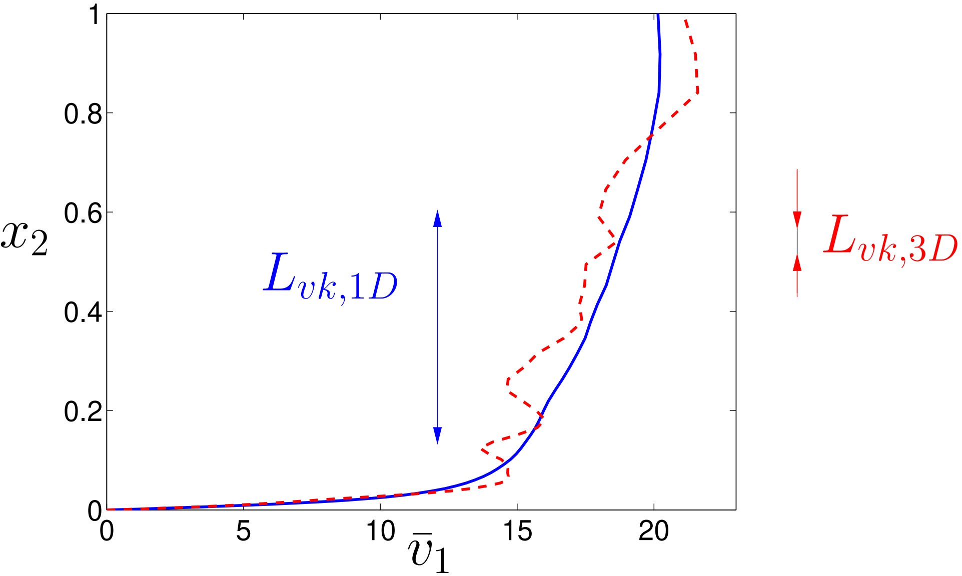

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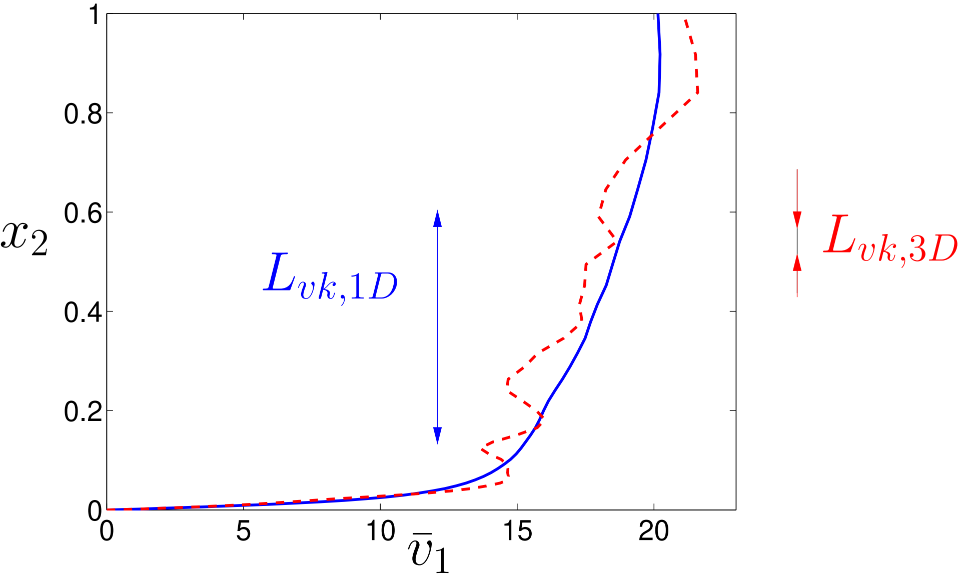

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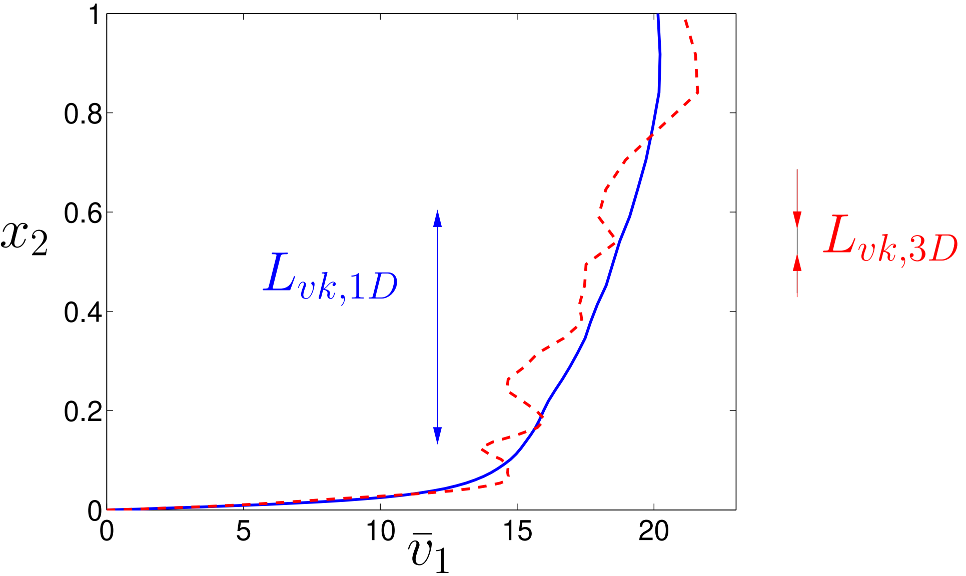

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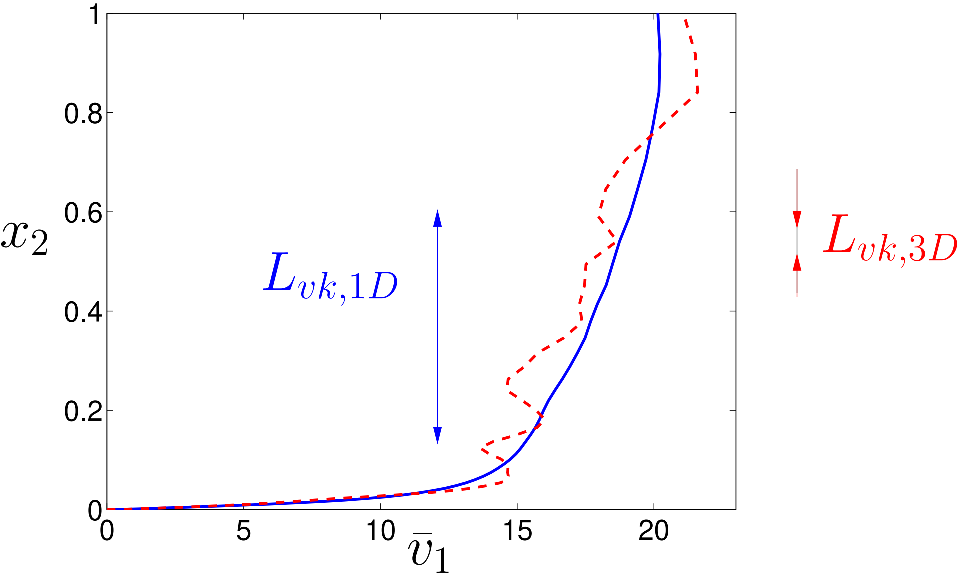

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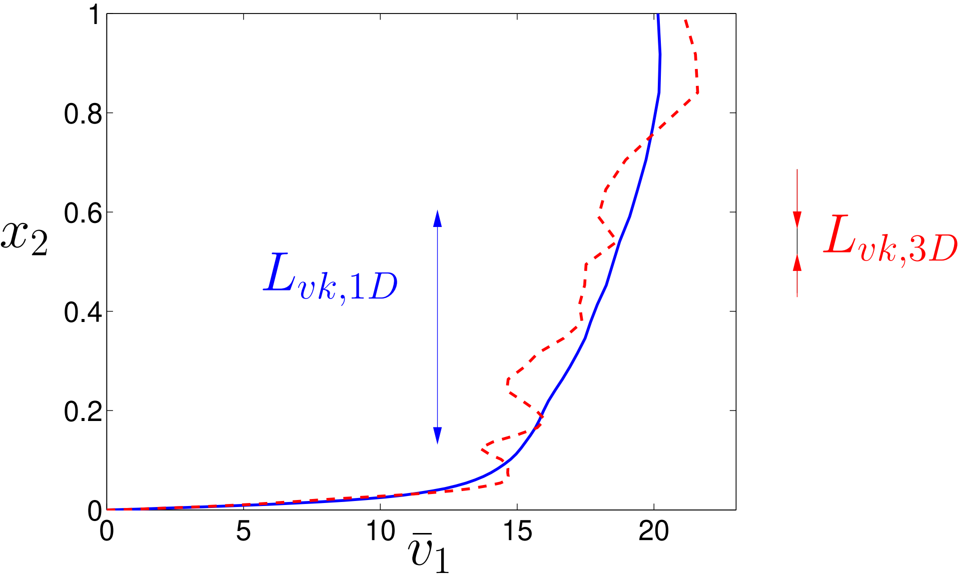

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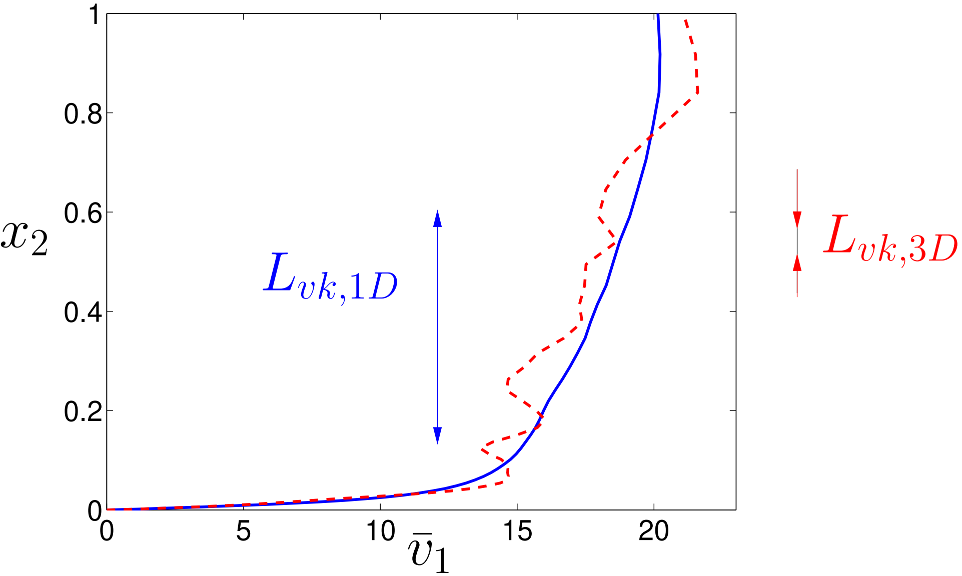

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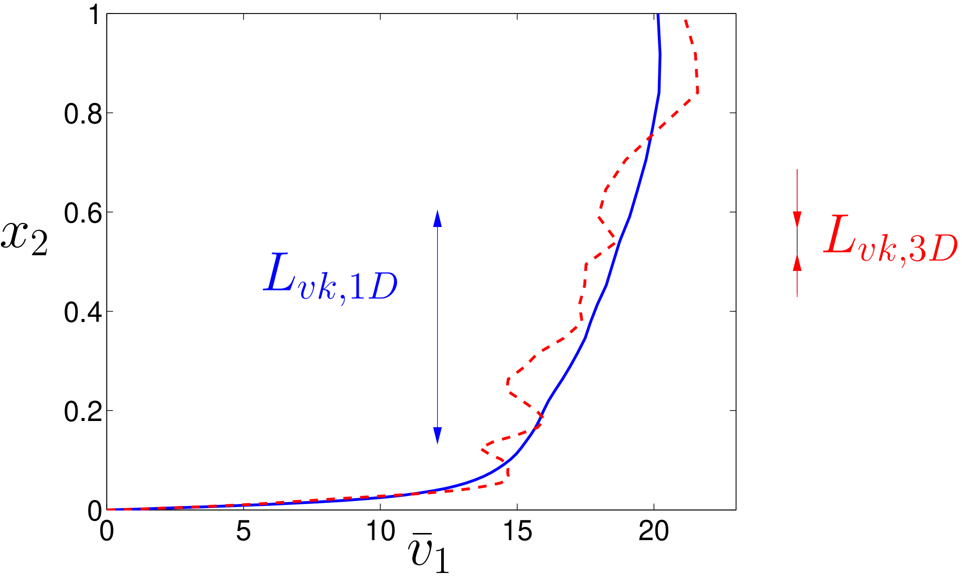

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▶ k and ω equations

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$$C^k =$$

► k and ω equations

$$C^k \;=\; P^k +$$

▶ k and ω equations

$$C^k = P^k + D^k -$$

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$$
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- In URANS (without SAS), resolved fluctuations are damped.

On-line Lecture 11

¶See Section 23, The PANS Model

On-line Lecture 11

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▶PANS: Partial-Averaging Navier-Stokes. It is a hybrid LES-RANS model based on the $k - \varepsilon$ model.

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On-line Lecture 11

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- Multiply the RANS k_{RANS} equation ($k_{tot} = k_{RANS} = k + k_{res}$) by f_k

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$$f_k \left\{ \frac{\partial k_{tot}}{\partial t} + \bar{V}_j \frac{\partial k_{tot}}{\partial x_j} \right\} = \frac{\partial k}{\partial t} + \bar{V}_j \frac{\partial k}{\partial x_j} \simeq \frac{\partial k}{\partial t} + \bar{v}_j \frac{\partial k}{\partial x_j}$$

▶Right side, diffusion term

$$f_k \left\{ \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_{t,tot}}{\sigma_k} \right) \frac{\partial k_{tot}}{\partial x_j} \right] \right\} = \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_{t,tot}}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] = \frac{\partial}{\partial x_j} \left[\left(\nu + \frac{\nu_t}{\sigma_{ku}} \right) \frac{\partial k}{\partial x_j} \right]$$

where $\sigma_{ku} = \sigma_k f_k^2 / f_\varepsilon, \quad \nu_t = c_\mu k^2/\varepsilon, \quad \nu_{t,tot} = c_\mu k_{tot}^2/\varepsilon_{tot}$

▶Right side, production and dissipation term

▶$P^{k,tot}$ and ε_{tot} are replaced by P_k and ε, i.e.

$$f_k \left(P^{k,tot} - \varepsilon_{tot} \right) = P^k - \varepsilon \quad \Rightarrow \quad P^{k,tot} = \frac{1}{f_k}(P^k - \varepsilon) + \frac{\varepsilon}{f_\varepsilon}$$

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► Production and destruction terms

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►use Eq. 40.1, $k_{tot} = k/f_k$, $\varepsilon_{tot} = \varepsilon/f_\varepsilon$

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$$C_{\varepsilon 1}\frac{\varepsilon f_k}{k}\left(\frac{1}{f_k}(P^k-\varepsilon)+\frac{\varepsilon}{f_\varepsilon}\right)-C_{\varepsilon 2}\frac{\varepsilon^2 f_k}{f_\varepsilon k}$$

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▶When $f_k = 1$, the PANS eqns are in RANS mode

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial(\varepsilon \bar{v}_j)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_{\varepsilon u}}\right)\frac{\partial \varepsilon}{\partial x_j}\right] + C_{\varepsilon 1} P^k \frac{\varepsilon}{k} - C^*_{\varepsilon 2}\frac{\varepsilon^2}{k}$$

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- When $f_k = 1$, the PANS eqns are in RANS mode
- When $f_k < 1$ (say, $f_k = 0.4$) then:

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial(\varepsilon \bar{v}_j)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_{\varepsilon u}}\right)\frac{\partial \varepsilon}{\partial x_j}\right] + C_{\varepsilon 1} P^k \frac{\varepsilon}{k} - C^*_{\varepsilon 2}\frac{\varepsilon^2}{k}$$

$$C^*_{\varepsilon 2} = C_{\varepsilon 1} + \frac{f_k}{f_\varepsilon}(C_{\varepsilon 2} - C_{\varepsilon 1}) = 1.5 + \frac{f_k}{f_\varepsilon}(1.9 - 1.5), \quad f_\varepsilon = 1, \quad \nu_t = c_\mu \frac{k^2}{\varepsilon}$$

- ▶When $f_k = 1$, the PANS eqns are in RANS mode
- When $f_k < 1$ (say, $f_k = 0.4$) then:

$C^*_{\varepsilon 2}\frac{\varepsilon^2}{k}$ is reduced

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial (\varepsilon \bar{v}_j)}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_{\varepsilon u}}\right)\frac{\partial \varepsilon}{\partial x_j}\right] + C_{\varepsilon 1} P^k \frac{\varepsilon}{k} - C^*_{\varepsilon 2}\frac{\varepsilon^2}{k}$$

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$\Rightarrow \varepsilon$ is increased

$\Rightarrow k$ is decreased

$\Rightarrow \nu_t$ is decreased (both small k and large ε)

$\Rightarrow$ the momentum eqns go into LES mode.

¶See Section 23.2, Zonal PANS: different treatments of the RANS-LES interface

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► In the previous slides we assumed that f_k is constant. ($f_k = k/k_{tot}$, $k_{tot} = k_{res} + k$)

(40.2)

¶See Section 23.2, Zonal PANS: different treatments of the RANS-LES interface

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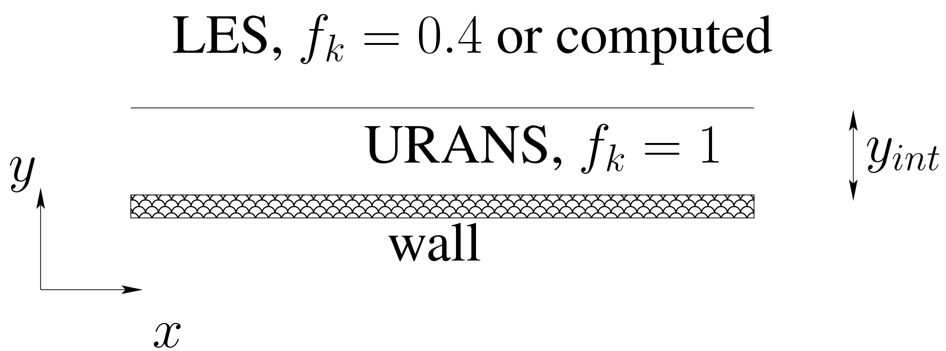

The URANS and the LES regions near a wall (horizontal interface).

