where the first on the second line is zero because of continuity. Let's verify that

$$\left(\frac{\partial^2 v_j}{\partial x_j \partial x_i} - \frac{\partial^2 v_i}{\partial x_j \partial x_j}\right) = \varepsilon_{inm}\varepsilon_{mjk}\frac{\partial^2 v_k}{\partial x_j \partial x_n} \tag{4.3}$$

Use the $\varepsilon - \delta$-identity (see Table C.1 at p. 328)

$$\varepsilon_{inm}\varepsilon_{mjk}\frac{\partial^2 v_k}{\partial x_j \partial x_n} = (\delta_{ij}\delta_{nk} - \delta_{ik}\delta_{nj})\frac{\partial^2 v_k}{\partial x_j \partial x_n} = \frac{\partial^2 v_k}{\partial x_i \partial x_k} - \frac{\partial^2 v_i}{\partial x_j \partial x_j} \tag{4.4}$$

which shows that Eq. 4.3 is correct. At the right side of Eq. 4.3 we recognize the vorticity, $\omega_m = \varepsilon_{mjk}\partial v_k/\partial x_j$, so that

$$\frac{\partial^2 v_i}{\partial x_j \partial x_j} = -\varepsilon_{inm}\frac{\partial \omega_m}{\partial x_n} \tag{4.5}$$

In vector notation the identity Eq. 4.5 reads

$$\nabla^2 \mathbf{v} = \nabla(\nabla \cdot \mathbf{v}) - \nabla \times \nabla \times \mathbf{v} = -\nabla \times \boldsymbol{\omega} \tag{4.6}$$

Using Eq. 4.5, Eq. 4.1 reads

$$\frac{\partial \tau_{ji}}{\partial x_j} = -\mu\varepsilon_{inm}\frac{\partial \omega_m}{\partial x_n} \tag{4.7}$$

Let's look at Eq. 4.7 for the v_1 equation in two dimensions. Setting $i = 1$ gives

$$\frac{\partial \tau_{j1}}{\partial x_j} = -\mu\varepsilon_{1nm}\frac{\partial \omega_m}{\partial x_n} = -\mu\varepsilon_{123}\frac{\partial \omega_3}{\partial x_2} - \mu\varepsilon_{132}\frac{\partial \omega_2}{\partial x_3} = -\frac{\partial \omega_3}{\partial x_2} + \frac{\partial \omega_2}{\partial x_3} = -\frac{\partial \omega_3}{\partial x_2} \tag{4.8}$$

since $\omega_2 = 0$. Inserting Eq. 1.12 gives

$$\frac{\partial \tau_{j1}}{\partial x_j} = -\frac{\partial}{\partial x_2}\left(\epsilon_{3jk}\frac{\partial v_k}{\partial x_j}\right) = -\frac{\partial}{\partial x_2}\left(\epsilon_{321}\frac{\partial v_1}{\partial x_2} + \epsilon_{312}\frac{\partial v_2}{\partial x_1}\right) = -\frac{\partial}{\partial x_2}\left(-\frac{\partial v_1}{\partial x_2} + \frac{\partial v_2}{\partial x_1}\right)$$

Changing the order of derivatation for the second term gives

$$\frac{\partial \tau_{j1}}{\partial x_j} = \frac{\partial}{\partial x_2}\left(\frac{\partial v_1}{\partial x_2}\right) - \frac{\partial}{\partial x_1}\left(\frac{\partial v_2}{\partial x_2}\right)$$

Using the continuity equation for the last term gives

$$\frac{\partial \tau_{j1}}{\partial x_j} = \frac{\partial}{\partial x_2}\left(\frac{\partial v_1}{\partial x_2}\right) + \frac{\partial}{\partial x_1}\left(\frac{\partial v_1}{\partial x_1}\right)$$

and now we have shown – again – that Eqs. 4.7 and 4.8 are indeed correct.

Thus, there is a one-to-one relation between the viscous term and vorticity: no viscous terms means no vorticity and vice versa. An imbalance in shear stresses (left side of Eq. 4.7) causes a change in vorticity, i.e. generates vorticity (right side of Eq. 4.7). Hence, inviscid flow (i.e. friction-less flow) has no rotation. (The exception is when vorticity is transported *into* an inviscid region, but also in that case no vorticity is generated or destroyed: it stays constant, unaffected.) Inviscid flow is often called *irrotational* flow (i.e. no rotation) or *potential* flow. The vorticity is always created at *boundaries*, see Section 4.3.1. **potential**

The main points that we have learnt in this section are: