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MTF071 Computational Fluid Dynamics of Turbulent Flow http://www.tfd.chalmers.se/gr-kurs/MTF071

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Task K2

In Task K1 you worked with the diffusion equation. Now we add the convective terms (left-hand side, see Eq. 1) and put the diffusion terms on the right-hand side. Equation 1 is called the convection-diffusion equation, or the transport equation for temperature. You should write a computer program to solve Eq. 1 in two dimensions with finite volume methods. Use the hybrid upwind/central differencing scheme, see Eq. 5.43 in [1]. Note that the continuity error $\Delta F = 0$. It is recommended that you use Matlab. The velocity field and the grid are given for each group and are available on the net; there is an m-file (`pl_vect.m`) by which you can read and plot the vector field of the velocity. The task should be carried out in groups of two (or one), and it should be presented both orally and written.

The two-dimensional transport equation for temperature reads

$$\frac{\partial}{\partial x}\left(\rho U T\right) + \frac{\partial}{\partial y}\left(\rho V T\right) = \frac{\partial}{\partial x}\left(\Gamma \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\Gamma \frac{\partial T}{\partial y}\right) + S$$

$$\Gamma = \frac{k}{c_p}.$$

(1)

The algebraic equation system should be solved using both

- Gauss-Seidel and

- TDMA.

How to proceed

- Fetch the grid and velocity field from the net.

- Plot the velocity field using the m-file `pl_vect.m`. From the supplied velocity field you can find out the location and extent of the inlet and outlet.

- Start to write you program to solve Eq. 1

- Use the coarse grid when you are writing/debugging your program. When the program works you should also use the fine grid.

Note that when prescribing a heat flux you must divide by c_p since Eq. 1 is an equation for temperature rather than for energy. Use $\partial T/\partial n = 0$ (n denotes the coordinate normal to the wall) at walls if no boundary condition is given (see tables).

Convergence

It is very important to verify that a converged solution has been obtained. At each iteration compute the residual as

$$\varepsilon = \frac{1}{F}\left(\sum_{\text{all cells}} |a_E T_{i+1,j} + a_W T_{i-1,j} + a_N T_{i,j+1} + a_S T_{i,j-1} + S_C - a_P T_{i,j}|\right) \quad (2)$$

where F is a temperature flux used to normalize the residual. The temperature flux F should be representative of the total flux in the domain. In the present work it is suitable to take F as the inlet mass flux multiplied by the difference of the temperature flux at inlet and outlet, i.e.

$$F = (\rho U h)_A \Delta T, \quad (3)$$

where ΔT is the temperature difference between inlet and outlet. The solution is consider as converged when $\varepsilon < 0.001$.

Presentation of the work

The work should be presented both orally and in a form of a short report. The oral presentation should be approximately 10 minutes (use some slides). Present the problem, how you carried out the work, problems encountered and how you solved them. The presentation and the report must include the following points:

1. Sensitivity to boundary conditions. Make (an interesting) change of one boundary condition.

2. Sensitivity to convergence. Does the temperature change if you use another convergence criterium? Test $\varepsilon = 0.01$ and $\varepsilon = 0.0001$.

3. Sensitivity to heat conductivity. If you decrease Γ (i.e. k) the convective transport will increase relative to the diffusive one. Increase and decrease Γ by a factor of 100. Discuss how the results are affected.

4. Plot the heat flux (along a Dirichlet boundary) or the wall temperature (along a Neumann boundary) as a function of the distance along the wall.

5. Check if you have global conservation, i.e. does the heat flux through all boundaries (inlet, outlet and walls) sum up to zero (as it should if you don't have any source term).

GRID 1: Case 1 to 5

Physical data: $\rho = 1$, $k/c_p = 1/500$, $h_A/H = h_C/H = 0.068$.
  Boundary conditions: $U_A = 1$, $U_B = 0$, $U_C = 1$, $U_D = 0$, $T_A = 20^o C$.

GRID 2: Case 6 to 10

Physical data: $\rho = 1$, $k/c_p = 1/500$.
  Boundary conditions: $U_A = 1$, $U_B = U_A h_A/h_B$, $U_C = 0$, $U_D = 0$, $T_A = 20^o C$.

GRID 3: Case 11 to 15

Physical data: $\rho = 1$, $k/c_p = 1/200$.
  Boundary conditions: $U_A = 0$, $U_B = 0.4 V_D h_D/h_B$, $U_C = 0.6|V_D|h_D/h_B$, $V_D = -1$, $T_D = 10^o C$.

GRID 4: Case 16 to 20

Physical data: $\rho = 1$, $k/c_p = 1/200$.
  Boundary conditions: $U_A = 0$, $U_B = 0.2 V_D h_D/h_B$, $U_C = 0.8|V_D|h_D/h_B$, $V_D = -1$, $T_D = -10^o C$.

GRID 5: Case 21 to 25

Physical data: $\rho = 1$, $k/c_p = 1/100$.
  Boundary conditions: $U_A = 0$, $U_B = 0$, $U_C = |V_D|h_D/h_B$, $V_D = -1$, $T_D = 0^o C$.