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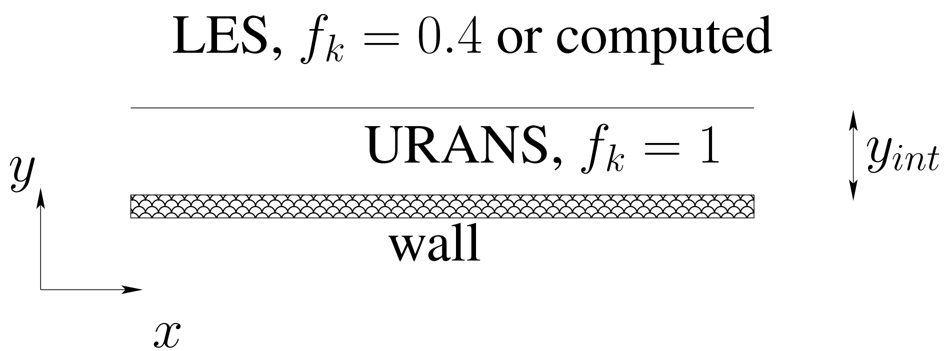

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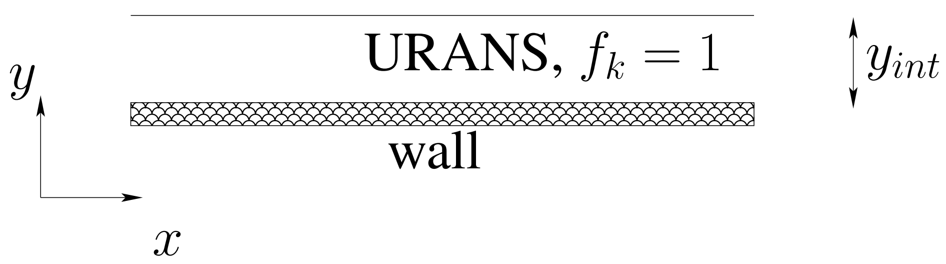

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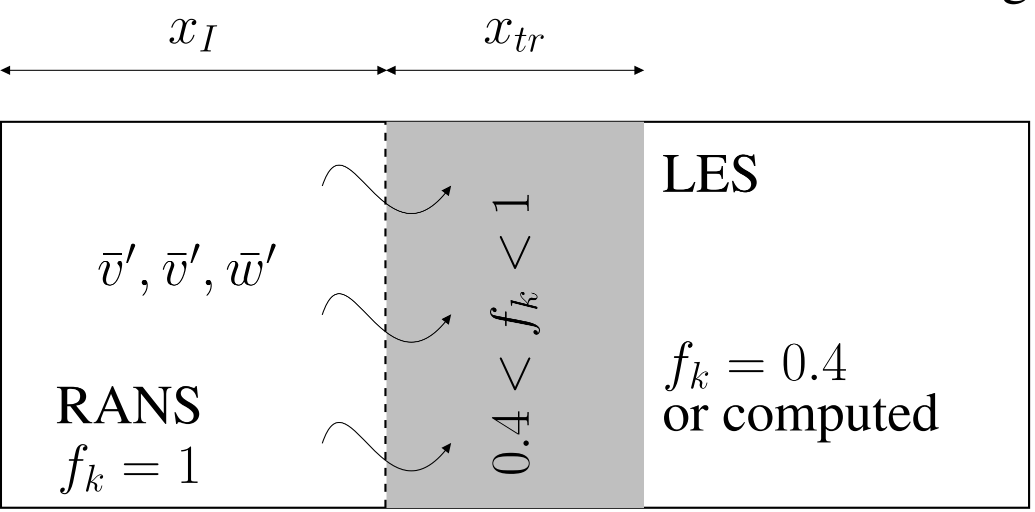

Embedded LES. RANS-LES interface at x_I.

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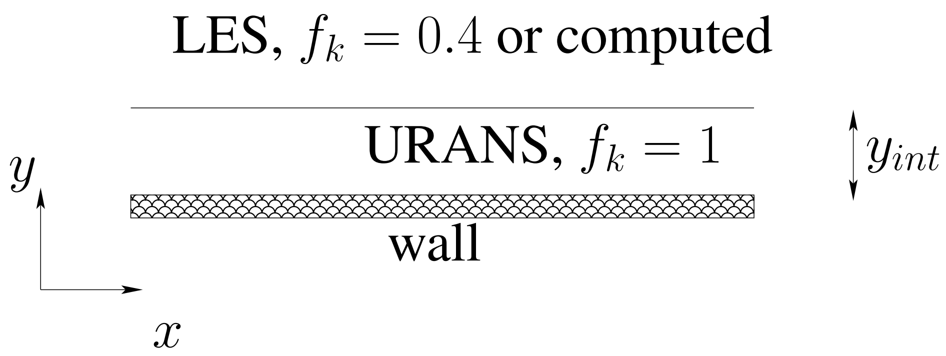

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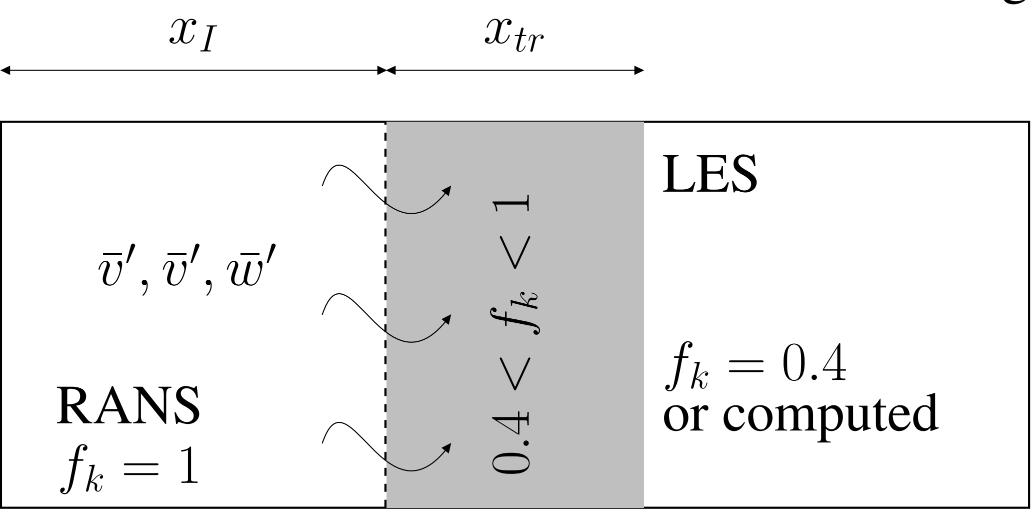

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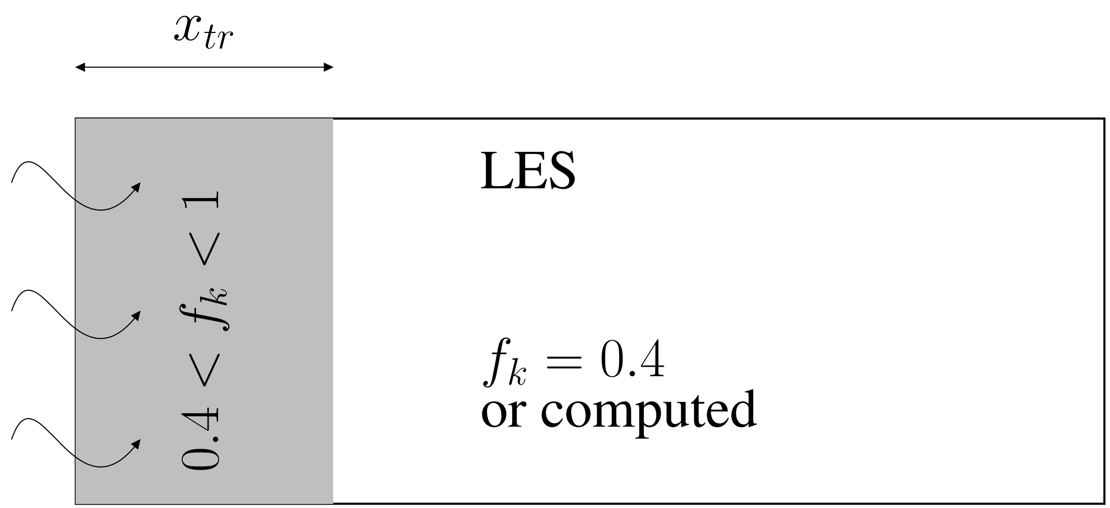

RANS-LES interface at inlet.

¶See Section 23.2, Zonal PANS: different treatments of the RANS-LES interface

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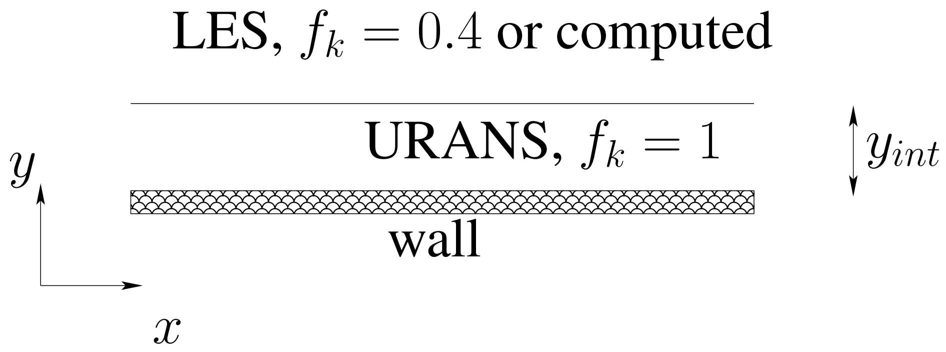
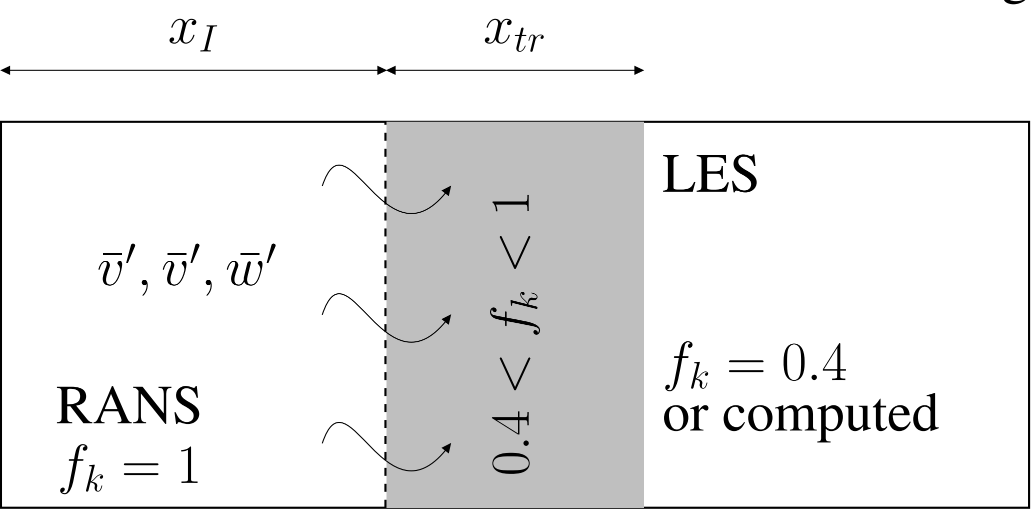


The URANS and the LES regions near a wall (horizontal interface).



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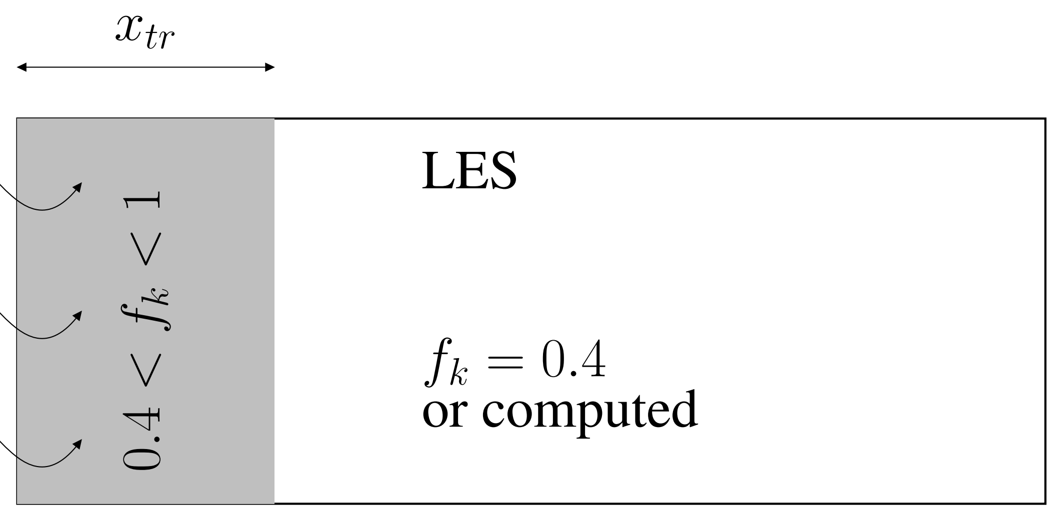

RANS-LES interface at inlet.

►A gradient of f_k at RANS-LES interface since f_k varies in space; Eq. 40.2 replaced by

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which agrees with $-k_{tot}\frac{df_k}{dt}$ on right side of k_{res} eq.

¶See Section 23.3, A new formulation of f_k for the PANS model

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▶How to compute f_k?

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First, some old formulations.

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1, 2 : $f_k = C_\mu^{-1/2}\left(\frac{\Delta}{L_t}\right)^{2/3}, L_t = \frac{k_{tot}^{3/2}}{\varepsilon}$ using $\Delta = \Delta_{min}$ or $\Delta = (\Delta V)^{1/3}$.

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►A formula derived an expression from the Kolmogorov energy spectrum

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►A formula derived an expression from the Kolmogorov energy spectrum

$$4: \quad f_k = 1 - \left[\frac{(\Lambda/\Delta)^{2/3}}{0.23 + (\Lambda/\Delta)^{2/3}}\right]^{9/2}$$

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▶DES

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▶DES

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▶DES

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\begin{aligned}
C^k &= P^k + D^k - \psi\varepsilon \\
C^\varepsilon &= C_{\varepsilon 1}\frac{\varepsilon}{k}P^k + D^\varepsilon - C_{\varepsilon 2}\frac{\varepsilon^2}{k}
\end{aligned}
$$

►DES

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►DES

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\psi &= \max\left(1, \frac{k^{3/2}/\varepsilon}{C_{DES}\Delta_{max}}\right)
\end{aligned}
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▶PANS

$$
C^k = P^k + D^k - \varepsilon
$$

▶DES

$$
\begin{aligned}
C^k &= P^k + D^k - \psi\varepsilon \\
C^\varepsilon &= C_{\varepsilon 1}\frac{\varepsilon}{k}P^k + D^\varepsilon - C_{\varepsilon 2}\frac{\varepsilon^2}{k} \\
\psi &= \max\left(1, \frac{k^{3/2}/\varepsilon}{C_{DES}\Delta_{max}}\right)
\end{aligned}
$$

▶PANS

$$
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C^k &= P^k + D^k - \varepsilon \\
C^\varepsilon &=
\end{aligned}
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▶PANS

$$
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C^k &= P^k + D^k - \varepsilon \\
C^\varepsilon &= C_{\varepsilon 1}\frac{\varepsilon}{k}P^k +
\end{aligned}
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▶PANS

$$C^k = P^k + D^k - \varepsilon$$

$$C^\varepsilon = C_{\varepsilon 1}\frac{\varepsilon}{k}P^k + D^\varepsilon - C^*_{\varepsilon 2}\frac{\varepsilon^2}{k}$$

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▶PANS

$$C^k = P^k + D^k - \varepsilon$$

$$C^\varepsilon = C_{\varepsilon 1}\frac{\varepsilon}{k}P^k + D^\varepsilon - C^*_{\varepsilon 2}\frac{\varepsilon^2}{k}$$

$$C^*_{\varepsilon 2} = C_{\varepsilon 1} + \frac{f_k}{f_\varepsilon}(C_{\varepsilon 2} - C_{\varepsilon 1})$$

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f_{k,obs} &= \frac{k_{model}}{k_{model} + k_{res}}
\end{aligned}
$$

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▶PANS

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\begin{aligned}
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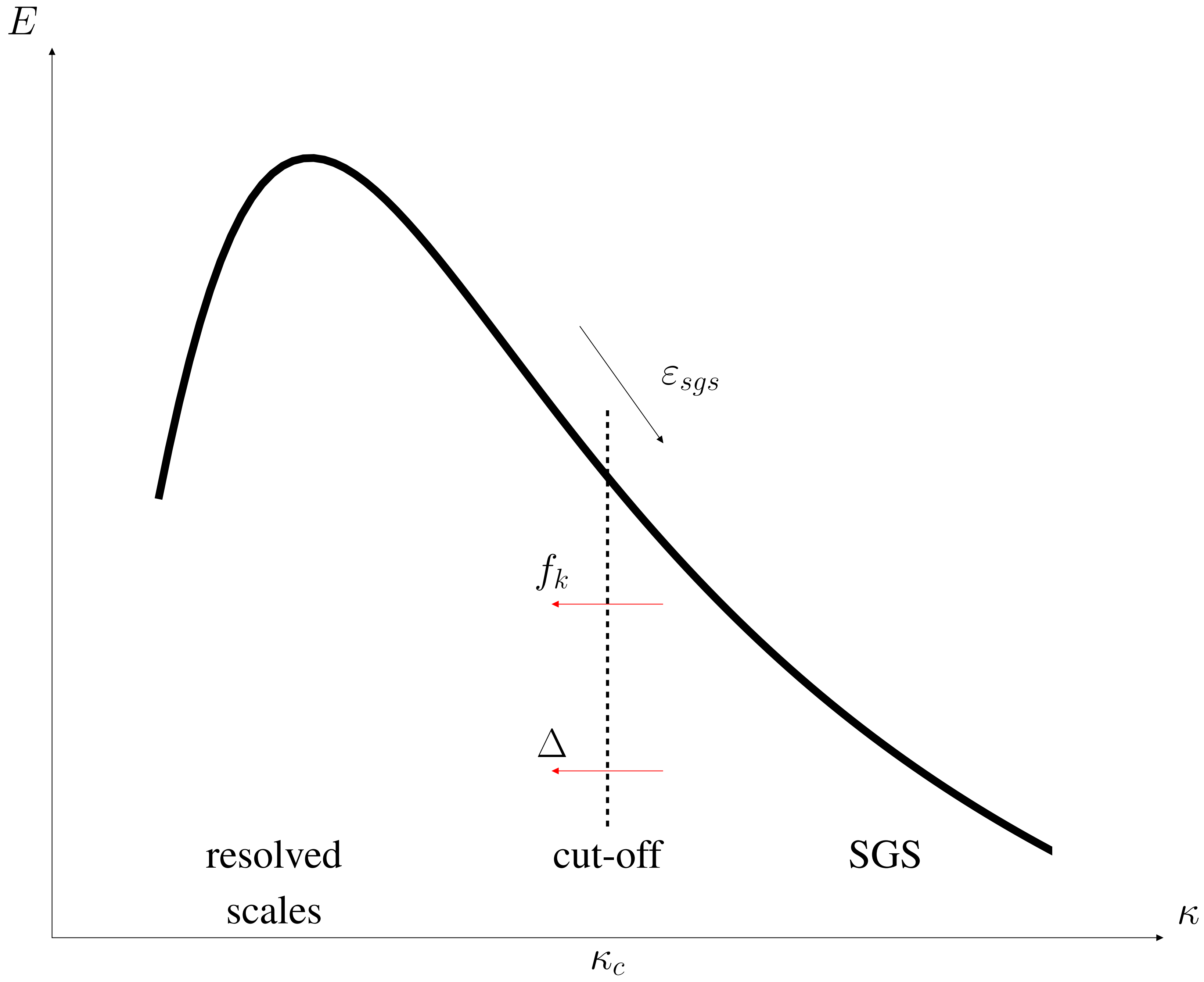
C^*_{\varepsilon 2} &= C_{\varepsilon 1} + \frac{f_k}{f_\varepsilon}(C_{\varepsilon 2} - C_{\varepsilon 1}) \\
f_{k,obs} &= \frac{k_{model}}{k_{model} + k_{res}}, \quad f_\varepsilon \simeq 1
\end{aligned}
$$



Spectrum of velocity fluctuations.

