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MTF270 Turbulence Modelling

Tentamen torsdagen den 27 maj 2010, f.m.

OBS! No means of assistance, i.e. no calculator, no mathematical handbook etc. (Inga hjälpmedel) except the formula sheet appended to this exam

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T1. a) Derive the transport equation for the $\overline{v_i'\theta'}$ equation. Explain the physical meaning of the different terms. (5p)

b) Use physical reasoning to derive a model for the pressure-strain term for the normal stress components. How is the model for the shear stress components obtained? (5p)

c) The exact equation for the fluctuating pressure p' reads (5p)

$$\frac{1}{\rho}\frac{\partial^2 p'}{\partial x_j \partial x_j} = -2\frac{\partial \bar{v}_i}{\partial x_j}\frac{\partial v_j'}{\partial x_i} - \frac{\partial^2}{\partial x_i \partial x_j}\left(v_i'v_j' - \overline{v_i'v_j'}\right)$$

For a Poisson equation

$$\frac{\partial^2 \varphi}{\partial x_j \partial x_j} = f$$

there exists an exact analytical solution

$$\varphi(\mathbf{x}) = -\frac{1}{4\pi}\int_V \frac{f(\mathbf{y})dy_1 dy_2 dy_3}{|\mathbf{y}-\mathbf{x}|}$$

Use these equations to derive the exact analytical solution for the pressure strain term

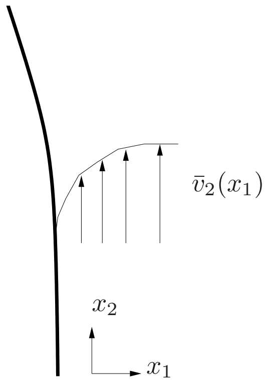
$$\overline{p'\left(\frac{\partial v_i'}{\partial x_j} + \frac{\partial v_j'}{\partial x_i}\right)}$$

Which terms are called the "slow" and "rapid" terms? Why are they called slow and rapid?

d) One realizability constraint is that the normal Reynolds stresses should not go negative. Show that the Boussinesq assumption does allow negative normal stresses. In which coordinate system is that risk largest? Derive an expression how to avoid negative normal stresses by reducing the turbulent viscosity. (5p)

T2. a) Describe the SST model. The SST model is a combination of two models: which ones? In which region is each model being used and why? How is the switch between the two models done? (5p)

b) Consider the flow over a curved wall (see figure below), in which the turbulence is reduced by streamline curvature. Show that Reynolds stress models can reproduce this reduction. (5p)



c) By filtering the Navier-Stokes equations in two different ways the following formula is obtained (5p)

$$\widehat{\bar{v}_i \bar{v}_j} - \widehat{\bar{v}}_i \widehat{\bar{v}}_j + \widehat{\tau}_{ij} = T_{ij}, \quad \mathcal{L}_{ij} = \widehat{\bar{v}_i \bar{v}_j} - \widehat{\bar{v}}_i \widehat{\bar{v}}_j$$

Show how it is derived. Use the Smagorinsky model to derive the following expression

$$\mathcal{L}_{ij} - \frac{1}{3}\delta_{ij}\mathcal{L}_{kk} = -2C\left(\widehat{\Delta}^2 |\widehat{\bar{s}}|\widehat{\bar{s}}_{ij} - \Delta^2 \widehat{|\bar{s}|\bar{s}_{ij}}\right)$$

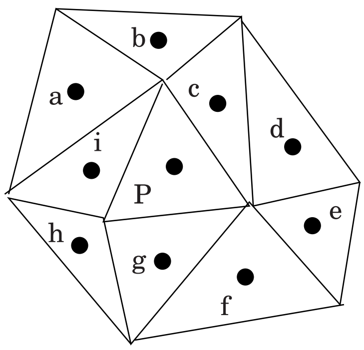
T3. a) Discuss the energy path in connection to the source and sink terms in the $\bar{k}$, $\bar{K}$ and the k_{sgs} equations. Give the expressions for the source and sink terms. How are $\bar{k}$ and k_{sgs} computed from the energy spectrum? (5p)

b) Describe hybrid LES-RANS based on one-equation models. Give all equations related to the turbulence model. (5p)

c) In scale-similarity models in LES the subgrid stresses are modelled as (5p)

$$\tau_{ij} = \overline{\bar{v}_i \bar{v}_j} - \bar{\bar{v}}_i \bar{\bar{v}}_j$$

Assume you're using LES on an unstructured mesh, see figure below. All velocities, $\bar{v}_i$, are stored in the center of the control volumes (black large dots). Cell p has volume V_p, cell a has volume V_a etc. Suggest a way to filter the velocities to get the first part of τ_{12} for node P.



