MTF271 Turbulence Modelling
Assignment 1, Part II: Machine Learning

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The Assignment, slides and recorded lecture can be found at

https://www.tfd.chalmers.se/~lada/comp_turb_model/assignment_1/index.html

Machine Learning

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- In my case, input and output are numerical values. Regression methods should then be used [2]; I use support vector regression (SVR) methods available in Python.

Training: I need a target database

$$
\begin{aligned}
\frac{\partial v_i}{\partial x_i} &= 0 \\
\frac{\partial v_i}{\partial t} + \frac{\partial}{\partial x_j}(v_i v_j) &= -\frac{\partial p}{\partial x_i} + \frac{\partial^2 v_i}{\partial x_j \partial x_j}
\end{aligned}
$$

- Fully-developed Channel flow
- Database can be found here. Reynolds number is 5 200.
- The DNS data are averaged in time, x_1 and x_3.

SVR (Support Vector Regression)

At this site you find the figure below

SVR data predictions

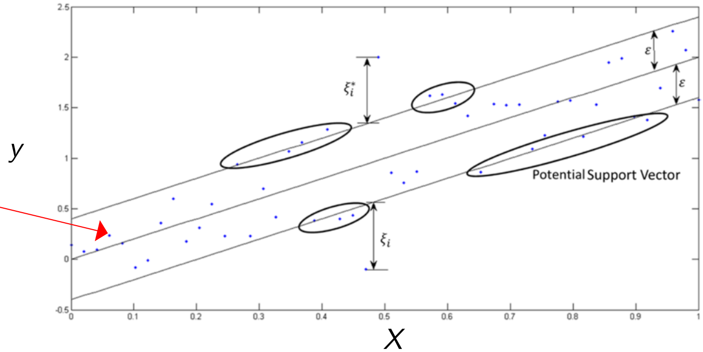

Figure: Hyperplane, ε tube and slack, ξ_i. $\cdot$: predicted (SVR) data point

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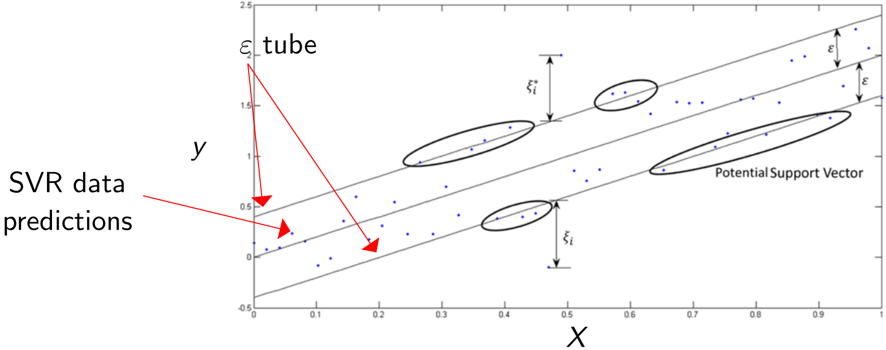

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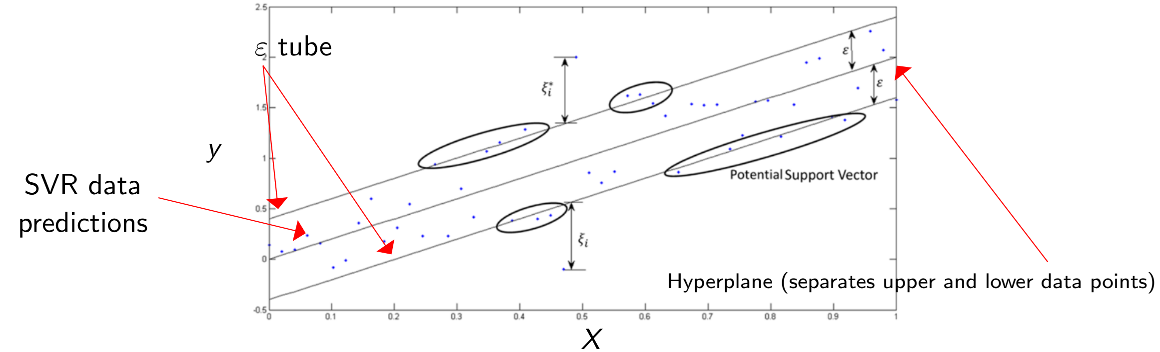