▶DES and PANS I

▶DES and PANS I

$$C^k - D^k \simeq$$

►DES and PANS I

$$C^k - D^k \simeq C^\varepsilon - D^\varepsilon \simeq 0$$

▶DES and PANS I

$$C^k - D^k \simeq C^\varepsilon - D^\varepsilon \simeq 0, \quad \gamma = \frac{P^k}{Sk}$$

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$$\psi =$$

►DES and PANS I

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▶PANS

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▶PANS

$$T P^\varepsilon - P^k = T C^*_{\varepsilon 2}\frac{\varepsilon^2}{k} - \varepsilon \Rightarrow$$

▶DES and PANS I

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▶PANS

$$\begin{aligned} T P^\varepsilon - P^k &= T C^*_{\varepsilon 2}\frac{\varepsilon^2}{k} - \varepsilon \Rightarrow \\ \gamma(C_{\varepsilon 1} - 1) Sk &= (C^*_{\varepsilon 2} - 1)\varepsilon \end{aligned}$$

▶DES and PANS I

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▶PANS

$$\begin{aligned} TP^\varepsilon - P^k &= TC^*_{\varepsilon 2}\frac{\varepsilon^2}{k} - \varepsilon \Rightarrow \\ \gamma(C_{\varepsilon 1} - 1)Sk &= (C^*_{\varepsilon 2} - 1)\varepsilon \end{aligned}$$

▶Differentiation yields:

►DES and PANS I

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►Differentiation yields:

$$\frac{\delta\gamma}{\gamma} +$$

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▶PANS

$$\begin{aligned} {\color{red}T}P^\varepsilon - P^k &= {\color{red}T}C^*_{\varepsilon 2}\frac{\varepsilon^2}{k} - \varepsilon \Rightarrow \\ \gamma(C_{\varepsilon 1} - 1)Sk &= (C^*_{\varepsilon 2} - 1)\varepsilon \end{aligned}$$

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DES and PANS I

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PANS

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Differentiation yields:

$$\frac{\delta\gamma}{\gamma} + \frac{\delta S}{S} + \frac{\delta k}{k} =$$

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$$=$$

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$$\begin{aligned} T P^\varepsilon - P^k &= T C^*_{\varepsilon 2}\frac{\varepsilon^2}{k} - \varepsilon \Rightarrow \\ \gamma(C_{\varepsilon 1} - 1)Sk &= (C^*_{\varepsilon 2} - 1)\varepsilon \end{aligned}$$

▶Differentiation yields:

$$\begin{aligned} \frac{\delta\gamma}{\gamma} + \frac{\delta S}{S} + \frac{\delta k}{k} &= \frac{\delta C^*_{\varepsilon 2}\varepsilon}{(C_{\varepsilon 1} - 1)\gamma Sk} \\ &= \frac{\delta C^*_{\varepsilon 2}}{C^*_{\varepsilon 2} - 1} \end{aligned}$$

DES and PANS I

$$C^k - D^k \simeq C^\varepsilon - D^\varepsilon \simeq 0, \quad \gamma = \frac{P^k}{Sk}, \quad S = (2\bar{s}_{ij}\bar{s}_{ij})^{1/2}, \quad T = \frac{k}{\varepsilon}$$

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▶DES

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Differentiation yields:

$$\frac{\delta\gamma}{\gamma} + \frac{\delta S}{S} + \frac{\delta k}{k} = -\frac{\delta\psi\varepsilon}{(C_{\varepsilon 1} - 1)Sk\gamma} = -\frac{\delta\psi}{C_{\varepsilon 2} - \psi}$$

▶DES and PANS I

$$C^k - D^k \simeq C^\varepsilon - D^\varepsilon \simeq 0, \quad \gamma = \frac{P^k}{Sk}, \quad S = (2\bar{s}_{ij}\bar{s}_{ij})^{1/2}, \quad T = \frac{k}{\varepsilon}$$

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▶DES and PANS II

$$\frac{dC_{\varepsilon 2}^*}{C_{\varepsilon 2}^* - 1} = \frac{-d\psi}{C_{\varepsilon 2} - \psi}$$

▶DES and PANS II

$$\frac{dC^*_{\varepsilon 2}}{C^*_{\varepsilon 2} - 1} = \frac{-d\psi}{C_{\varepsilon 2} - \psi}$$

▶DES and PANS II

$$\frac{dC^*_{\varepsilon 2}}{C^*_{\varepsilon 2}-1}=\frac{-d\psi}{C_{\varepsilon 2}-\psi}$$

▶Integrate from RANS ($C_{\varepsilon 2}$ and $\psi = 1$)

▶DES and PANS II

$$\frac{dC_{\varepsilon 2}^*}{C_{\varepsilon 2}^* - 1} = \frac{-d\psi}{C_{\varepsilon 2} - \psi}$$

▶Integrate from RANS ($C_{\varepsilon 2}$ and $\psi = 1$) to LES ($C_{\varepsilon 2}^*$ and ψ) conditions

▶DES and PANS II

$$\frac{dC^*_{\varepsilon 2}}{C^*_{\varepsilon 2} - 1} = \frac{-d\psi}{C_{\varepsilon 2} - \psi}$$

▶Integrate from RANS ($C_{\varepsilon 2}$ and $\psi = 1$) to LES ($C^*_{\varepsilon 2}$ and ψ) conditions

$$\int_{C_{\varepsilon 2}}^{C^*_{\varepsilon 2}} \frac{dC^*_{\varepsilon 2}}{C^*_{\varepsilon 2} - 1} =$$

▶DES and PANS II

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  - The PANS model is based on a rigorous derivation whereas DES is based on an ad-hoc modification of RANS models

On-line Lecture 12

¶See Section 27.1, Synthesized turbulence

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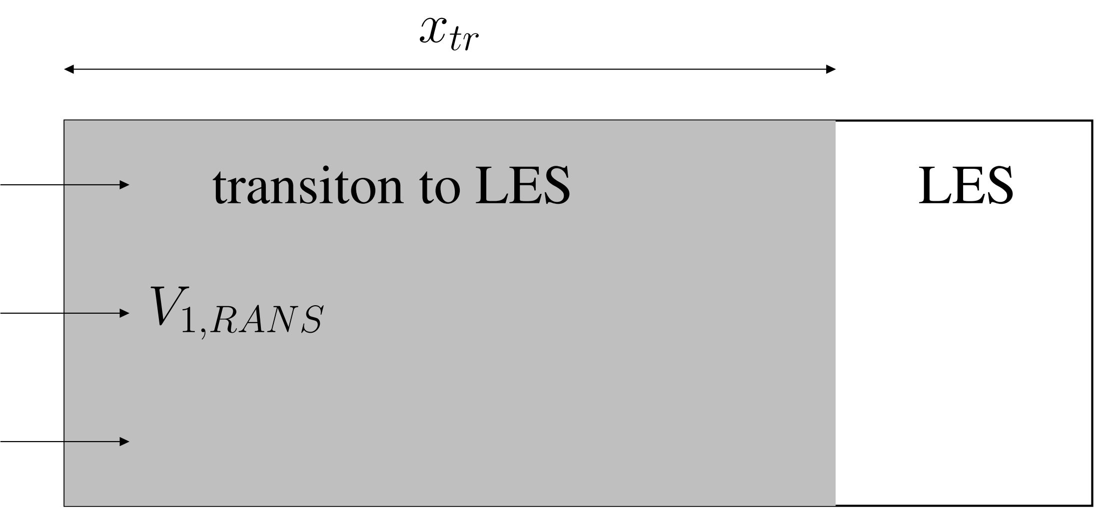

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On-line Lecture 12

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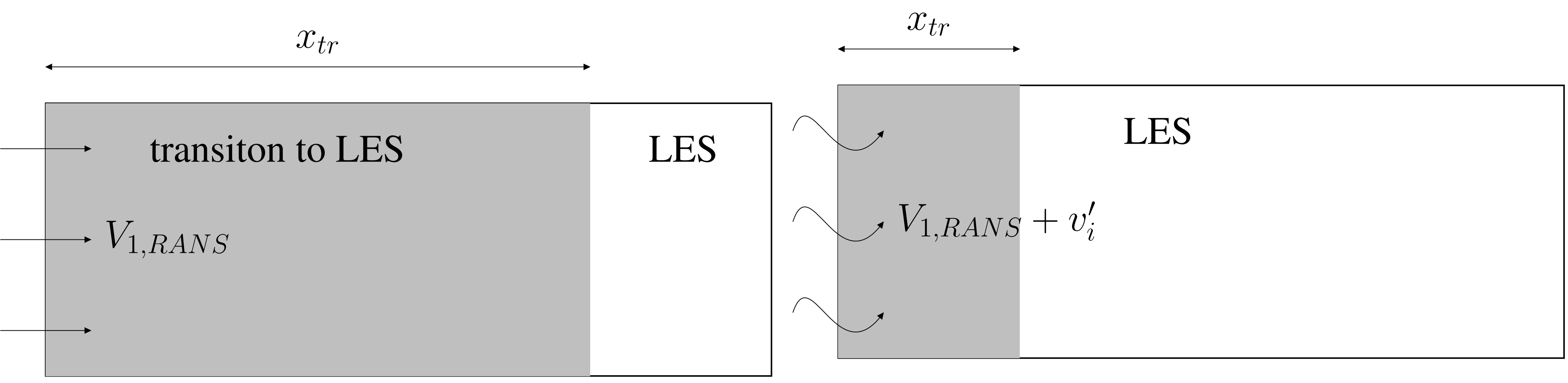

No inlet fluctuations, large x_{tr}.   Realistic, synthetic inlet fluctuations, small x_{tr}.

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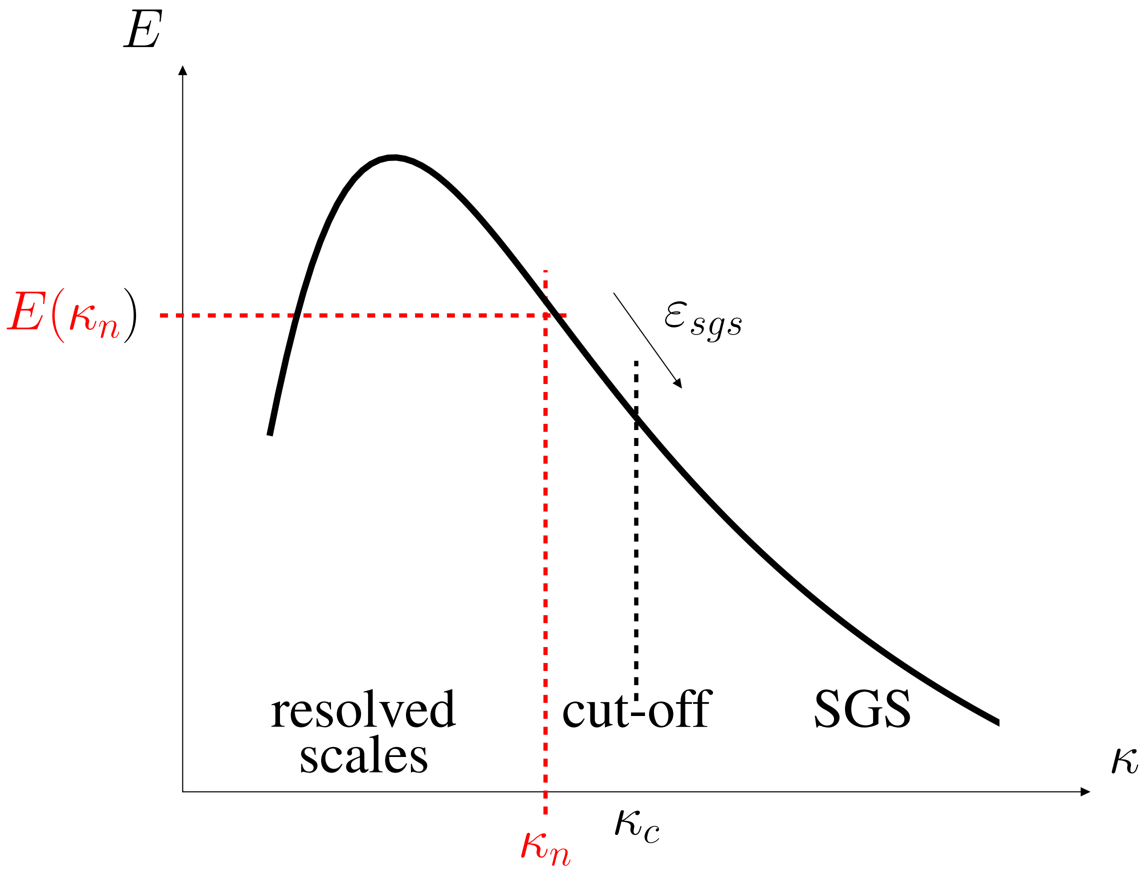

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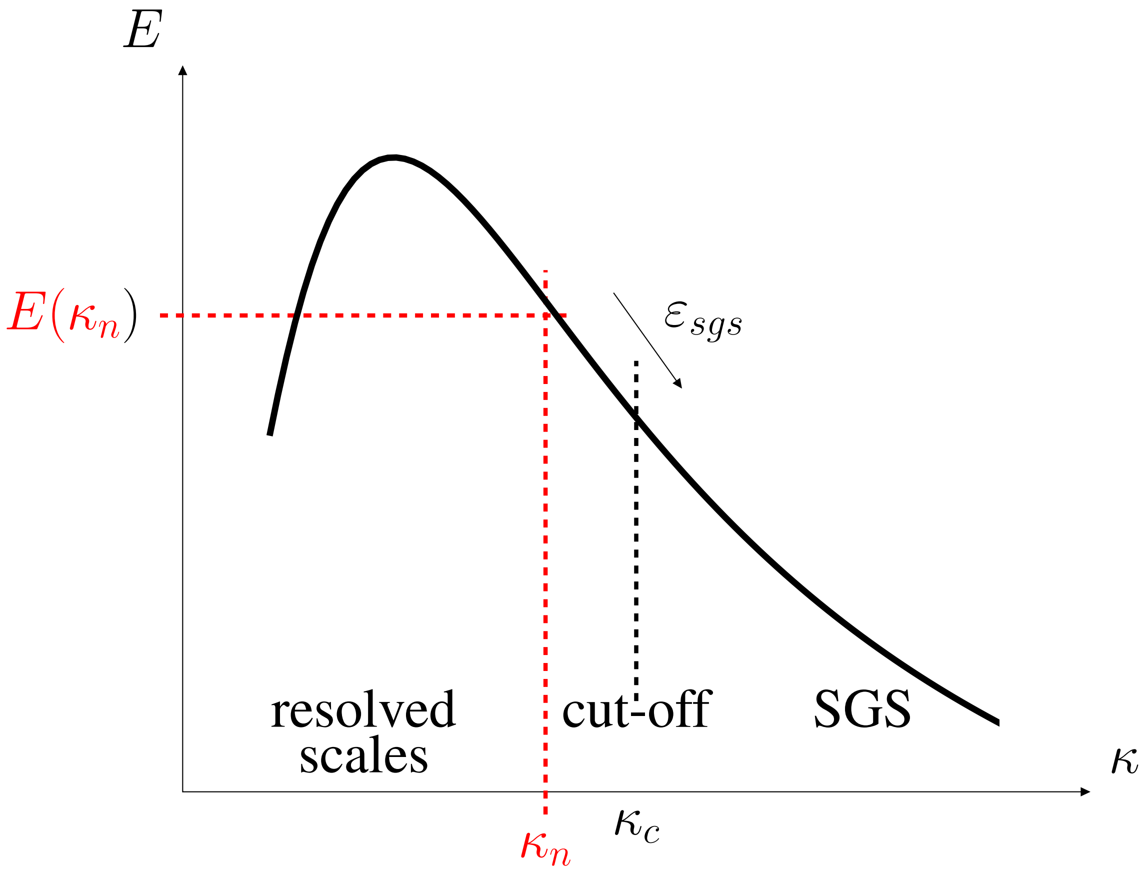

