IMPROVING AN EXPLICIT ALGEBRAIC STRESS MODEL USING NEURAL NETWORK

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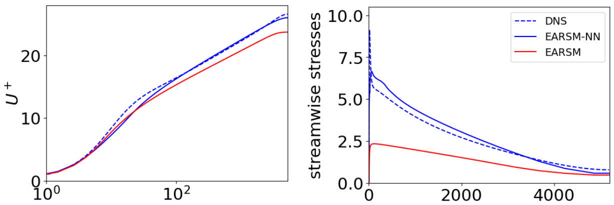
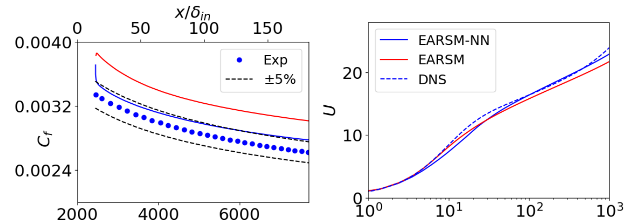


(a) Velocity. **(b)** Streamwise stress.

Figure 1: Channel flow at $Re_\tau = 5\,200$.



(a) Skin friction. **(b)** Velocity. $Re_\theta = 5\,500$.

Figure 2: Flat-plate boundary layer.

The EARSM

In 2D, the EARSM reads [1]

$$a_{ij} = \beta_1 \bar{s}^*_{ij} + \beta_2 \left(\bar{s}^*_{ik}\bar{s}^*_{kj} - \frac{1}{3}\bar{s}^*_{mn}\bar{s}^*_{nm}\delta_{ij} \right) + \beta_4(\bar{s}^*_{ik}\bar{\Omega}^*_{kj} - \bar{\Omega}^*_{ik}\bar{s}^*_{kj}) \quad (1)$$

where

$$\beta_1 = -\frac{A_1 N}{Q}, \quad \beta_2 = 2\frac{A_1 A_2}{Q}, \quad \beta_4 = -\frac{A_1}{Q} \qquad Q = N^2 - 2II_\Omega - \frac{2}{3}A_2^2 II_S \quad (2)$$

N is given by a cubic equation, solved analytically.

Instead of computing β_1, β_2 and β_4 from Eqs. 2, I will in the present work make them functions of some input parameter(s) (to be determined) using Neural Network (NN). The process can be depicted as:

1. The production term, $P^{k,+}$ and y^+ are chosen as input parameters
2. The output (target) parameters are β_1, β_2, β_4
3. Train the NN model in fully-developed channel flow, $Re_\tau = 10\,000$.
4. Use the NN model to compute β_1, β_2, β_4 in the EARSM (k and ω predicted with the $k-\omega$ model) in the **pyCALC-RANS** CFD code
5. The new model is denoted EARSM-NN.

Figure 1 presents the predicted velocity and streamwise Reynolds stress using the EARSM (Eqs. 1 and 2) and the EARSM-NN model and the agreement for the EARSM-NN is much better.

Next, the flat-plate boundary layer is studied. A precursor $k-\omega$ simulation of a flat-plate boundary layer is carried out and U, V, k and ω are stored at $Re_\theta = 2\,500$ where θ denotes the boundary-layer momentum thickness. These stored data are used as inlet boundary condition in the subsequent flat-plate boundary-layer simulations using the two EARSM models. Figure 2 show the predicted skin friction. The EARSM-NN predicts C_f within 5% of experimental data whereas the EARSM over-predicts is by 14%. More results can be found in [2].

Conclusions

An Explicit Algebraic Reynolds Stress Model (together with Wilcox $k-\omega$ model) has been improved using Neural Network (NN). The NN model is trained in channel flow at $Re_\tau = 10\,000$. It is found that target data cannot be taken from DNS because the stress-strain relation and the turbulent kinetic energy are not the same in the DNS and the $k-\omega$ predictions. Hence the target data are taken both from DNS ($\overline{v_1'^2}$ and $\overline{v_2'^2}$) and a $k-\omega$ simulation ($\frac{\partial \bar{v}_1}{\partial x_2}$, $\overline{v_1'v_2'}$, k, $\varepsilon = C_\mu k\omega$). In this way the strain-stress relation and the turbulent kinetic energy are the same in the training process and the CFD-NN predictions. Since k in the training process is taken from the $k-\omega$ results it means that $k \neq 0.5(\overline{v_1'^2}_{DNS} + \overline{v_2'^2}_{DNS} + \overline{v_3'^2}_{DNS})$. In the NN model, the spanwise Reynolds stress adapts to satisfy $a_{ii} = 0$ which means that $\overline{v_3'^2}$ is not correctly predicted (it even goes negative in the near-wall region). Hence, the EARSM-NN model is applicable only to two-dimensional flow where $\overline{v_3'^2}$ is not used. One way to make the model applicable in three-dimensional flow is to develop a $k-\omega$ (or $k-\varepsilon$ model) which accurately predicts the turbulent kinetic energy.

References

[1] S. Wallin, A. V. Johansson, A new explicit algebraic Reynolds stress model for incompressible and compressible turbulent flows, JFM (2000)

[2] L. Davidson, Using Neural Network for Improving an Explicit Algebraic Stress Model in 2D Flow, CUSF, Murray Edwards College, Cambridge, UK (2024) (to appear)