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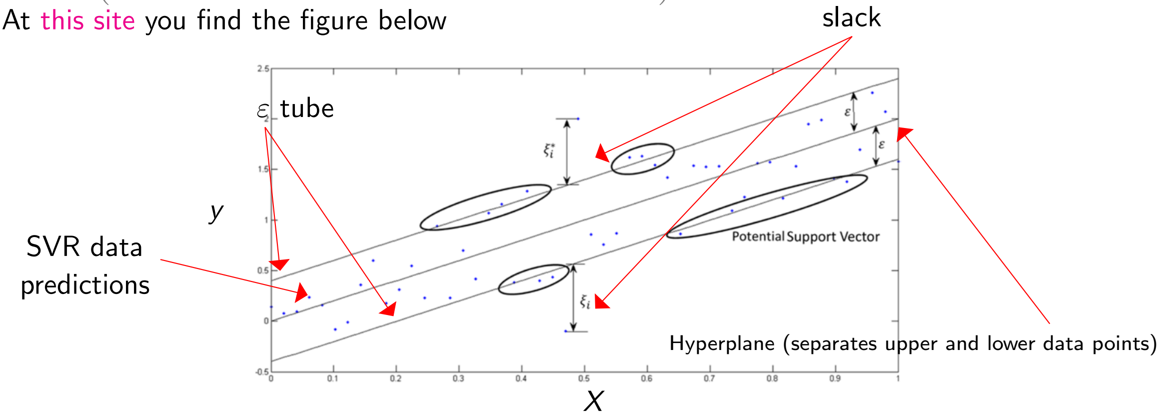

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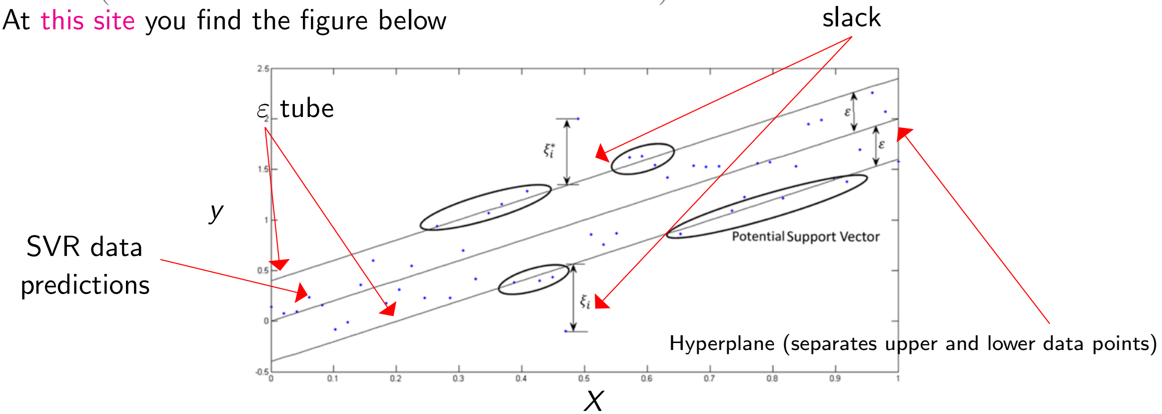