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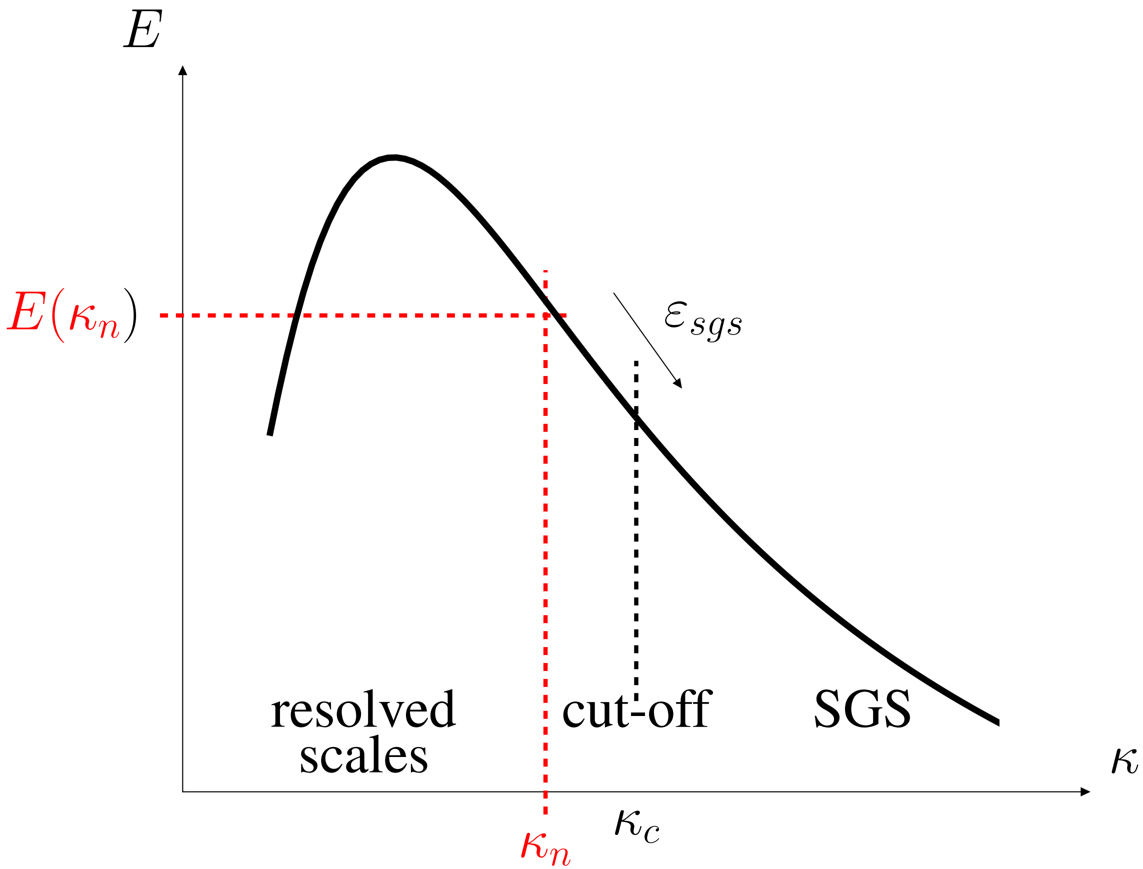

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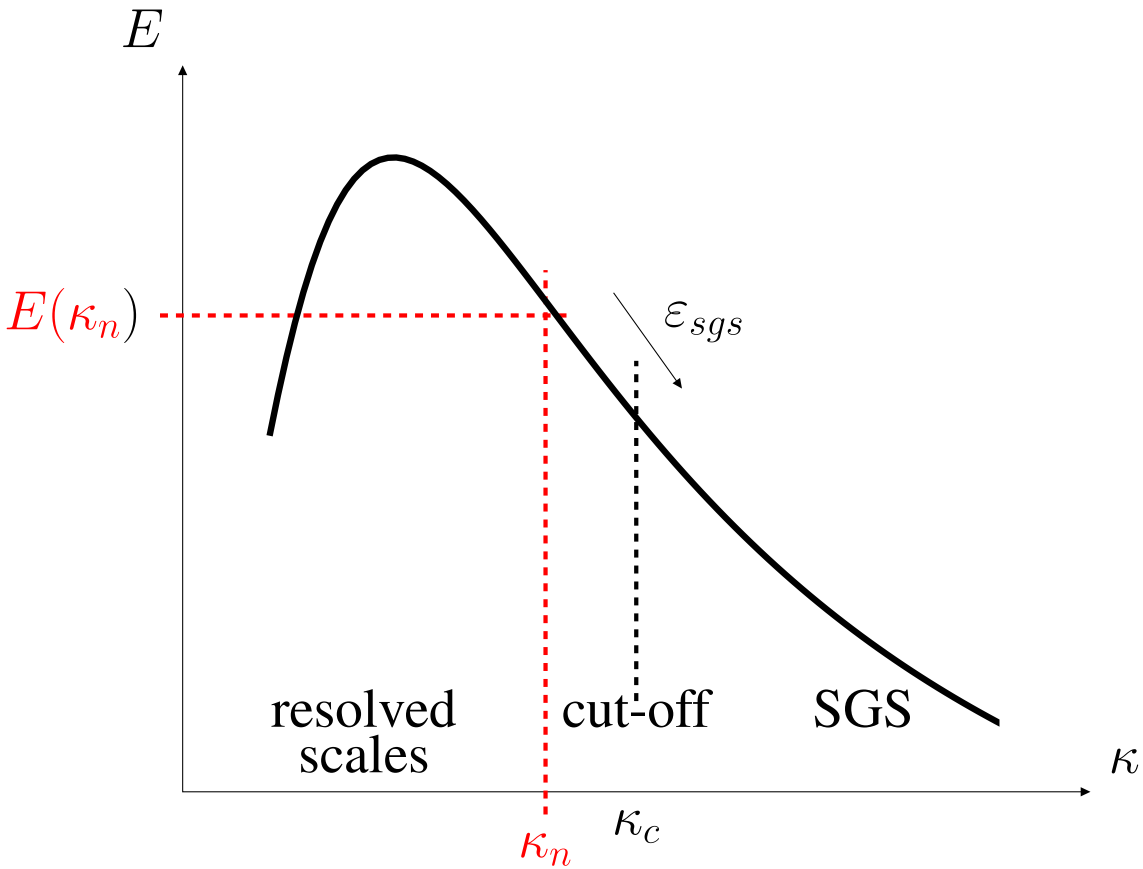

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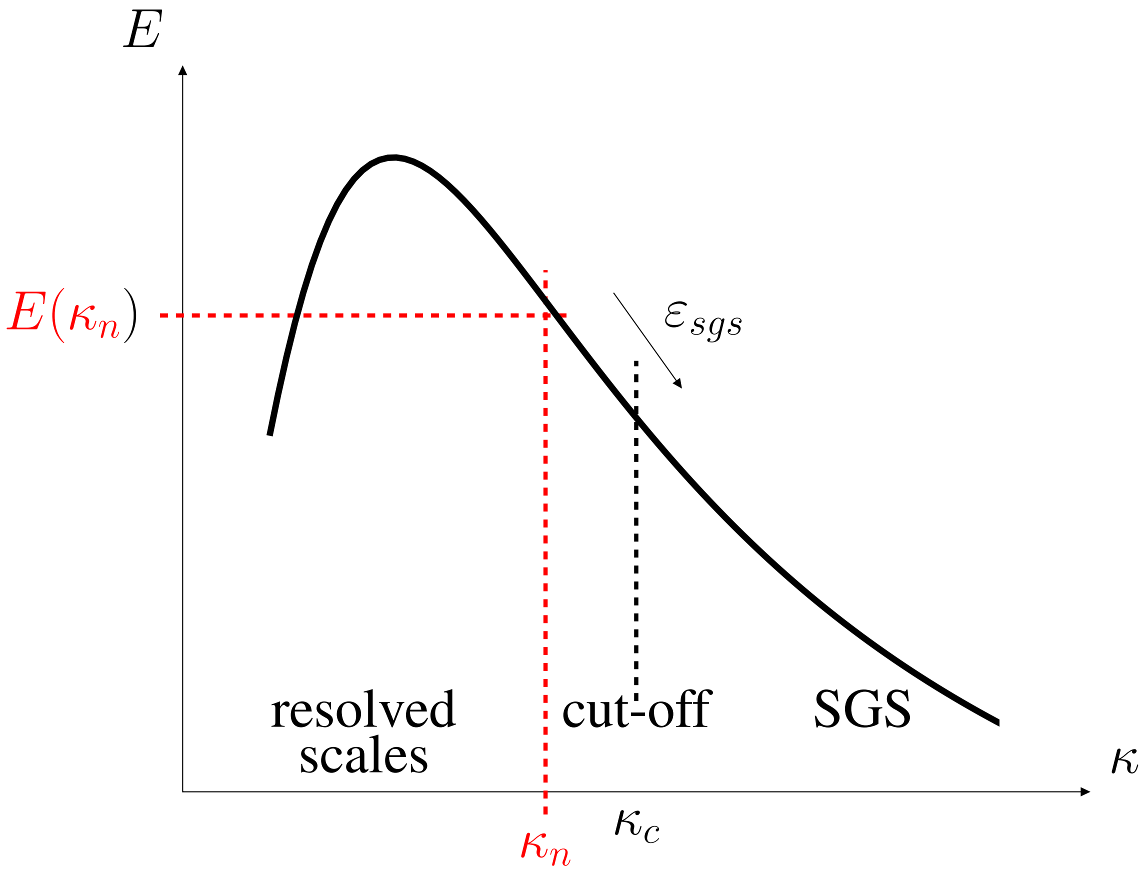

Spectrum of velocity fluctuations.

▶Usually we generate energy spectra from turbulent fluctuations. ▶Here we prescribe a spectrum and generate turbulent fluctuations. ▶$-5/3$ spectrum: ▶this gives the amplitude $\hat{u}^n$ for wavenumber κ_n

¶See Section 27.2, Random angles

$$\mathbf{v}'(\mathbf{x}) = 2\sum_{n=1}^{N} \hat{u}^n \cos(\boldsymbol{\kappa}^n \cdot \mathbf{x} + \psi^n)\boldsymbol{\sigma}^n, \qquad \text{Fourier serie}$$

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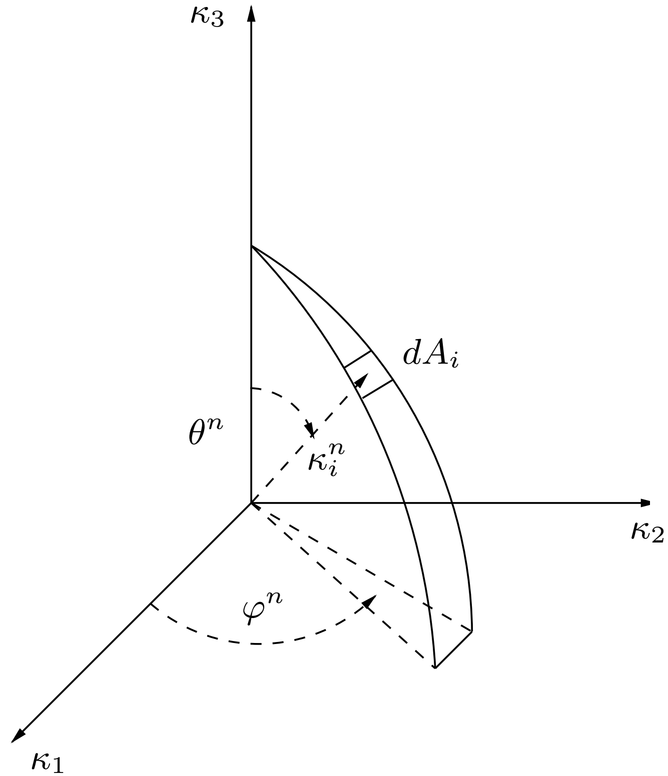
Probability distributions of the random variables. α^n is the angle for $\boldsymbol{\sigma}^n$.



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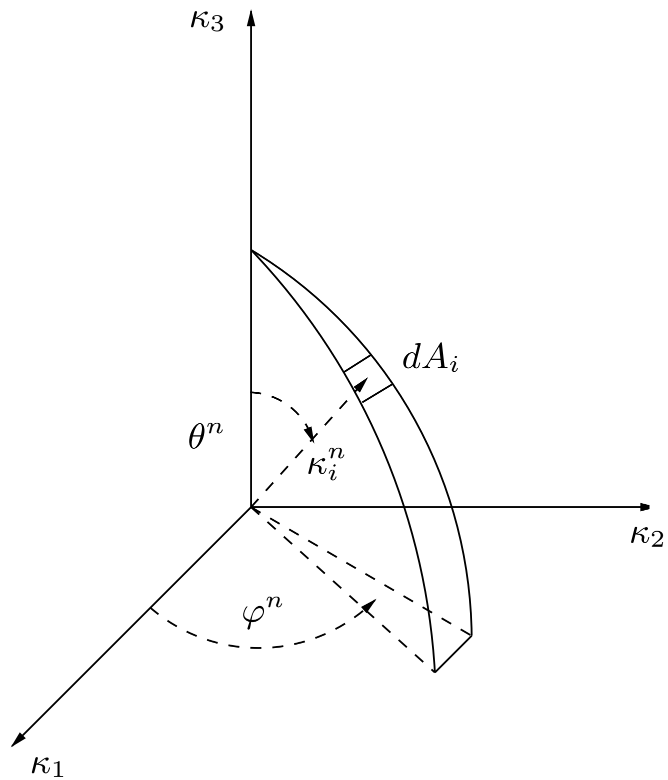

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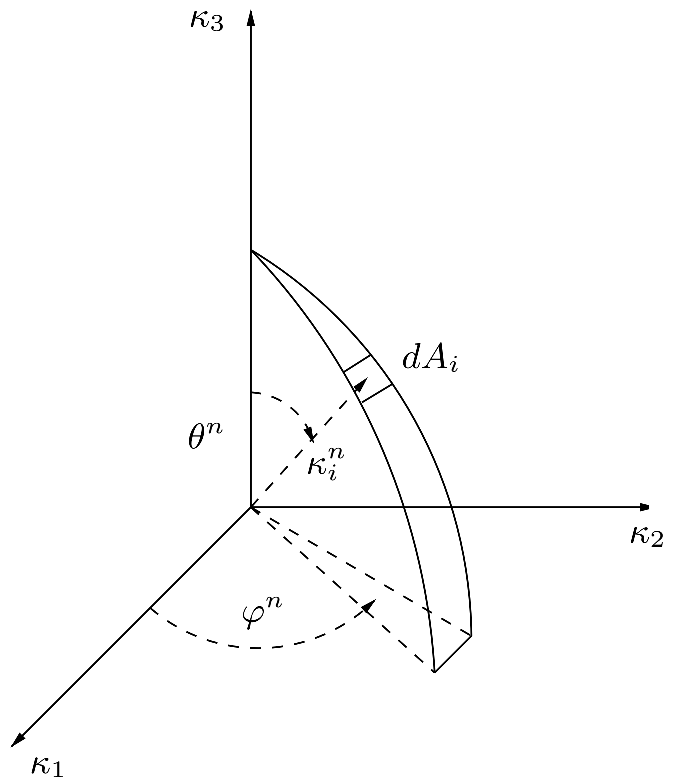

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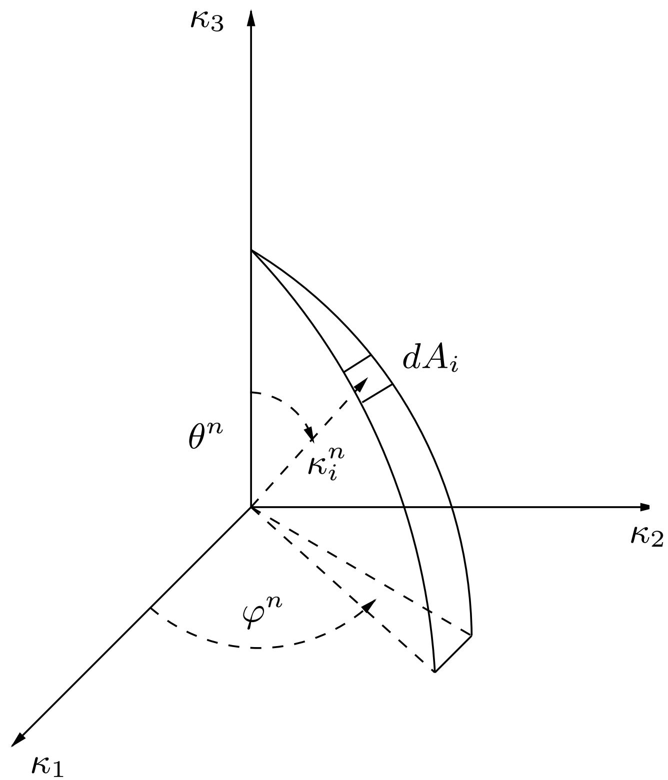

