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MTF071 Computational Fluid Dynamics of Turbulent Flow (5.0 p)

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Task K3

In this task, you should write a computer program to solve for a fully developed turbulent channel flow based on a chosen turbulence model. Since the flow 1D, the transport equation system may be simplified into three coupled 1D-diffusion equations for the velocity U and the two turbulent quantities, respectively. It is recommended that you use Matlab.

The task should be carried out in groups of two (or one), and it should be presented both orally and in the form of a report.

The $k - \varepsilon$ Model

The governing equations in the $k - \varepsilon$ model read (compare Eqs. 3.10, 3.12a, 3.12b, 3.35, 3.36 (Versteegh & Malalasekera, 1995) and Eqs. 2.29, 3.2 (Davidson, 1997)

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0$$

$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\partial}{\partial x}\left[(\nu + \nu_t)\frac{\partial U}{\partial x}\right] + \frac{\partial}{\partial y}\left[(\nu + \nu_t)\frac{\partial U}{\partial y}\right]$$

$$U\frac{\partial V}{\partial x} + V\frac{\partial V}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \frac{\partial}{\partial x}\left[(\nu + \nu_t)\frac{\partial V}{\partial x}\right] + \frac{\partial}{\partial y}\left[(\nu + \nu_t)\frac{\partial V}{\partial y}\right]$$

$$U\frac{\partial k}{\partial x} + V\frac{\partial k}{\partial y} = \frac{\partial}{\partial x}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x}\right] + \frac{\partial}{\partial y}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial y}\right] + P_k - \varepsilon$$

$$U\frac{\partial \varepsilon}{\partial x} + V\frac{\partial \varepsilon}{\partial y} = \frac{\partial}{\partial x}\left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x}\right] + \frac{\partial}{\partial y}\left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial y}\right] + \frac{\varepsilon}{k}\left(c_1 P_k - c_2 \varepsilon\right). \tag{1}$$

Since the flow is fully developed we have $V = \partial U/\partial x = \partial k/\partial x = \partial \varepsilon/\partial x = 0$. Hence the above equations can be simplified as

$$
\begin{aligned}
0 &= -\frac{1}{\rho}\frac{\partial P}{\partial x} + \frac{\partial}{\partial y}\left[(\nu + \nu_t)\frac{\partial U}{\partial y}\right] \\
0 &= \frac{\partial}{\partial y}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial y}\right] + P_k - \varepsilon \\
0 &= \frac{\partial}{\partial y}\left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial y}\right] + \frac{\varepsilon}{k}\left(c_1 P_k - c_2 \varepsilon\right)
\end{aligned}
\tag{2}
$$

We find that the equations reduce to simple 1D diffusion equations with some complicated source terms. Thus these equations are of the same form as that in Task K1, but the source terms are more complicated. The turbulent viscosity and production term have the form

$$
\begin{aligned}
\nu_t &= c_\mu \frac{k^2}{\varepsilon} \\
P_k &= \nu_t \left(\frac{\partial U}{\partial y}\right)^2 .
\end{aligned}
\tag{3}
$$

The model constants are given in Eq. 3.37 (Versteegh & Malalasekera, 1995).

The horizontal channel has a height of $y_{max} = 2\delta$ between the bottom flat plate (at $y = 0$) and the top flat plate (at $y = y_{max}$). The flow is driven by the pressure gradient $\frac{\partial P}{\partial x}$, as shown in the above momentum equation. Since the channel flow is fully developed, $\frac{\partial P}{\partial x}$ should be a constant and be balanced by the wall shear stresses.

The flow is symmetric about the centerline of the channel. As a consequence, you may take only a half of the channel as the computation domain in your solution (from the wall to the centerline).

For comparison of your results with existing DNS (Direct Numerical Simulation) data, the task is to compute a fully developed channel flow with $Re_\tau = 395$, where $Re_\tau = u_\tau \delta/\nu$. Note that u_τ is the friction velocity and $u_\tau = \sqrt{\tau_w/\rho}$ with $\tau_w = \mu \frac{\partial U}{\partial y}|_{wall}$.