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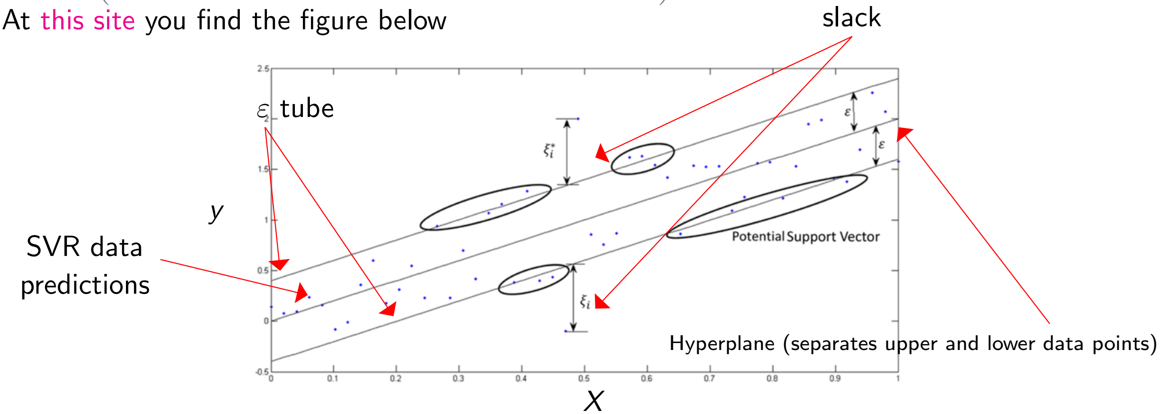

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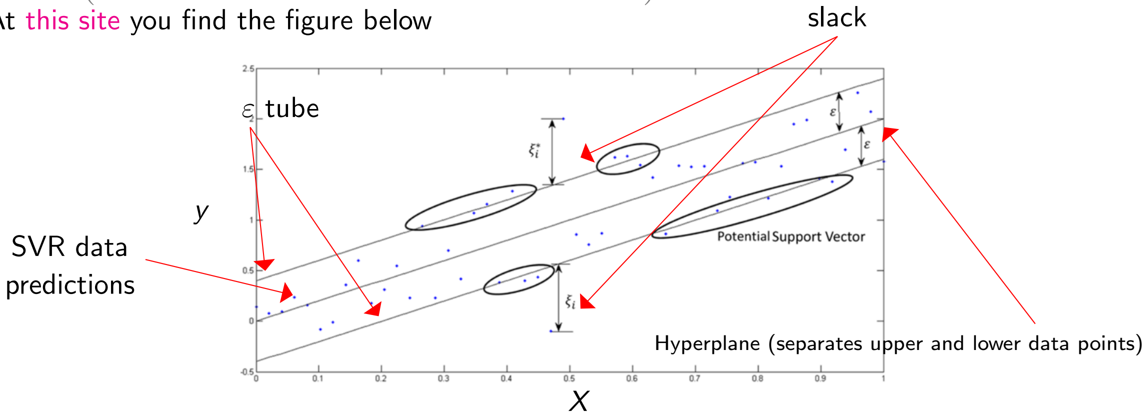

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- How far outside of the ε tube can a predicted point be?

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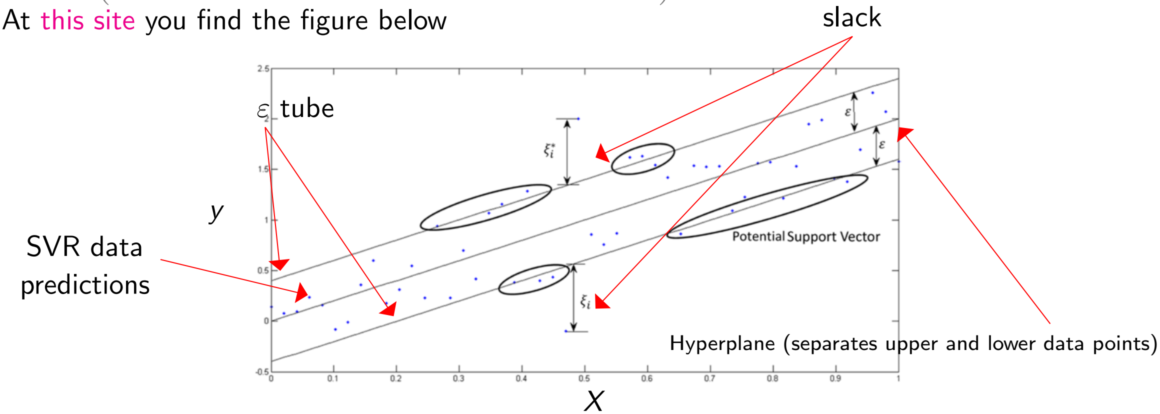
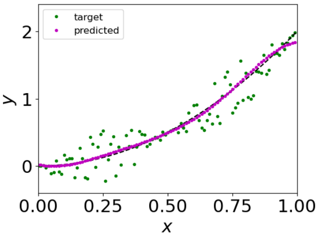


