CHALMERS TEKNISKA HÖGSKOLA
Termo- och Fluiddynamik
412 96 Göteborg

MTF071 Computational Fluid Dynamics of Turbulent Flow http://www.tfd.chalmers.se/gr-kurs/MTF071

Lars Davidson, tel. 772 1404 2001-02-04

Task K1

We are going to study the diffusion equation for temperature T (i.e. heat conduction equation), which can be written

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+b=0$$

Discretise this equation according to the textbook (Eq. 4.57)

$$a_P T_P = a_E T_E + a_W T_W + a_N T_N + a_S T_S + b\Delta x\Delta y.$$

and solve it using Gauss-Seidel

The task should be carried out (using MATLAB) in groups of one or two students. Below the computational domain, boundary conditions, etc. are given for 20 cases. Pick a case and register your choice by the teacher. The task should be presented both orally and in form of a small report. The oral presentation should be approximately 10 minutes (use some slides). Try to discuss the results from a physical and numerical point of view. Present the results for example as contour plots of the temperature. The presentation and the report must include the following parts:

1. Use different meshes to solve the problem (i.e. 10×10, 20×20 and 40×40). Make the mesh finer in regions where you expect large gradients.
2. How sensitive is the solution to the coefficient of conductivity k? Increase and reduce k by a factor 100. Explain why the solution is changed!
3. What happens if you change the boundary conditions? If you have Neumann b.c. somewhere: change Neumann to Dirichlet (given T) along one side. If you don't have Neumann b.c. anywhere: change Dirichlet to Neumann along one side. Discuss how/why the temperature field is changed.
4. In order to illustrate the heat flow, plot the heat flux vector $\dot{q}_x, \dot{q}_y$

   $$\dot{q}_x=-k\frac{\partial T}{\partial x},\ \dot{q}_y=-k\frac{\partial T}{\partial y}$$

   as a vector plot. Discuss and investigate the relation of the heat flow to the contours of constant T.

Case	$\mathbf{T_1}$	$\mathbf{T_2}$	$\mathbf{T_3}$
1	10	$10 + 20\sin(\pi y/H)$	10
2	15	$10 + 5(1 - y/H) + 15\sin(\pi y/H)$	10
3	15	$15\cos(2\pi y/H)$	15
4	10	$10 + 5y/H + 10\sin(\pi y/H)$	15
5	15	$-5y/H + 15\cos(2\pi y/H)$	10

Table 1: Definition of case 1 - 5. $L = 1$, $H = 0.5$, $dT/dx = 0$ *at boundary 4. Constant source term (per area) b=-1.5. Coefficient of conductivity* $k = 5(1 + 100x/L)$.

Case	$\mathbf{T_2}$	$\mathbf{T_4}$
6	$10 + 20\sin(\pi y/H)$	10
7	$10 + 20\sin(\pi y/H)$	30
8	$5(y/H - 1) + 15\cos(\pi y/H)$	15
9	$5(y/H - 1) + 15\cos(\pi y/H)$	30
10	$10 + 5\sin(\pi y/H)$	10

Table 2: Definition of case 6 - 10. $L = 1.5$, $H = 0.5$, $T_1 = 10$, $dT/dy = 0$ *at boundary 3. Coefficient of conductivity* $k = 0.01$ *in the region* $0.7 < x < 1.1$, $0.3 < y < 0.4$, *and in the remaining of the computational domain* $k = 20$. *The source* $b = 0$.

Case	$\mathbf{c_1}$	$\mathbf{c_2}$	$\mathbf{T_3}$
11	20	0.2	$20x/L$
12	25	0.1	$10(1 + 2x/L)$
13	25	0.3	$15 + 5x/L$
14	20	0.4	$5 + 15x/L$
15	25	0.25	$5 + 3(1 + 5x/L)$

Table 3: Definition of case 11 - 15. $L = 1$, $H = 1$, $T_1 = 10$, $T_2 = 20$, $dT/dx = 0$ *at boundary 4. Coefficient of conductivity* $k = 2(1 + 20T/T_1)$. *Heat source over the whole computational domain* $b = 15(c_1 - c_2T^2)$.

Case	$\mathbf{c_1}$	$\mathbf{c_2}$	$\mathbf{T_3}$
16	20	0.2	$20x/L$
17	25	0.1	$10(1 + 2x/L)$
18	25	0.3	$15 + 5x/L$
19	20	0.4	$5 + 15x/L$
20	25	0.25	$5 + 3(1 + 5x/L)$

Table 4: Definition of case 16 - 20. $L = 1$, $H = 7$, $T_1 = 10$, $T_2 = 20$, $dT/dx = 0$ *at boundary 4. Coefficient of conductivity* $k = 16(y/H + 30T/T_1)$. *Heat source over the whole computational domain* $b = 15(c_1 - c_2T^2)$.

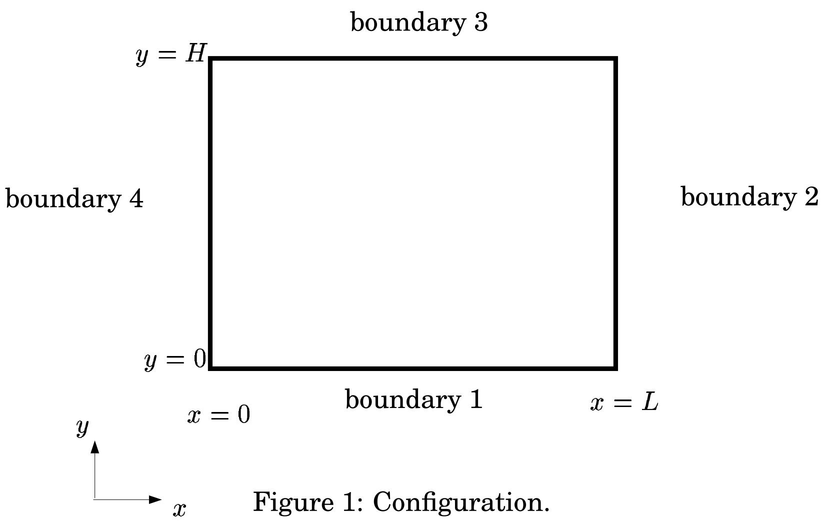

Figure 1: Configuration.