Letting $\delta = 1$ (i.e. $y_{max} = 2$), we have $-\frac{\partial P}{\partial x} = \tau_w$. Show this in your report of this task. If we further let $\rho = \delta = u_\tau = 1$ we have $-\frac{\partial P}{\partial x} = 1$.

Make a clear description in the task report about the boundary conditions you have implemented in your computation.

I: Common task

All groups start by using the $k - \varepsilon$ model described above. At the wall the boundary conditions

$$\begin{aligned} U = k = 0 \\ \frac{\partial \varepsilon}{\partial y} = 0 \end{aligned} \tag{4}$$

should be used. Please note that this model is inappropriate for near-wall flow resolution. We're using it all the way to the wall – through the viscous sublayer – without using any damping functions. However, it is a good starting point for the next exercise presented below.

As mentioned above you should solve the U velocity and the two turbulence quantities (k and ε or k and ω or k and ν_t, depending on which model you are going to use). On the www-page you find the Matlab file `turb.m` which reads U as well as several other physical quantities to be used for comparison.

When implementing the source terms, you should split the source term into S_U and S_P as usual

$$S = S_P \Phi_P + S_U \tag{5}$$

where $S_P < 0$. It is very important the the turbulent quantities stay positive during the iteration process. For example, if ε goes negative the turbulent viscosity becomes negative, and usually this leads to rapid divergence. Also, if k becomes negative, the production term in the ε equation will be negative, which also will cause convergence problems. The remedy is to make sure that *all* negative source terms are put in S_P. For example, the dissipation term in the k equation should be put in S_P so that

$$S_P = -\rho \varepsilon \delta V / k. \tag{6}$$

Solve the equation system using Gauss-Seidel. You'll find that the convergence is rather slow, so probably you need a fairly large number of iterations. Compare U, k and ε with DNS data.

II: Low Reynolds Number Model

In this part you should use your own LRN (Low Reynolds Number) model, see Table 1, below.

Case	Model	Reference
1,13	$k-\varepsilon$	Launder & Sharma (1974); Davidson (1997)
2,14	$k-\omega$	Bredberg *et al.* (2002); Davidson (1997)
3,15	$k-\omega$	Wilcox (1988); Davidson (1997)
4,16	$k-\omega$	Peng *et al.* (1997); Davidson (1997)
5,17	$k-\nu_t$	Peng & Davidson (2000)
6,18	$k-\varepsilon$	Rahman & Siikonen (2000)
7,19	$k-\varepsilon$	Chien (1982)
8,20	$k-\varepsilon$	Yang & Shih (1993)
9,21	$k-\varepsilon$	Hwang & Lin (1998)
10,22	$k-\varepsilon$	Nagano & Tagawa (1990)
11,23	$k-\varepsilon$	Abe *et al.* (1994)
12,24	$k-\varepsilon$	Lien & Leschziner (1994)

The paper by Patel *et al.* (1985) is useful for learning more about low-Re models.

The following items must be covered in your presentation:

- Note that the equations converge *very* slowly (why?) for a fine grid. At least 1500 iterations is needed. Compare your results for U, k and ε with DNS data. Look especially what happens close the wall. What is the asymptotic behaviour for k and ε (or ω or ν_t) close to the wall ($y^+ < 5$)? Be careful when you compare ε; which ε does your model predict, ε or $\tilde{\varepsilon}$? (ε_{DNS} is the true dissipation).
- Compare the shear stress $\overline{uv}$. What is the asymptotic behaviour?
- Compute the Kolmogorov's velocity, length and time scales (see Section 1.2 in Davidson (1997)). Compare them with the large turbulent scales.
- Look at the budget of your k equation (i.e. plot the different terms). Compare with DNS data. If your model performs poorly, try to understand why. Is any special term in the k equation poorly predicted? In the `DNS_data`-file, the six columns correspond to (all scaled by u_* and ν)

  1. y^+
  2. dissipation ε
  3. production $-\overline{uv}\partial\bar{U}/\partial y$
  4. diffusion due to pressure-velocity fluctuations $-\partial/\partial y(\overline{pv}/\rho)$
  5. diffusion due to triple velocity fluctuations $-\partial/\partial y(\overline{vk'})$
  6. viscous diffusion $\nu\partial^2 k/\partial y^2$

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