FIGURE: Hyperplane, ε tube and slack, ξ_i. $\cdot$: predicted (SVR) data point

- How close to the Hyperplane should the predicted data points be? Answer: ε
- How far outside of the ε tube can a predicted point be? Answer: ξ (large C, promotes small ξ)

Testing SVR for a line

$$y = 2x^2 + \sin(2\pi) \cdot rand(-0.4, 0.4)$$



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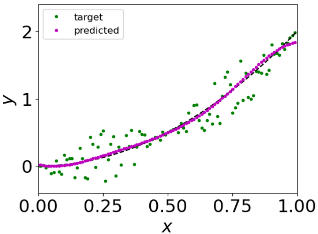

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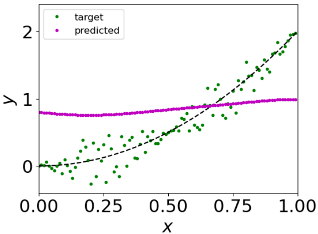

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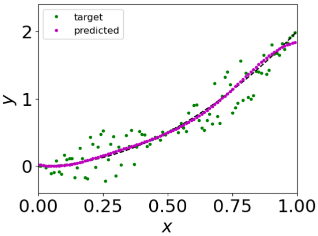

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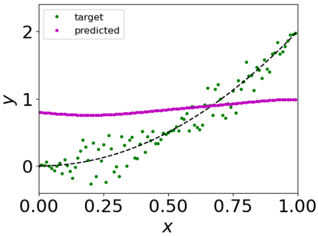

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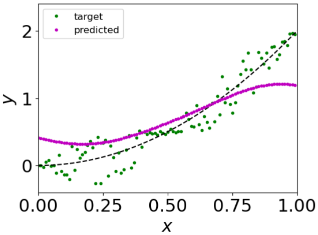

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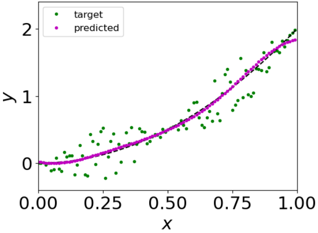

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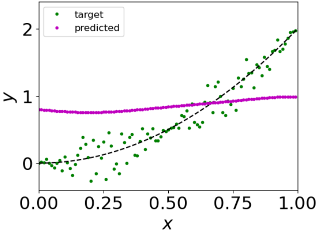

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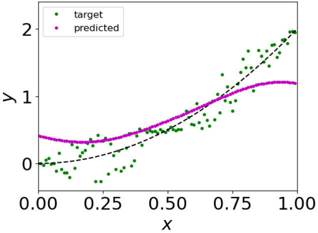

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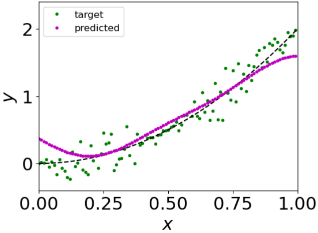

(D) $C = 1.0,\ \varepsilon = 0.4$

Channel flow

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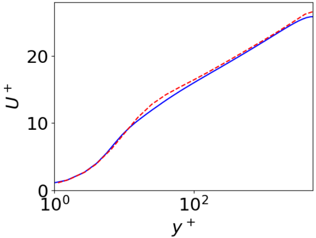

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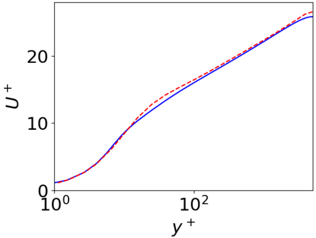

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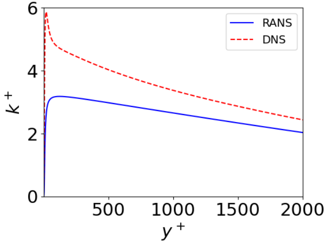

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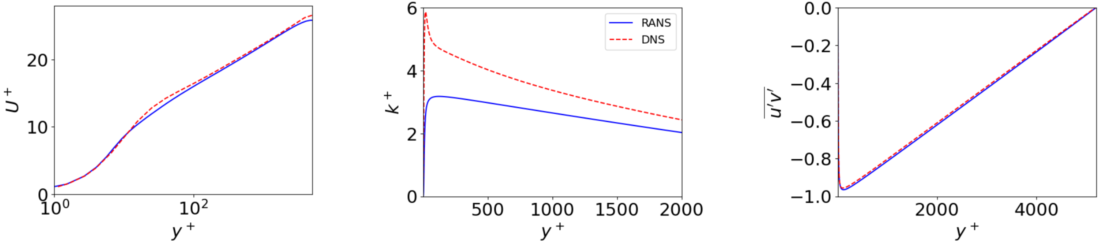

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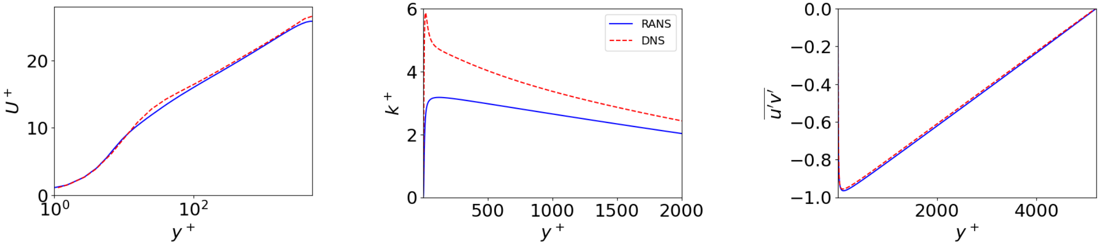

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- The $k-\omega$ model predicts accurate Reynolds shear stress, $-\overline{v_1'v_2'} = \nu_t \dfrac{\partial \bar{v}_1}{\partial x_2}$ (all turbulence models predict a linear total shear stress, see Eq. 6.20 in the eBook)

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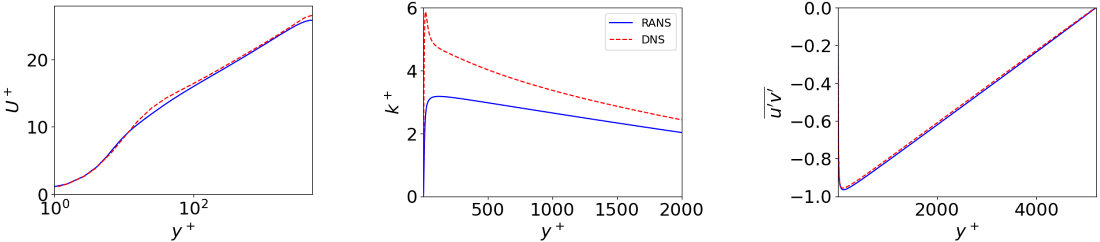