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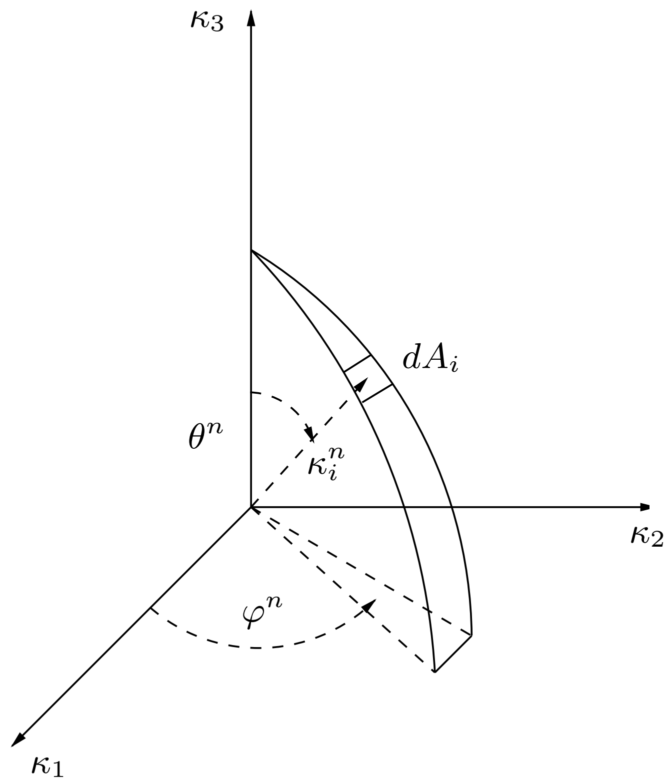

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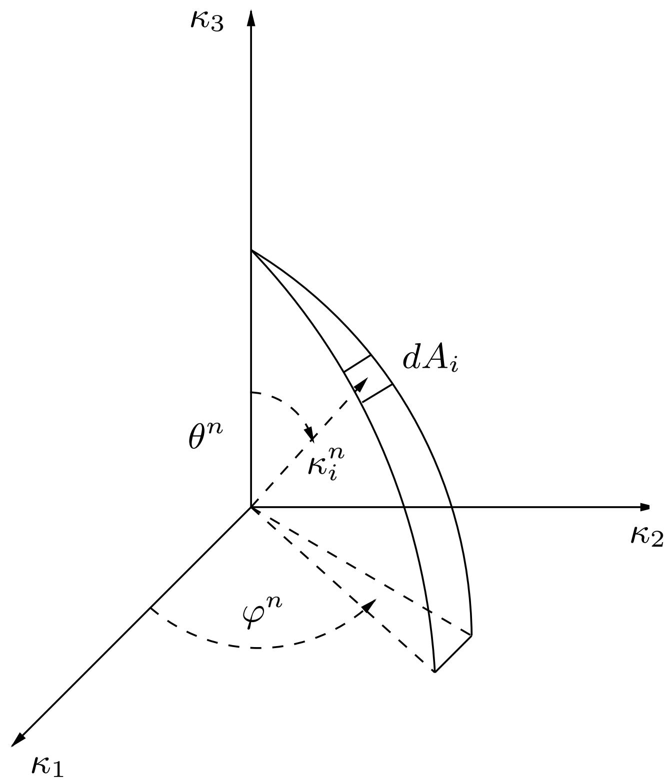

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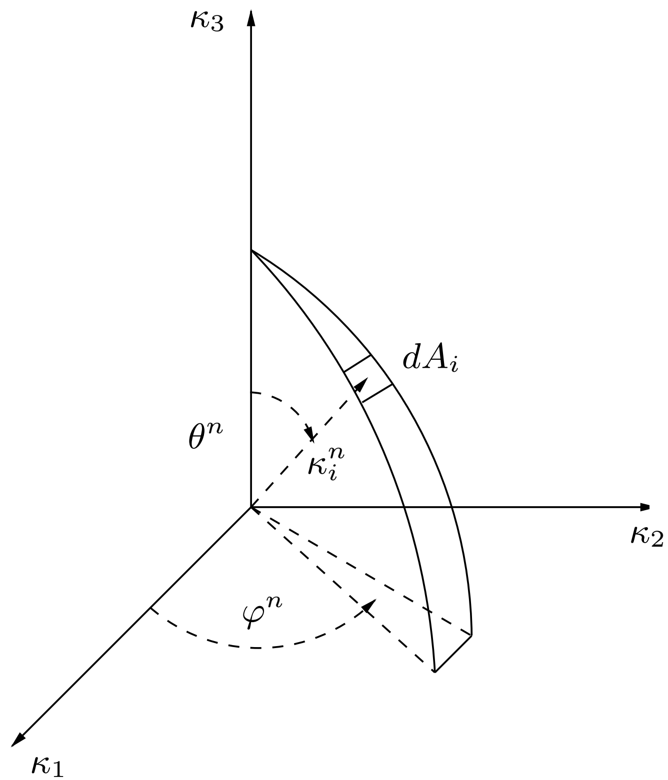

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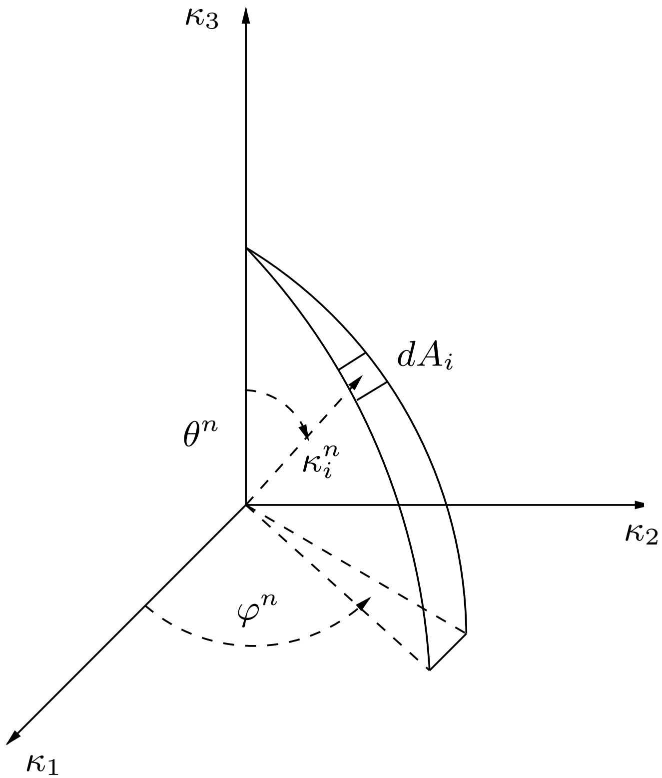

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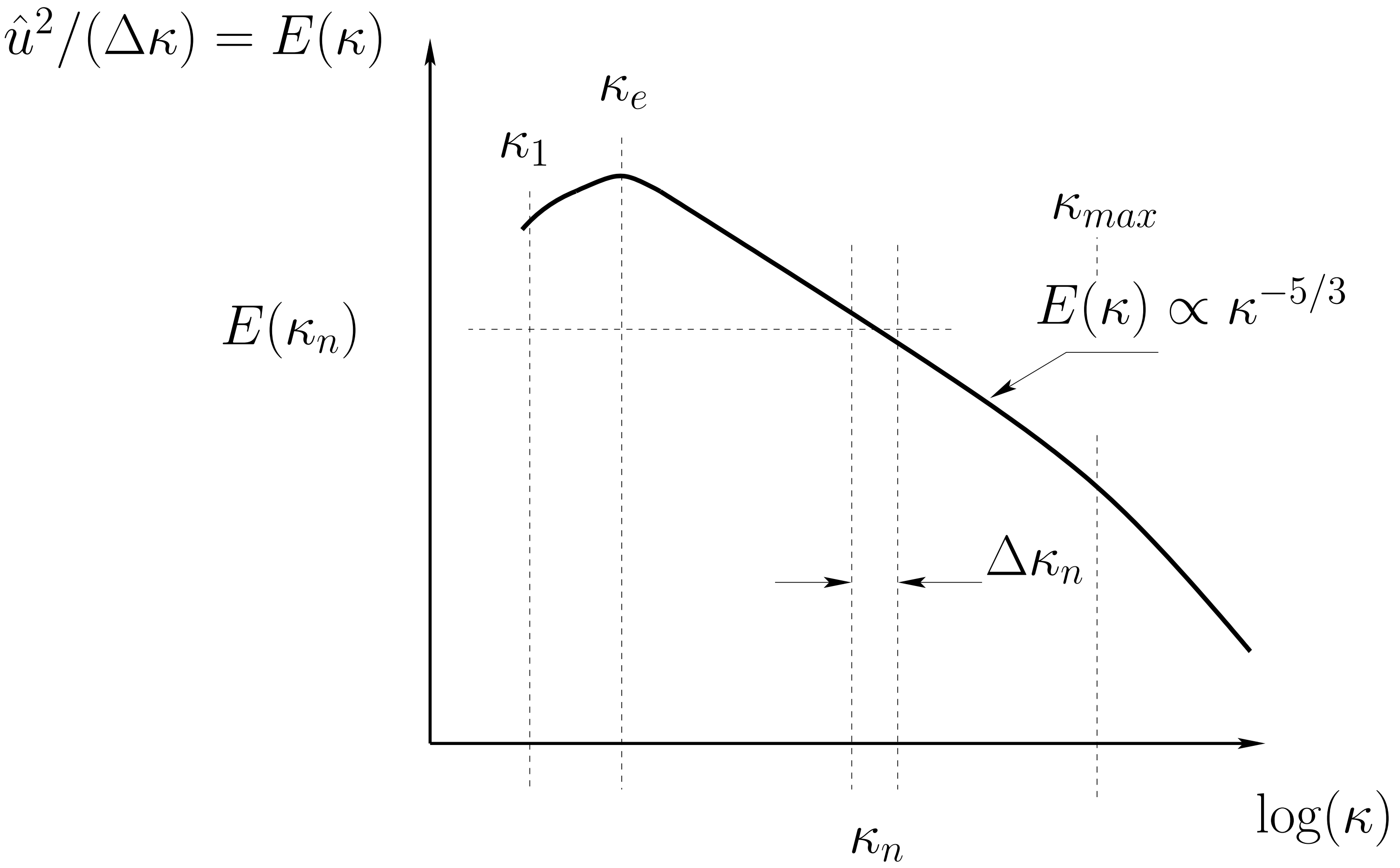
- Amplitude $\hat{u}^n$ related to energy spectrum: $\hat{u}^n = (E(\kappa)\Delta\kappa)^{1/2}$



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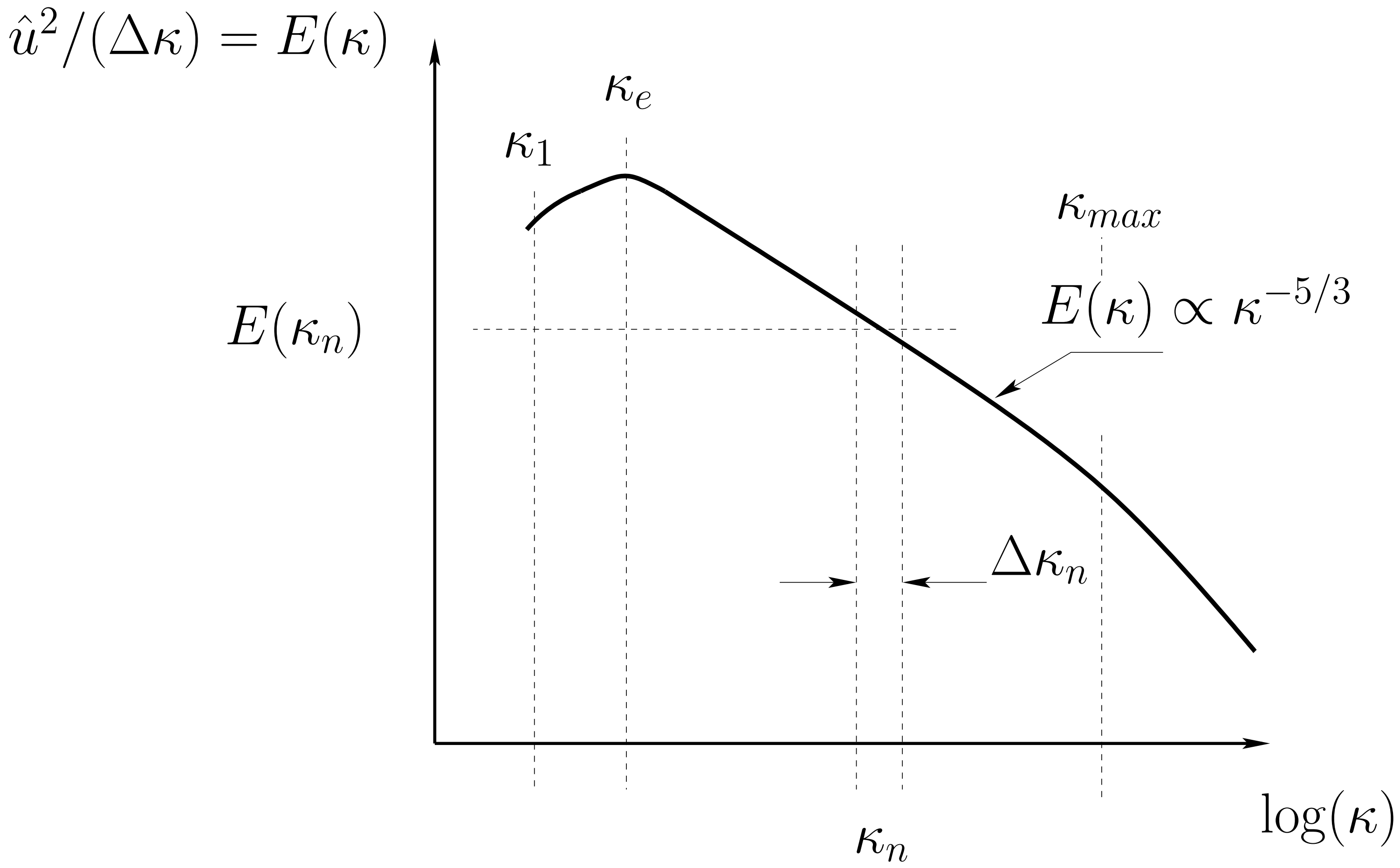

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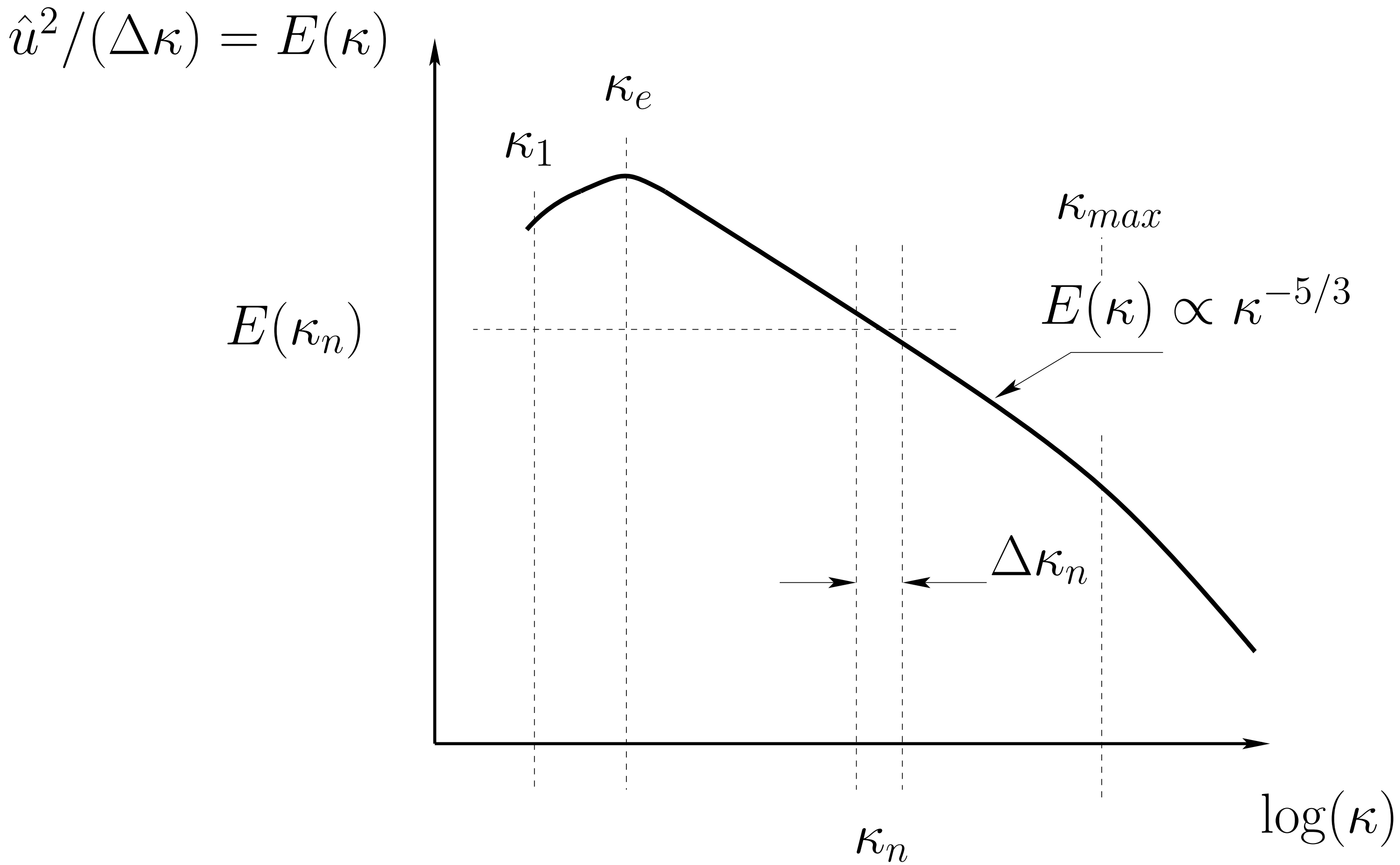

