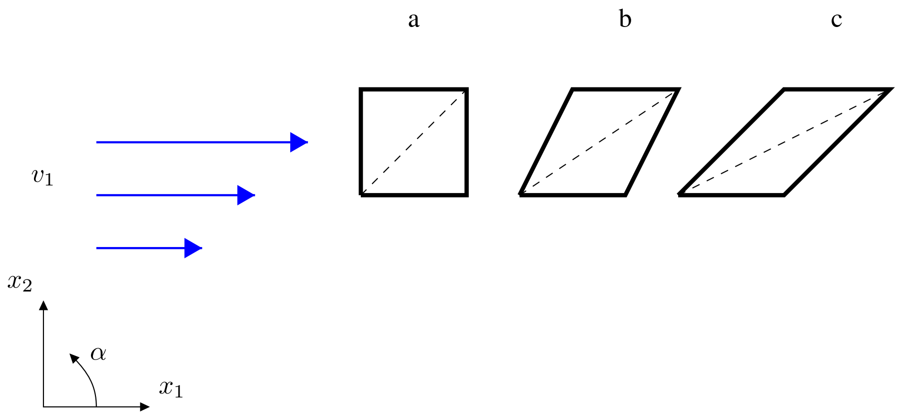

Figure 1.10: A shear flow. A fluid particle with vorticity. $v_1 = cx_2^2$.

1.7.2 Shear flow

Another example – which is rotational – is the lower half of fully-developed channel flow for which the velocity reads (see Eq. 3.28)

$$\frac{v_1}{v_{1,max}} = \frac{4x_2}{h}\left(1 - \frac{x_2}{h}\right), \quad v_2 = 0 \tag{1.35}$$

where $x_2 < h/2$, see Fig. 1.10. The vorticity vector for this flow reads

$$\omega_1 = \omega_2 = 0, \quad \omega_3 = \frac{\partial v_2}{\partial x_1} - \frac{\partial v_1}{\partial x_2} = -\frac{4}{h}\left(1 - \frac{2x_2}{h}\right) \tag{1.36}$$

When the fluid particle is moving from position a, via b to position c its has vorticity. Its vertical too edge move faster than its bottom edge. The horizontal edges stay horizontal because $v_2 =$. Its vertical edges are rotating in clockwise direction. The diagonal is rotating which really is the definition of rotation. Note that the positive rotating direction is defined as the counter-clockwise direction, indicated by a in Fig. 1.10. This is why the vorticity, ω_3, in the lower half of the channel ($x_2 < h/2$) is negative. In the upper half of the channel the vorticity is positive because $\partial v_1/\partial x_2 < 0$. It may be noted that for the flow in Fig. 1.10 the magnitude of the shear, S_{12}, and the vorticity, Ω_{12}, are equal but of opposite sign, i.e. $S_{12} = -\Omega_{12}$.

1.8 Eigenvalues and eigenvectors: physical interpretation

See also [1], Chapt. 2.5.5.

Consider a two-dimensional fluid (or solid) element, see Fig. 1.11. In the left figure it is oriented along the $x_1 - x_2$ coordinate system. On the surfaces act normal stresses (σ_{11}, σ_{22}) and shear stresses (σ_{12}, σ_{21}). The stresses form a tensor, σ_{ij}. Any tensor has eigenvectors and eigenvalues (also called principal vectors and principal values). Since σ_{ij} is symmetric, the eigenvalues are real (i.e. not imaginary). The eigenvalues are obtained from the characteristic equation, see [1], Chapt. 2.5.5 or Eq. 13.5 at p. 165. When the eigenvalues have been obtained, the eigenvectors can be computed. Given