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- Download the Python script `ML-channel.py` and all datafiles. (in `ML-channel.py` I use $\partial \bar{v}_1/\partial x_2$ as input and $-\overline{v_1' v_2'}$ as output)

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y=uv_out
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uv_all_data=uv_DNS
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- and then I train (fit) the SVR

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X=np.zeros((n_svr,2))
y=uv_out
X[:,0]=dudy_in[:,0]
X[:,1]=vist_in[:,0]
```

- I choose type of SVR

```
# choose Machine Learning model
C=1
eps=0.001
model = SVR(kernel='rbf', epsilon = eps, C = C)
```

- and then I train (fit) the SVR

```
# Fit the model
svr = model.fit(X, y.flatten())
```

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X_test=np.zeros((n_test,2))
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X_test[:,1]=vist_test[:,0]
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(np.mean(uv_predict**2))**0.5
print('\nRMS error using ML turbulence model',uv_error)
```

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- One way to improve the accuracy is to use data only where the flow is fully turbulent, e.g. $100 < y^+ < 1000$

```
index=np.nonzero((yplus_DNS > 100 ) & (yplus_DNS< 2000 ))
```

ML model (SVR)

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```

- Next step is to import the SVR model into your Python CFD code (either `rans-k-omega.py`, pyCALC-RANS or your own)

ML model (SVR)

```
from sklearn.preprocessing import StandardScaler
from sklearn.svm import SVR
from joblib import dump, load

folder='./'
filename=str(folder)+'model-svr.bin'
model = load(tr(folder)+'model-svr.bin')
scaler_dudy = load(str(folder)+'scalar-dudy-svr.bin')
scaler_vist = load(str(folder)+'scalar-vist-svr.bin')
dudy_max,dudy_min,vist_max,vist_min = np.loadtxt(str(folder)+'min-max-svr.
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- Then you insert the SVR coding corresponding to the test/predict part in the previous slides. Note: you should NOT include the transform fitting, `scaler_dudy.fit_transform(dudy_in)`

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- You may find some useful Python coding at p. 14 in my report [1]

ML model: CFD code

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- Then we get C_ω as `cmu_omega=uv_predict/dudy/vist`
- Note that you must invert the scaling for `vist` and `dudy` in the expression above, e.g. `scaler_vist.inverse_transform(vist)`

ML model: Problem 1

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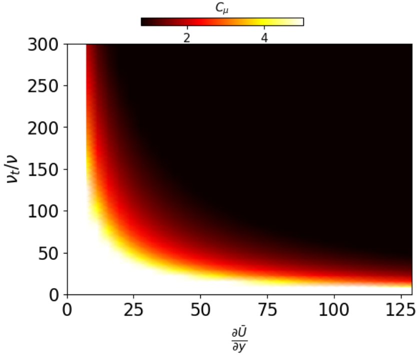

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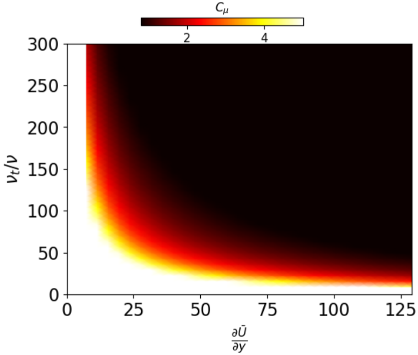

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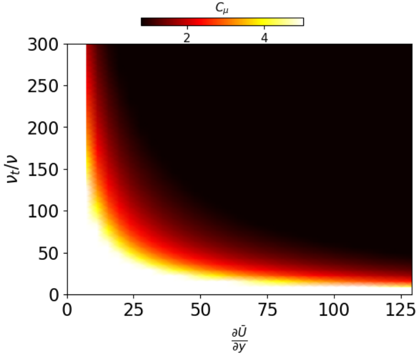

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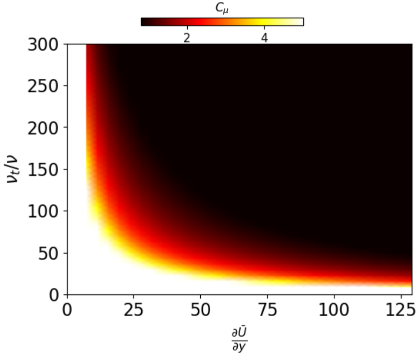

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- This may occur at the beginning of the CFD simulation and the solution is "stuck" (the product $\nu_t \partial \bar{v}_1/\partial x_2$ is correct but both ν_t and $\partial \bar{v}_1/\partial x_2$ are incorrect)
- The reason is poor/incorrect initial b.c. You can, e.g., use the profiles of a solution without ML/SVR as initial b.c.

Problem 1 how to plot the figure at the previous slide