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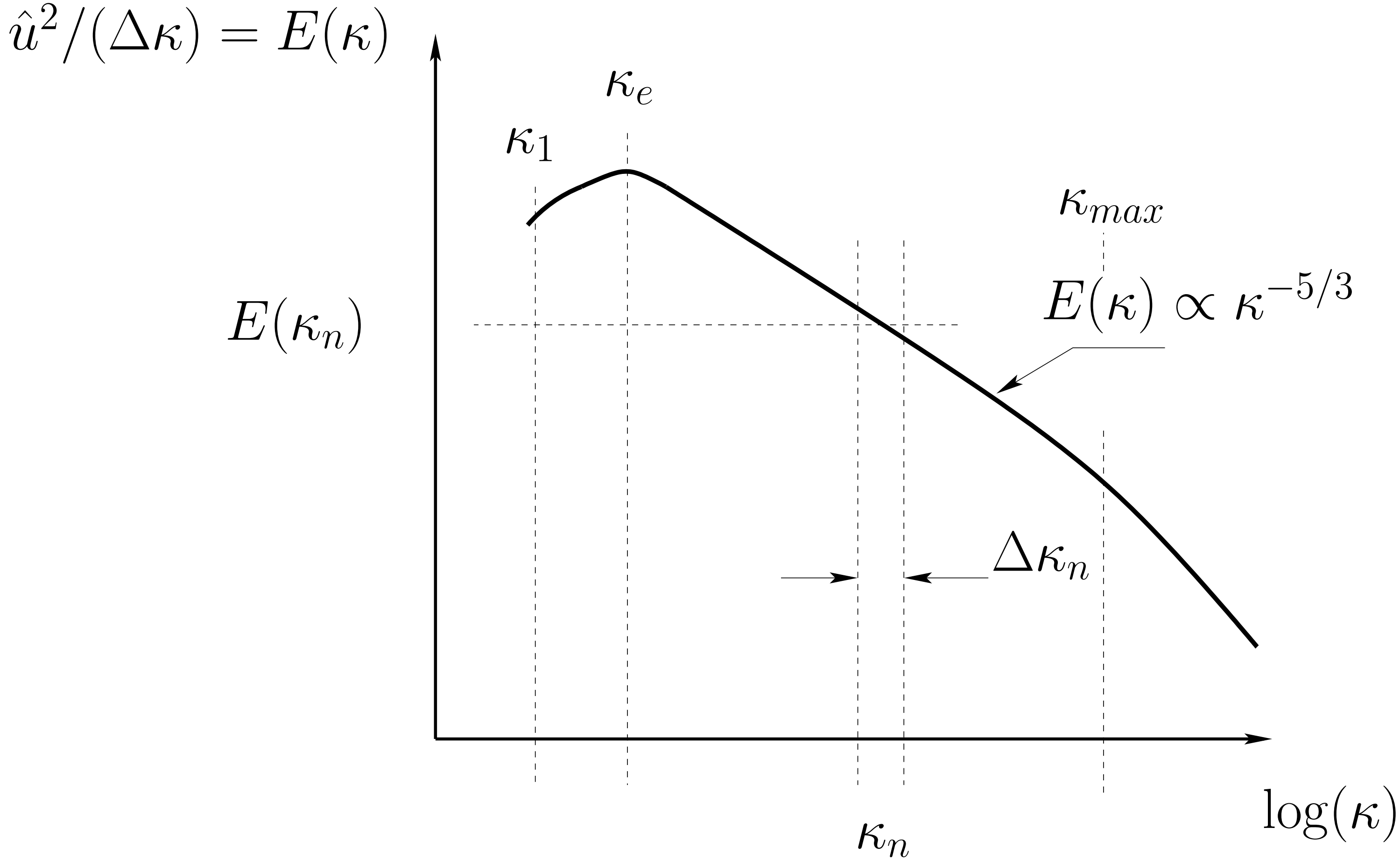
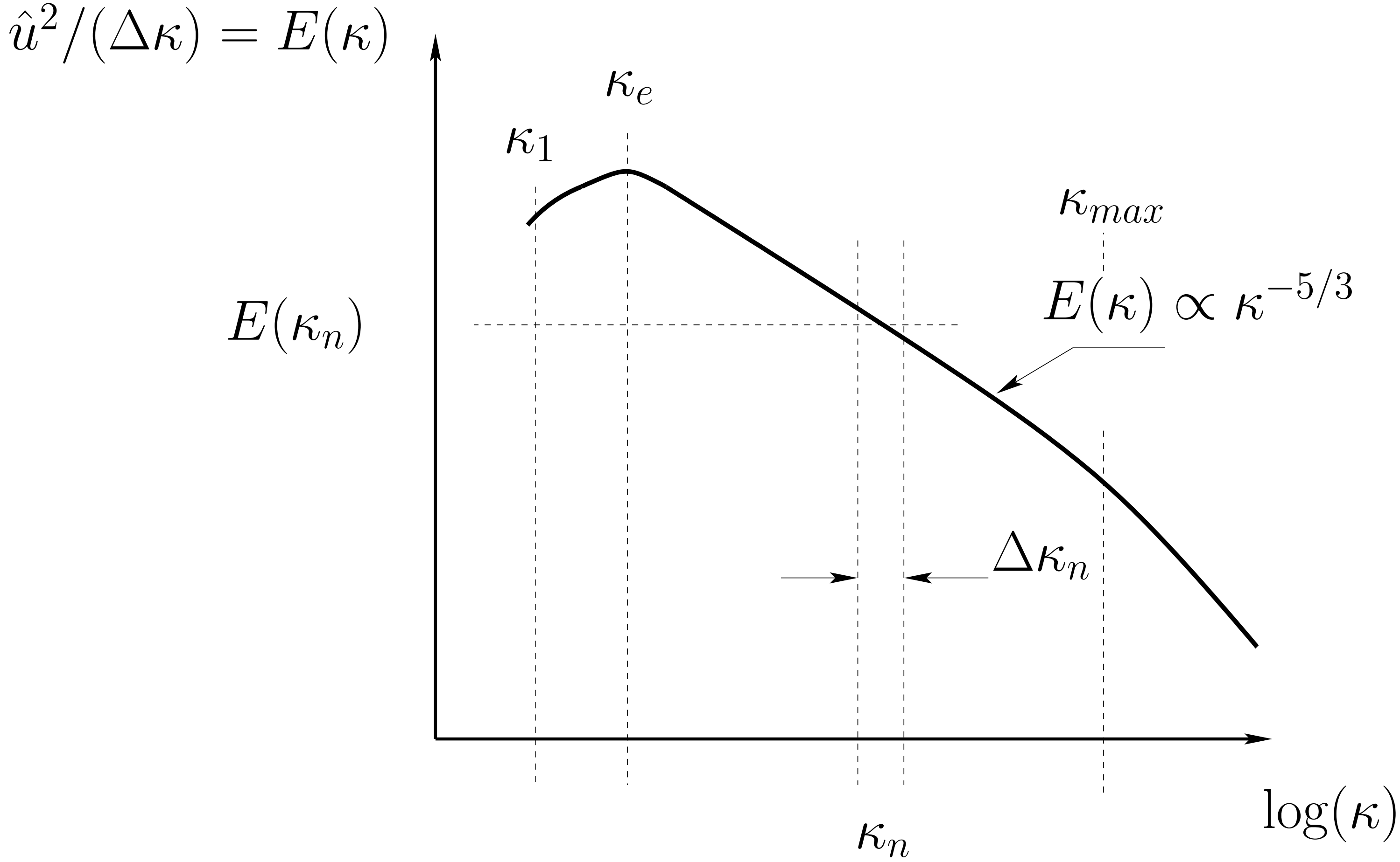


$\hat{u}^2/(\Delta\kappa) = E(\kappa)$
κ_e
κ_1
κ_{max}
$E(\kappa) \propto \kappa^{-5/3}$
$E(\kappa_n)$
$\Delta\kappa_n$
$\log(\kappa)$
κ_n



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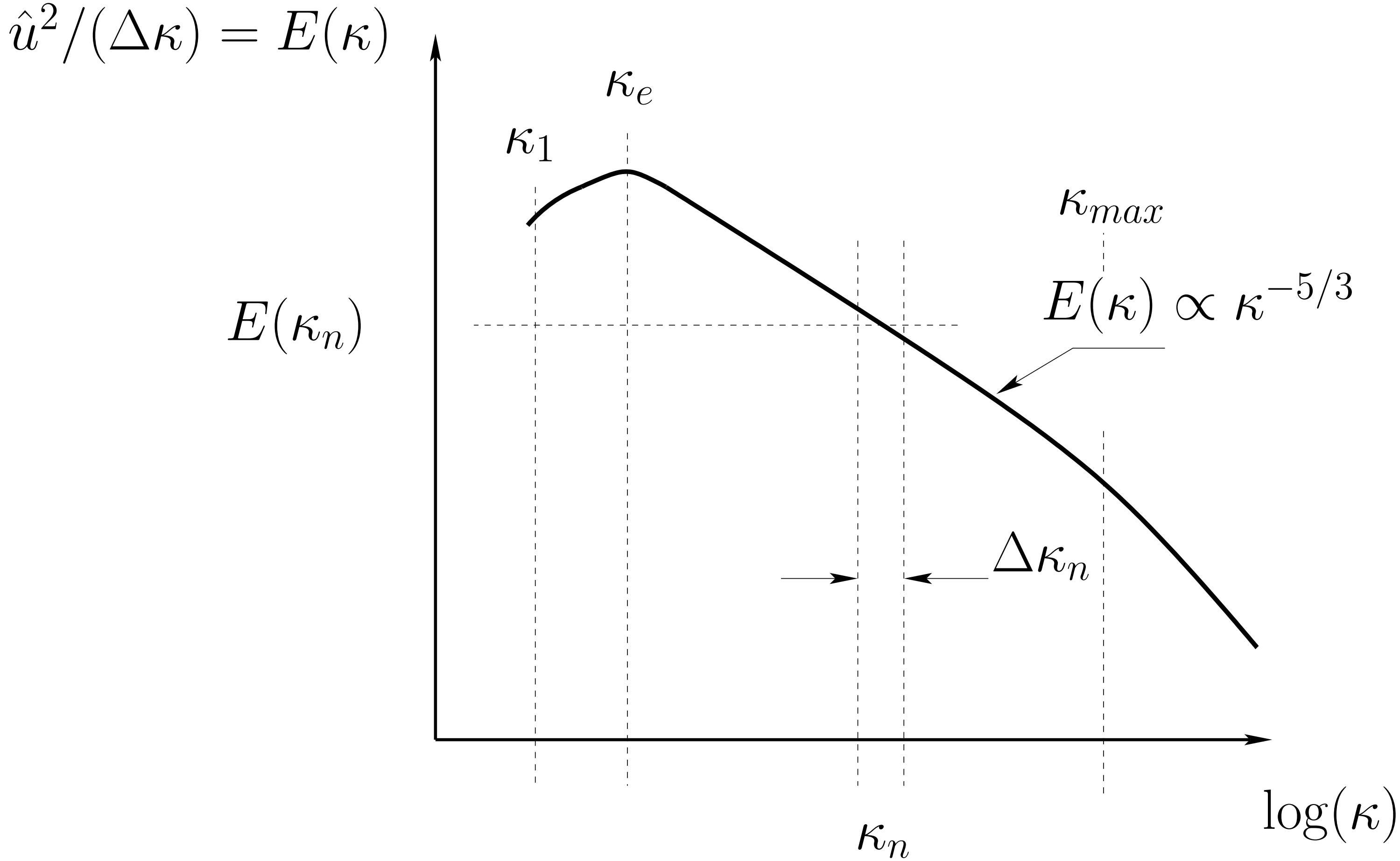
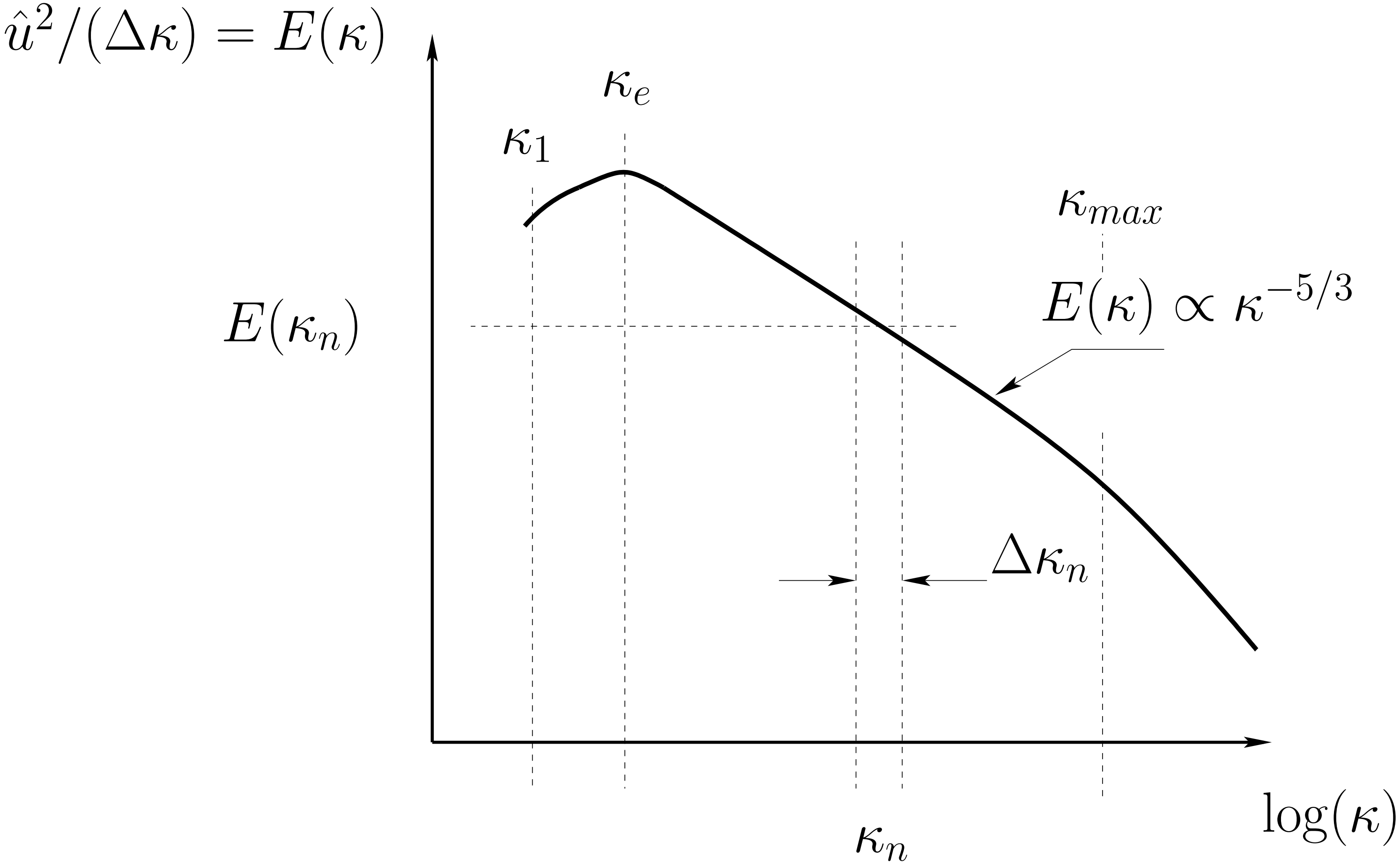


- Highest wave number: $\kappa_{max} = 2\pi/2\Delta$ from the cell size, $\Delta = \min(\Delta x_2)$



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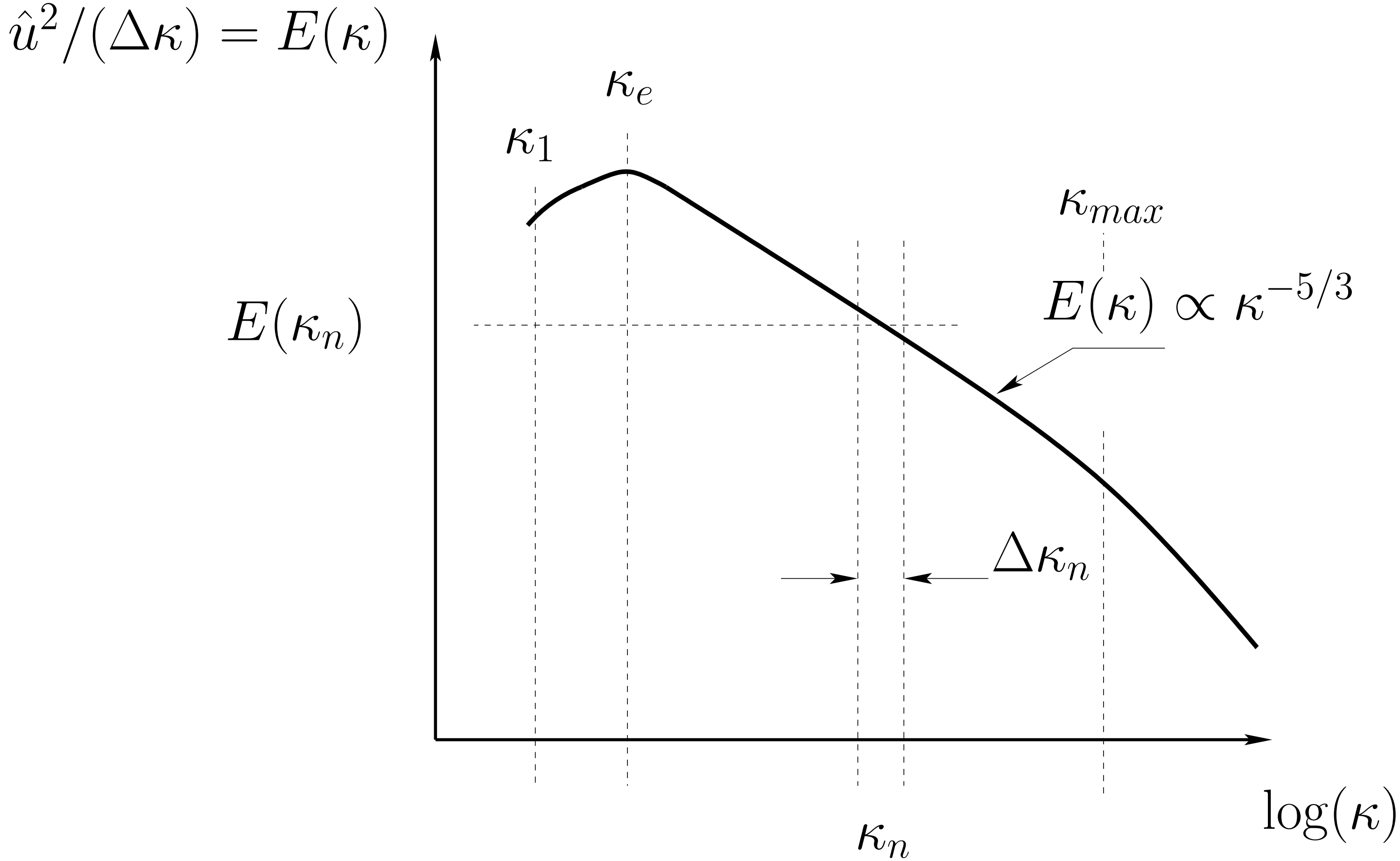