References

1. H.K. Versteegh and W. Malalasekera. *An Introduction to Computational Fluid Dynamics – The Finite Volume Method*. Longman Scientific & Technical, Harlow, England, 1995

Case	b.c.
1	$y = H : \ T = 10$
2	$y = H : \ q = 100 \ \mathrm{W/m^2}$
3	$x = L : \ T = 10$
4	$x = L : \ T = 50$
5	$y = 0 : \ q = 100 \ \mathrm{W/m^2}$

Table 1: Boundary conditions for Case 1 to 5.

Case	b.c.
6	$x = 0: \ T = 10$
7	$x = 0: \ T = 5$
8	$x = 0: \ 100\mathrm{W/m}^2$
9	$x = 0, y/H \geq 0.5: \ T = 10$
10	$x = 0, y/H \leq 0.5: \ T = 10$

Table 2: Boundary conditions for Case 6 to 10.

Case	b.c.
11	$x = 0: \ T = 20, \ x = L: \ T = 20, \ y = 0: \ T = 0$
12	$x = 0: \ T = 0, \ x = L: \ T = 20, \ y = 0: \ T = 0$
13	$x = L: \ T = 20, \ y = 0: \ T = 10$
14	$x = 0: \ T = 20, \ x = L: \ T = 20$
15	$y = 0: \ q = 100 \ \mathrm{W/m}^2$

Table 3: Boundary conditions for Case 11 to 15.

Case	b.c.
16	$x = 0: \ T = 0, \ x = L: \ T = -20, \ y = 0: \ T = 0$
17	$x = L: \ T = 20, \ y = 0: \ T = 0$
18	$x = 0: \ T = 0, \ x = L: \ q = -100 \ \mathrm{W/m}^2, \ y = 0, \ T = 10$
19	$x = 0: \ T = 50, \ x = L: \ T = -20, \ y = 0: \ T = 10$
20	$x = 0: \ T = 20, \ y = 0: \ q = 50 \ \mathrm{W/m}^2$

Table 4: Boundary conditions for Case 16 to 20.

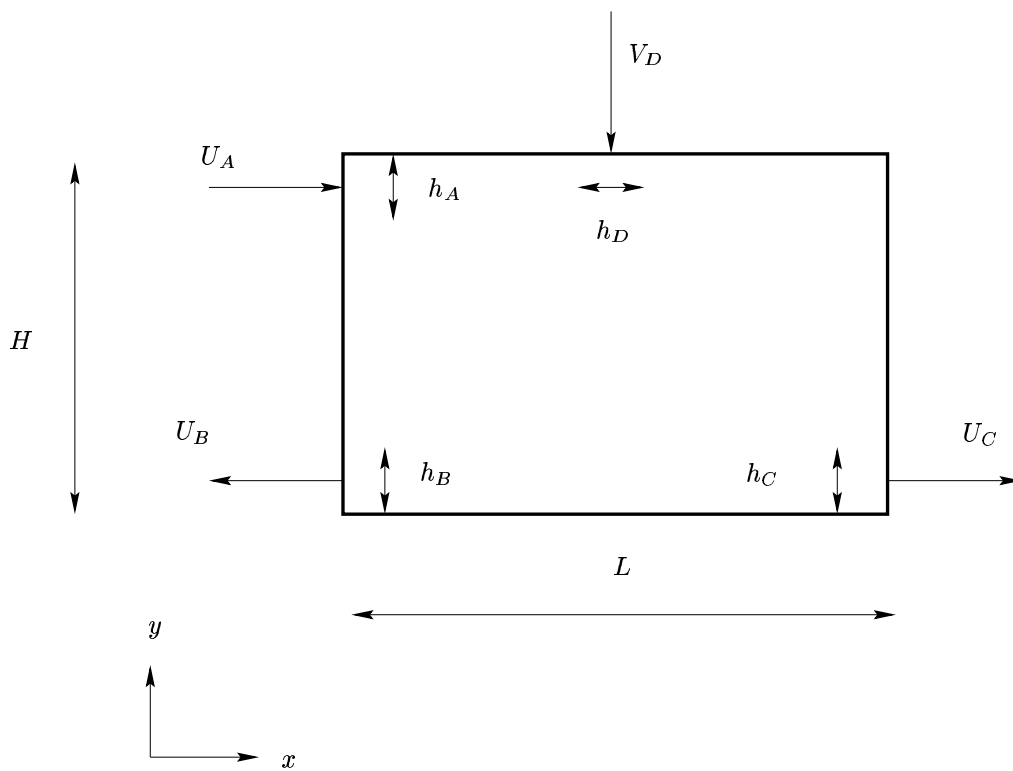| Case	b.c.
21	$x = 0: \ T = 0, \ y = 0: \ T = 0$
22	$x = 0: \ T = 0, \ x = L: \ T = 20, \ y = 0: \ T = 0$
23	$x = 0: \ T = 10, \ x = L: \ T = -10, \ y = 0: \ T = 0$
24	$x = L: \ T = 20, \ y = 0: \ T = 20$
25	$x = 0: \ T = 0, \ x = L: \ q = 50 \ \mathrm{W/m}^2, \ y = 0: \ T = 30$

Table 5: Boundary conditions for Case 21 to 25.

Figure 1: Configuration. The extent in the third coordinate direction is 1.