2019-01-08, Exam in

Mechanics of fluids, TME225: Answers

1

a) **Compute the vorticity tensor, Ω_{ij}, and the vorticity vector, ω_i.** See Eqs. 1.11 and 1.12

$$\Omega_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} - \frac{\partial v_j}{\partial x_i}\right), \quad \omega_i = \epsilon_{ijk}\frac{\partial v_k}{\partial x_j}$$

since the flow is 2D, the only non-zero components are

$$\Omega_{12} = -\Omega_{21} = \frac{1}{2}\left(\frac{\partial v_1}{\partial x_2} - \frac{\partial v_2}{\partial x_1}\right), \quad \omega_3 = \epsilon_{3jk}\frac{\partial v_k}{\partial x_j} = \frac{\partial v_2}{\partial x_1} - \frac{\partial v_1}{\partial x_2}$$

From $v_1 = x_1$ and $v_2 = -x_2$ we get $\partial v_1/\partial x_1 = 1$, $\partial v_2/\partial x_2 = -1$ and $\partial v_1/\partial x_2 = \partial v_1/\partial x_2 = 0$. Hence, $\underline{\Omega_{12} = \Omega_{21} = \omega_3 = 0}$.

b) **Show that the product of a symmetric and an antisymmetric tensor is zero.**

See Section 1.5.

2

a) **Derive the transport equation for the internal energy**

See Section 2.2.

b) **Derive the equation for two-dimensional boundary-layer flow expressed in the streamfunction**

See Eqs. 3.42–3.45

3

a) **Show the similarities between the vorticity and temperature transport equations in fully developed flow**

See Section 4.3.

b) **Compute the tangential, v_θ, and the radial, v_r, velocity components.**

See Section 4.4.6.

4

a) **Make a figure and show where these three (sub, buffer and log) regions are valid**.

See Section 6.2.

b) **Make a figure and show how the three terms vary across the channel.**

See Section 6.3.

5

a) **What are the relevant velocity and length scales in the viscous-dominated region ($x_2^+ \lesssim 5$)?**

See Section 6.2.

b) **give the equation for boundary-layer flow.**

See Section 8.3.

6

a) **How are the Reynolds stresses, $\overline{v_i' v_j'}$, modeled in the Boussinesq approach?**

See Section 11.6

b) **What is the wall boundary condition for the velocity?**

See Section 11.14.1.