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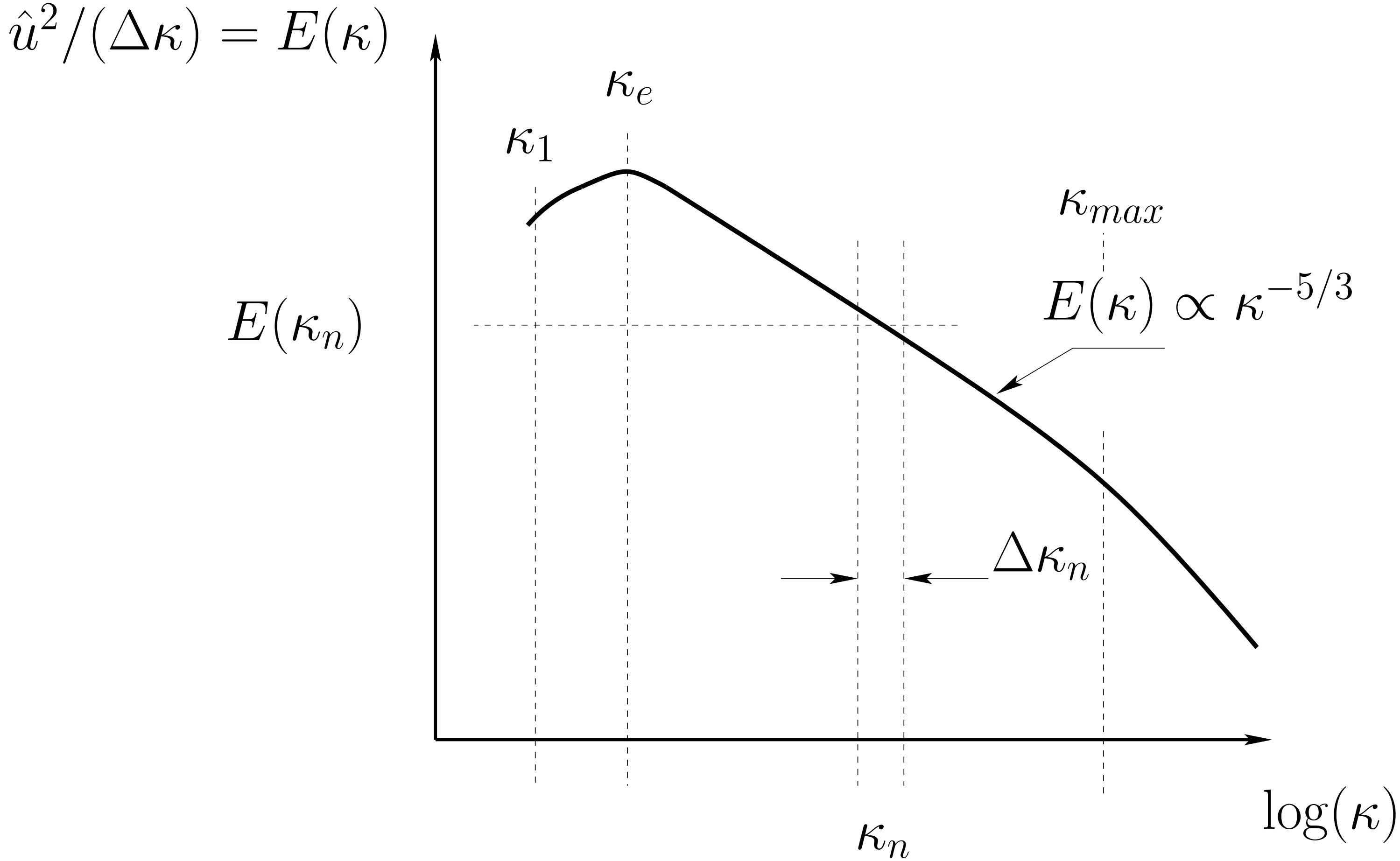

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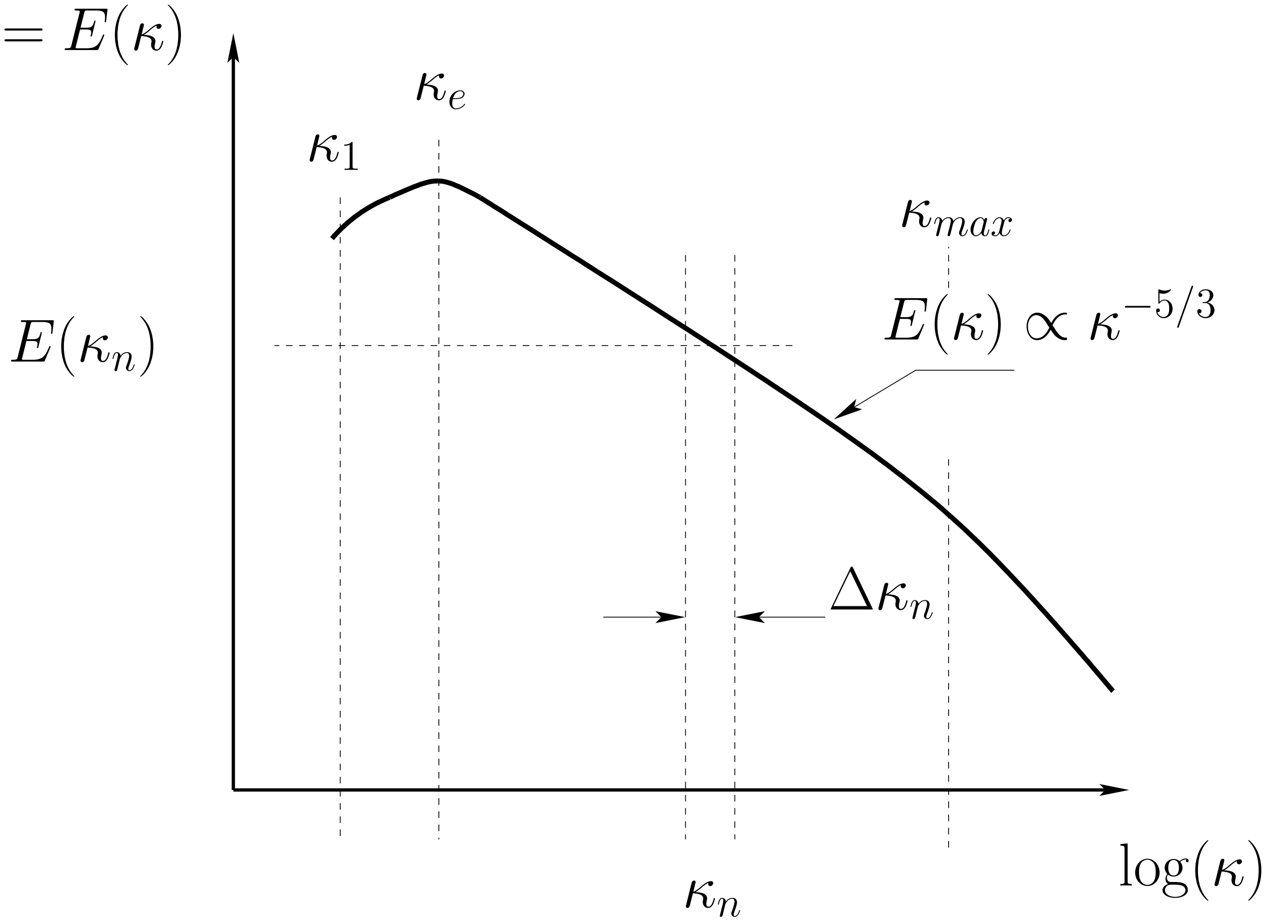
► Most energetic wave number: $\kappa_e \propto 1/L_t$: integral turbulent length scale. ► $\kappa_e = 0.75/L_t$



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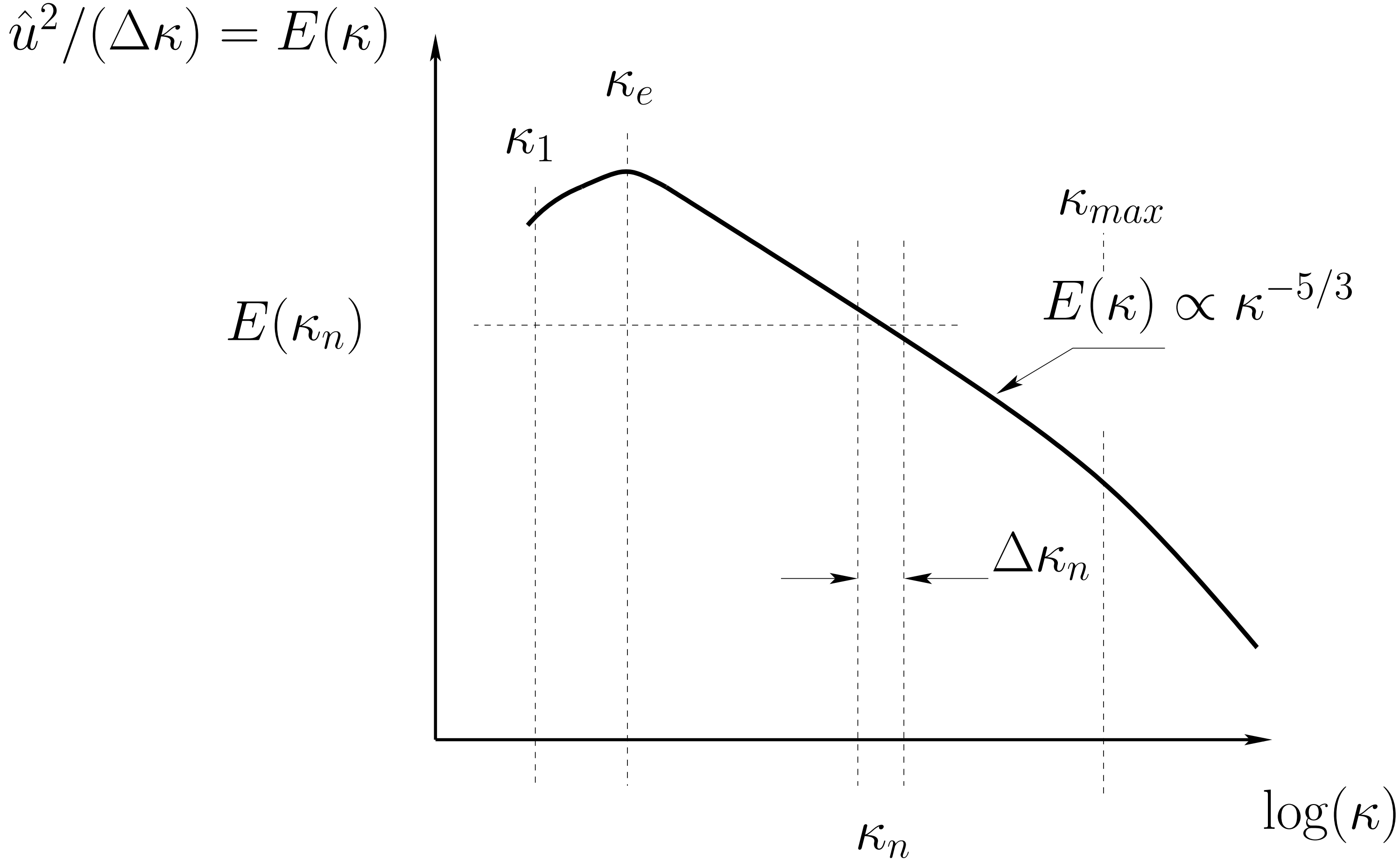

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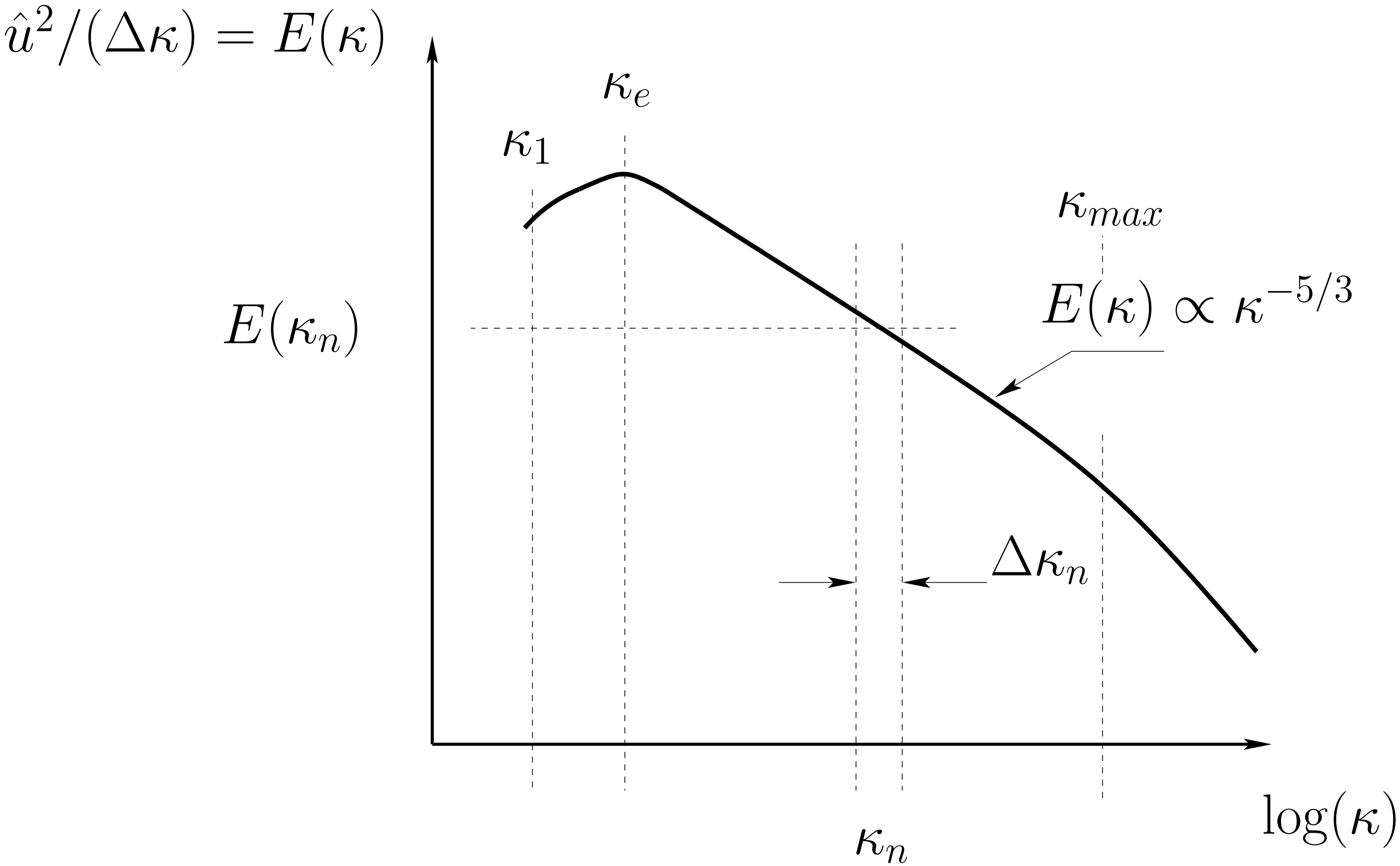

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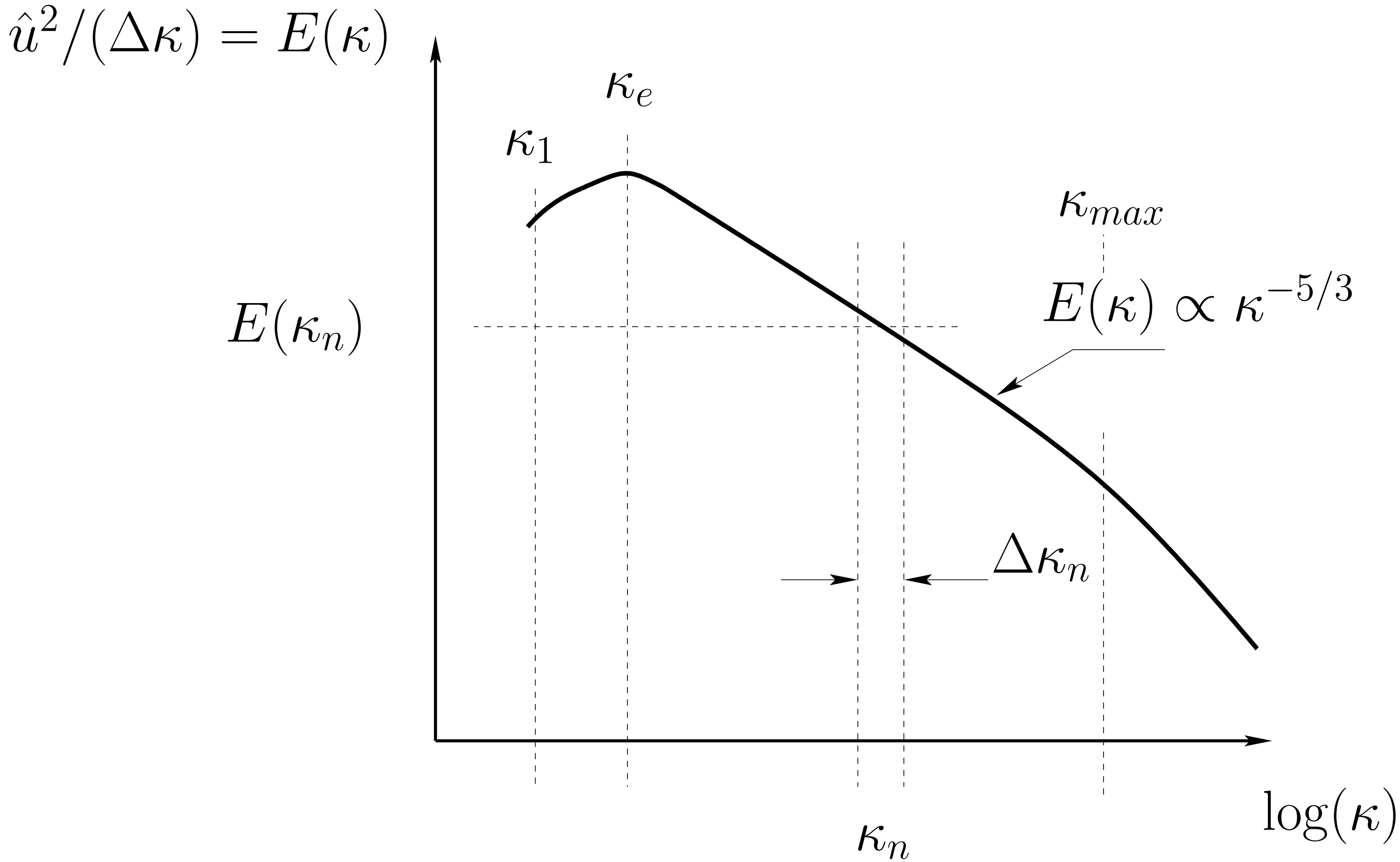

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- ▶Now the fluctuations, $\mathbf{v}'(\mathbf{x})$, can be computed

$$v_1' = 2\sum_{n=1}^{N} \hat{u}^n \cos(\beta^n)\sigma_1$$

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► Synthetic turbulent isotropic fluctuations at the inlet plane for all time steps.