```
fig1,ax1 = plt.subplots();ax=plt.gca()
# Set Increments between points in a meshgrid
mesh_size = 0.05
# Identify min and max values for input variables
x_min, x_max = X[:,0].min(), X[:,0].max()
y_min, y_max = X[:,1].min(), X[:,1].max()
# Return evenly spaced values based on a range between min and max
xrange = np.arange(x_min, x_max, mesh_size)
yrange = np.arange(y_min, y_max, mesh_size)
# Create a meshgrid
xx, yy= np.meshgrid(xrange, yrange)
# Use model to create a prediction plane --- SVR
pred_svr = model.predict(np.c_[xx.ravel(), yy.ravel()])
pred_svr = pred_svr.reshape(xx.shape)
xx_no_scale=scaler_dudy.inverse_transform(xx)
yy_no_scale=scaler_vist.inverse_transform(yy)
# Make the color plot (excl. fig1,ax1 = plt.subplots() ...)
ax_plot=plt.pcolormesh(xx_no_scale,yy_no_scale/viscos, pred_svr, vmin=0.8,vmax
    =1,cmap=plt.get_cmap('hot'),shading='gouraud')
```

ML model: Problem 2

- Make sure that predicted C_ω agrees well with the target (that is not that case in the figure to the right)

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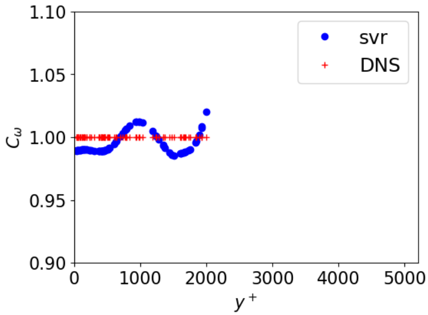

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- Use f_m as output and $|\partial \bar{v}_1/\partial x_2|$ and L_m^2 as input.

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- We take the mixing length L_m as the wall distance and then let our function f_m adapt to get a reasonable lengthscale.
- Use f_m as output and $|\partial \bar{v}_1/\partial x_2|$ and L_m^2 as input.
- You could also try ν_t as output instead of f_m.

References

[1] L. Davidson. Using Machine Learning for formulating new wall functions for Large Eddy Simulation: A second attempt. Technical report, Division of Fluid Dynamics, Dept. of Mechanics and Maritime Sciences, Chalmers University of Technology, Gothenburg, 2022.

[2] Andreas Lindholm, Niklas Wahlström, Fredrik Lindsten, and Thomas Schön. *Machine Learning: A First Course for Engineers and Scientists*. Cambridge University Press, 2022.

[3] Menneni Rachana, Jegadeesan Ramalingam, Gajula Ramana, Adigoppula Tejaswi, Sagar Mamidala, and G Srikanth. Fraud detection of credit card using machine learning. *GIS-Zeitschrift für Geoinformatik*, 8:1421–1436, 10 2021.

[4] Sudarshana S Rao and Santosh R Desai. Machine learning based traffic light detection and ir sensor based proximity sensing for autonomous cars. In *Proceedings of the International Conference on IoT Based Control Networks & Intelligent Systems – ICICNIS*, 2021.