Under tentamen kan ansvarig lärare (Lars Davidson) nås på telefon: 772 14 04 eller 0730-791 161

During the exam the responsible teacher (Lars Davidson) can be reached at 772 14 04 or 0730-791 161

Maximal score is 50. For grade *passed* a score of 20 is required; for grade 4, 30 points are required; for grade 5, 40 points are required.

MTF270 Turbulence modelling: Formula sheet

August 12, 2010

The continuity, Navier-Stokes and temperature equations for incompressible flow with constant viscosity read

$$
\begin{aligned}
\frac{\partial v_i}{\partial x_i} &= 0\\
\rho\frac{\partial v_i}{\partial t} + \rho\frac{\partial v_i v_j}{\partial x_j} &= -\frac{\partial p}{\partial x_i} + \mu\frac{\partial^2 v_i}{\partial x_j \partial x_j} - \rho_0\beta(\theta-\theta_0)g_i\\
\frac{\partial \theta}{\partial t} + \frac{\partial \bar{v}_i\theta}{\partial x_i} &= \alpha\frac{\partial^2\theta}{\partial x_i\partial x_i}
\end{aligned}
$$

The *time averaged* continuity equation, Navier-Stokes equation temperature equations read

$$
\begin{aligned}
\frac{\partial \bar{v}_i}{\partial x_i} &= 0\\
\rho\frac{\partial \bar{v}_i\bar{v}_j}{\partial x_j} &= -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\mu\frac{\partial \bar{v}_i}{\partial x_j} - \rho\overline{v_i'v_j'}\right) - \rho_0\beta(\bar{\theta}-\theta_0)g_i\\
\frac{\partial \bar{v}_i\bar{\theta}}{\partial x_i} &= \alpha\frac{\partial^2\bar{\theta}}{\partial x_i\partial x_i} - \frac{\partial\overline{v_i'\theta'}}{\partial x_i}
\end{aligned}
$$

The Boussineq assumption reads

$$
\overline{v_i'v_j'} = -\nu_t\left(\frac{\partial \bar{v}_i}{\partial x_j} + \frac{\partial \bar{v}_j}{\partial x_i}\right) + \frac{2}{3}\delta_{ij}k = -2\nu_t\bar{s}_{ij} + \frac{2}{3}\delta_{ij}k
$$

The modeled $\overline{v_i'v_j'}$ equation with IP model reads

$$
\begin{aligned}
\bar{v}_k\frac{\partial\overline{v_i'v_j'}}{\partial x_k} = & \quad \text{(convection)}\\
-\overline{v_i'v_k'}\frac{\partial \bar{v}_j}{\partial x_k} - \overline{v_j'v_k'}\frac{\partial \bar{v}_i}{\partial x_k} & \quad \text{(production)}\\
-c_1\frac{\varepsilon}{k}\left(\overline{v_i'v_j'} - \frac{2}{3}\delta_{ij}k\right) & \quad \text{(slow part)}\\
-c_2\left(P_{ij} - \frac{2}{3}\delta_{ij}P^k\right) & \quad \text{(rapid part)}\\
+c_{1w}\rho\frac{\varepsilon}{k}\left[\,\overline{v_k'v_m'}n_kn_m\delta_{ij} - \frac{3}{2}\overline{v_i'v_k'}n_kn_j\right. & \\
\left.-\frac{3}{2}\overline{v_j'v_k'}n_kn_i\,\right]f\left[\frac{\ell_t}{x_n}\right] & \quad \text{(wall, slow part)}\\
+c_{2w}\left[\,\Phi_{km,2}n_kn_m\delta_{ij} - \frac{3}{2}\Phi_{ik,2}n_kn_j\right. & \\
\left.-\frac{3}{2}\Phi_{jk,2}n_kn_i\,\right]f\left[\frac{\ell_t}{x_n}\right] & \quad \text{(wall, rapid part)}\\
+\nu\frac{\partial^2\overline{v_i'v_j'}}{\partial x_k\partial x_k} & \quad \text{(viscous diffusion)}\\
+\frac{\partial}{\partial x_k}\left[c_k\,\overline{v_k'v_m'}\,\frac{k}{\varepsilon}\frac{\partial\overline{v_i'v_j'}}{\partial x_m}\right] & \quad \text{(turbulent diffusion)}\\
-g_i\beta\overline{v_j'\theta'} - g_j\beta\overline{v_i'\theta'} & \quad \text{(buoyancy production)}\\
-\frac{2}{3}\varepsilon\delta_{ij} & \quad \text{(dissipation)}
\end{aligned}
$$