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- ►BUT:

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►The synthetic fluctuations are not correlated in time. An asymmetric time filter is used

$$(\mathcal{V}_1')^m = a(\mathcal{V}_1')^{m-1} + b(v_1')^m$$

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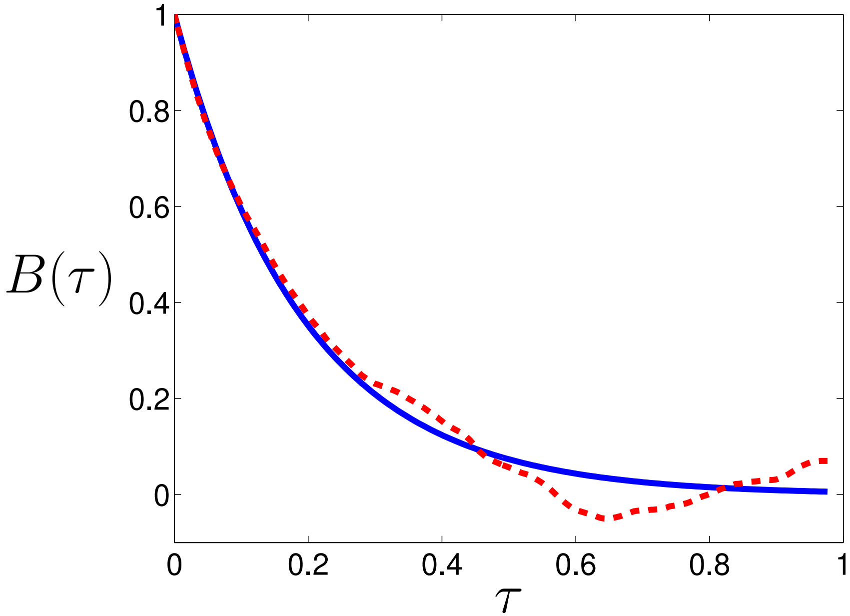

Auto correlation. —: Eq. 41.1; - - -: $B(\tau) = \langle \mathcal{V}_1'(t)\mathcal{V}_1'(t-\tau)\rangle_t$.

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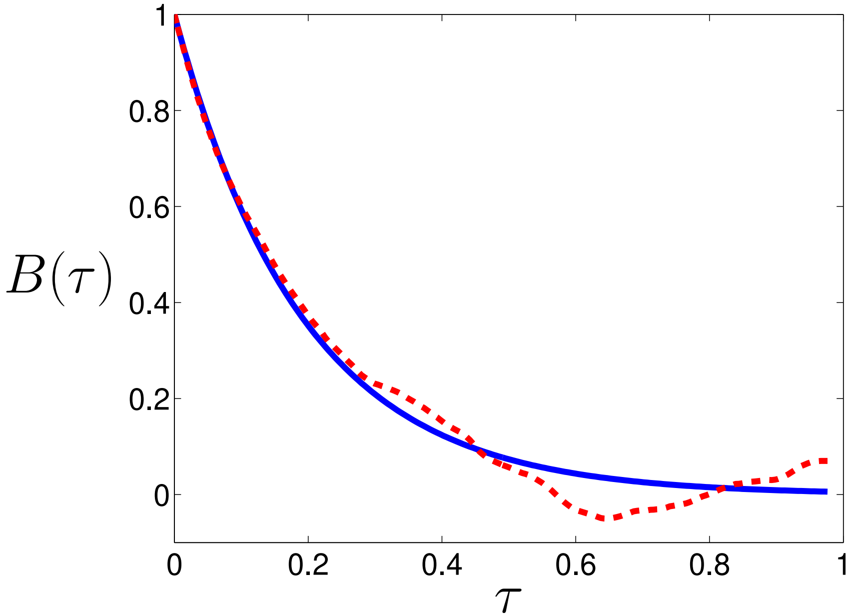

Auto correlation. —: Eq. 41.1; - - -: $B(\tau) = \langle \mathcal{V}_1'(t)\mathcal{V}_1'(t-\tau)\rangle_t$.

▶Finally, the turbulent synthetic fluctuations are superimposed to the inlet mean velocity.

¶See Section 23.2.1, The Interface Condition

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▶Embedded LES and inlet b.c. for k and ε using PANS

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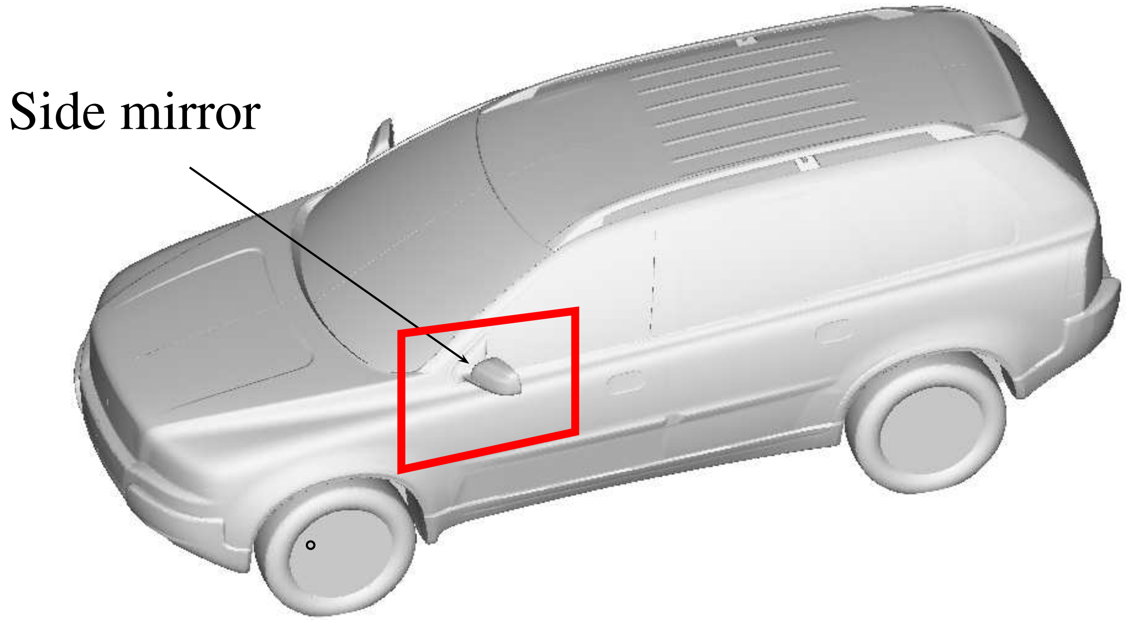

Vehicle geometry (from [116]). Colored rectangle shows embedded LES region

¶See Section 23.2.1, The Interface Condition

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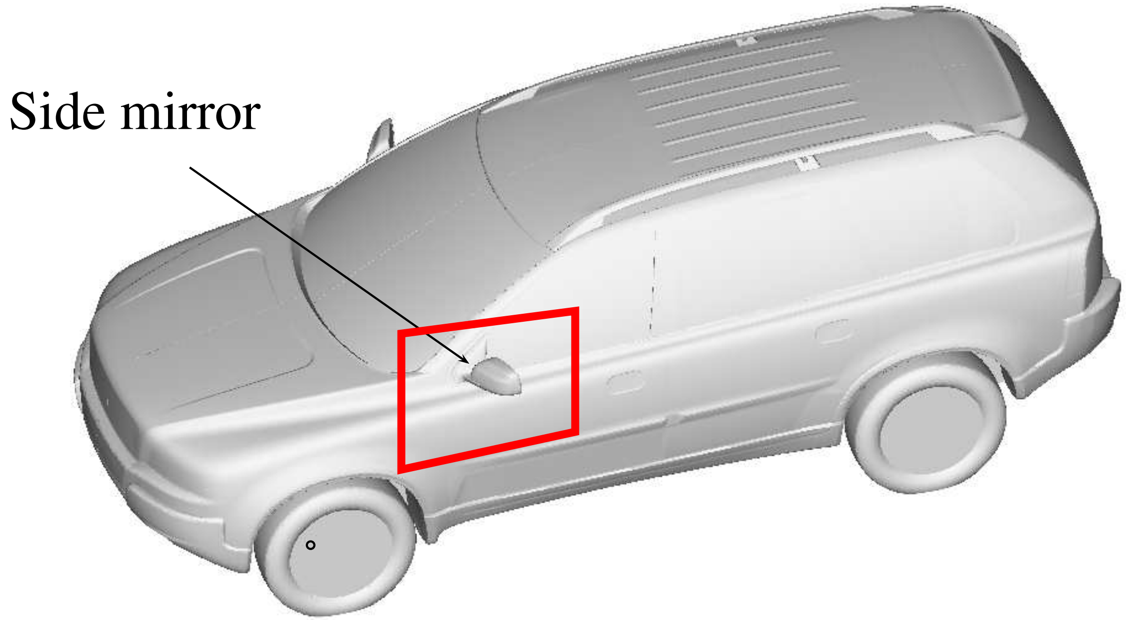

Vehicle geometry (from [116]). Colored rectangle shows embedded LES region

►An LES region (e.g. the side mirror, see figure above) is embedded in a steady RANS simulation.

¶See Section 23.2.1, The Interface Condition

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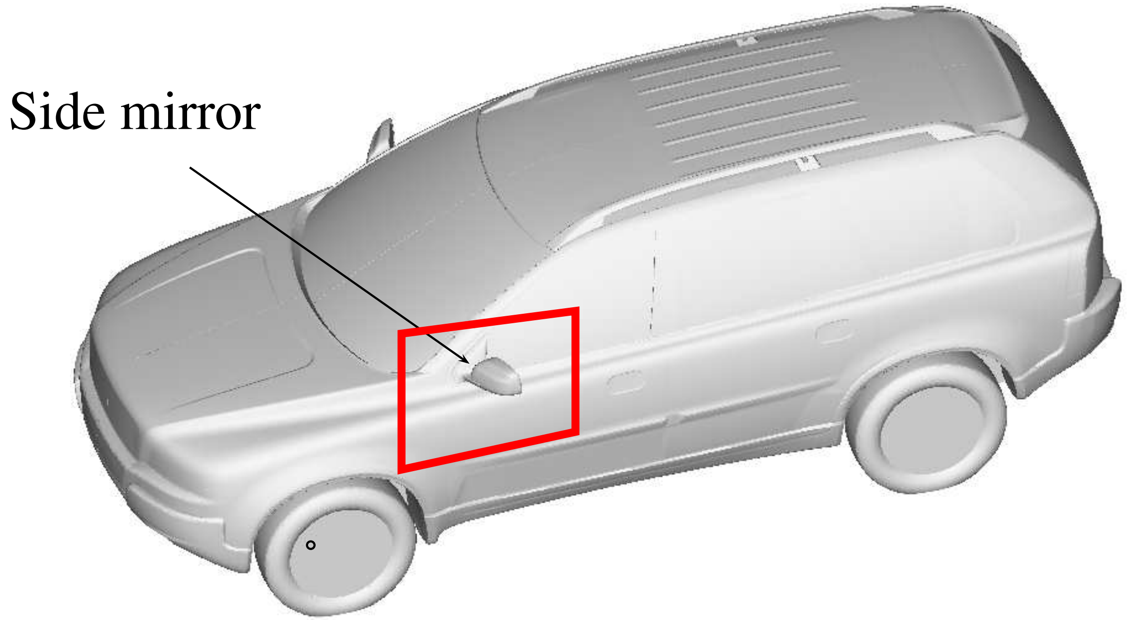

Vehicle geometry (from [116]). Colored rectangle shows embedded LES region

▶An LES region (e.g. the side mirror, see figure above) is embedded in a steady RANS simulation.

▶LES is used around the mirror in order to compute aeroacoustic sources (wind noise)

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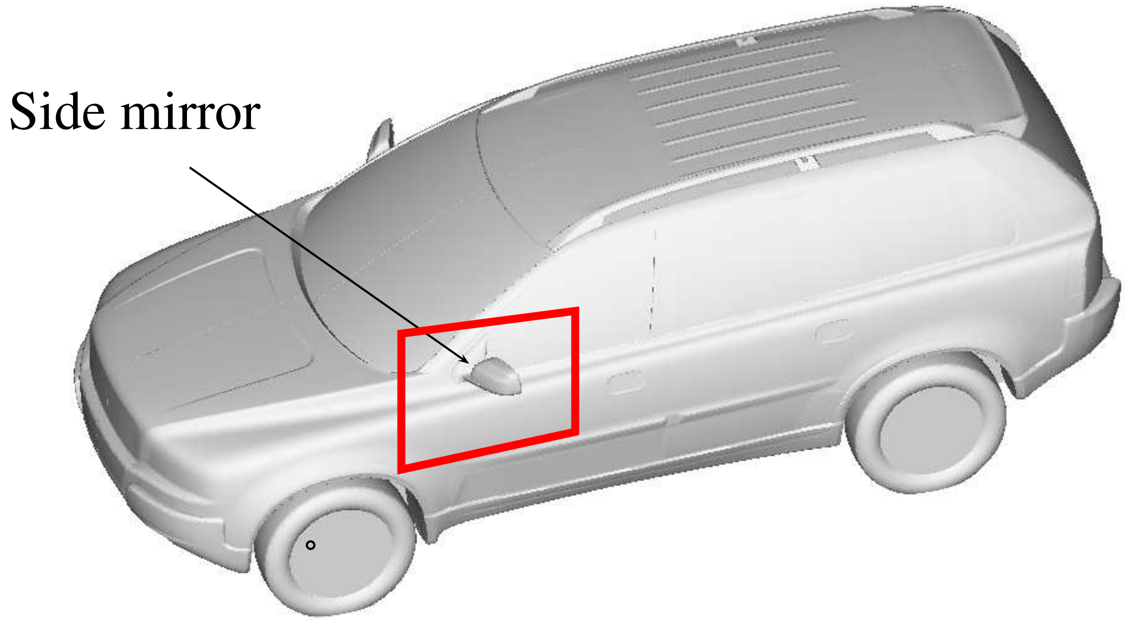

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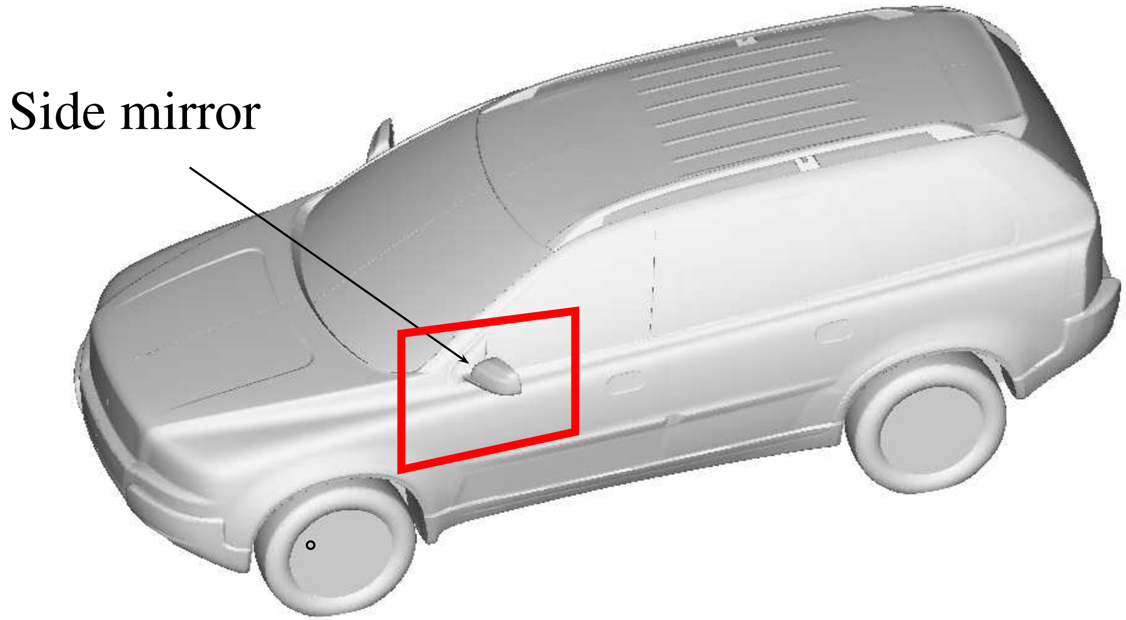

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►An LES region (e.g. the side mirror, see figure above) is embedded in a steady RANS simulation.

►LES is used around the mirror in order to compute aeroacoustic sources (wind noise)

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►Mean velocity, k and ε at the LES inlet region are taken from the RANS simulation

▶Summary

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- The source terms in the k equation (see Eq. 40.3) quickly reduces k from RANS values to LES values
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- For more detail, see Section 5 in [183]