J TME225 Learning outcomes 2019

Note that the questions related to the movies are not part of the learning outcomes. They are included to enhance you learning and understanding.

TME225 Learning outcomes 2019: week 1

1. Explain the difference between Lagrangian and Eulerian description of the motion of a fluid particle.
2. Consider the flow in Section 1.2. Show that $\partial v_1/\partial t$ is different from dv_1/dt.
3. Watch the on-line lecture *Eulerian and Lagrangian Description* at http://www.tfd.chalmers.se/~lada/MoF/flow_viz.html
   i. The first part (approx. the first 12 minutes) describes the difference between Lagrangian and Eulerian points and velocities.
   ii. The formula $\frac{\partial T}{\partial t} + v_i \frac{\partial T}{\partial x_i}$ is nicely explained in (after approx. 12 minutes) where the example is the flow in a river
4. Show which stress components, σ_{ij}, that act on a Cartesian surface whose normal vector is $n_i = (1, 0, 0)$. Show also the stress vector, $t_i^{\hat{n}}$. (see Eq. D.2 and Fig. 1.3 and the Lecture notes of Ekh [2])
5. Show that the product of a symmetric and an antisymmetric tensor is zero.
6. Explain the physical meaning of diagonal and off-diagonal components of S_{ij}
7. Explain the physical meaning of Ω_{ij}
8. What is the definition of irrotational flow?
9. What is the physical meaning of irrotational flow?
10. Derive the relation between the vorticity vector and the vorticity tensor
11. Start from Eq. 1.16 and express the vorticity tensor as a function of the vorticity vector (Eq. 1.18)
12. Explain the physical meaning of the eigenvectors and the eigenvalues of the stress tensor (see Section 1.8 and the Lecture notes of Ekh [2])
13. Watch the on-line lecture *Vorticity, part 1* at http://www.tfd.chalmers.se/~lada/MoF/flow_viz.html
   i. What is a vortex line?
   ii. The teacher talks of ω_A and ω_B, where $\omega = 0.5(\omega_A + \omega_B)$; what does it denote? (cf. Fig. 1.4 in the eBook)
   iii. The teacher shows the rotating tank (after 3 minutes into the movie). He puts the vorticity meter into the tank. The flow in the tank moves like a solid body. How does the vorticity meter move? This is a curved flow with vorticity.

iv. The teacher puts the vorticity meter into a flow in a straight channel (near a wall). What happens with the vorticity meter? (cf. Fig. 1.10)

v. After 4:20 minutes, the teacher shows the figure of a boundary layer. He says that one of the "vorticity legs" (ω_A in Item 13ii above) is parallel to the wall; what does he say about ω_B? What conclusion does is draw about ω? This is a straight flow with vorticity.

vi. After 4:30 minutes, the teacher introduces a spiral vortex tank (in the eBook this is called an ideal vortex). How does the vortex meter behave? (cf. Fig. 1.8 in the eBook). How does the teacher explain that the vorticity is zero (look at the figure he talks about after 6 minutes); the explanation uses the fact that the tangential velocity, v_θ, is inversely proportional to the radius, r, see Eq. 1.29 in the eBook.

vii. After 8:40 minutes, the teacher puts the vorticity meter at different locations in the boundary layer; he puts it near the solid wall, a bit further out and finally at the edge of the boundary layer. How does the vorticity meter move at the different locations? Where is the vorticity smallest/largest? Explain why.

viii. After 10:35, the vortex meter is shown in the spiral vortex tank. What happens with the vorticity when we get very close to the center? Does it still remain zero? What happens with the tangential velocity? (see Eq. 1.29)

ix. The teachers explains the concept of circulation, Γ, and its relation to vorticity (cf. Eqs. 1.23 and 1.25).

x. What is a vortex core?

xi. How large is the vorticity and the circulation in the rotating tank?

xii. How large is the vorticity and the circulation in the spiral vortex tank? Does it matter if you include the center?

xiii. The teacher presents a two-dimensional wing. Where is the pressure low and high, respectively? The Bernoulli equation gives then the velocity; where is it high and low, respectively? The velocity difference creates a circulation, Γ.

xiv. After 15:25 minutes, the teacher looks at the rotating tank again. He starts to rotate the tank; initially there is only vorticity near the outer wall. As time increases, vorticity (and circulation) spread toward the center. Finally, the flow in the entire tank has vorticity (and circulation). This illustrates that as long as there is an imbalance in the shear stresses, vorticity (and circulation) is changed (usually created), see Figs. 1.10 and 4.1.

14. A vortex lines are shown in experiments in the on-line lecture *Vorticity, part 2* (18 minutes into the movie) at
`http://www.tfd.chalmers.se/˜lada/MoF/flow_viz.html`

15. Show that the vorticity is non-zero in a boundary layer, see Section 1.7.2 (see also Item 13iv above)

16. Show that the vorticity is zero in an ideal vortex (see Item 13vi above)

Hint:

$$
\begin{aligned}
v_1 &= -v_\theta \frac{x_2}{(x_1^2 + x_2^2)^{1/2}} \\
v_2 &= v_\theta \frac{x_1}{(x_1^2 + x_2^2)^{1/2}}
\end{aligned}
$$

TME225 Learning outcomes 2019: week 2

1. Derive the Navier-Stokes equation, Eq. 2.5 (use the formulas in the Formula sheet which can be found on the course www page)
2. Simplify the Navier-Stokes equation for incompressible flow and constant viscosity (Eq. 2.8)
3. Derive the transport equation for the internal energy, u, Eq. 2.14 (again, use the Formula sheet). What is the physical meaning of the different terms?
4. Simplify the transport equation for internal energy to the case when the flow is incompressible (Eq. 2.17).
5. Derive the transport equation for the kinetic energy, $v_i v_i/2$, Eq. 2.22. What is the physical meaning of the different terms?
6. Explain the energy transfer between kinetic energy, k, and internal energy, u
7. Show how the left side of the transport equations can be written on conservative and non-conservative form
8. Starting from the Navier-Stokes equations (see Formula sheet), derive the flow equation governing the Rayleigh problem expressed in f and η; what are the boundary conditions in time (t) and space (x_2); how are they expressed in the similarity variable η?
9. Show how the boundary layer thickness can be estimated from the Rayleigh problem using f and η (Fig. 3.3)
10. Explain the flow physics at the entrance (smooth curved walls) to a plane channel (Fig. 3.5). Watch also the on-line lecture *Pressure field and acceleration* (22 minutes into the movie) at
    http://www.tfd.chalmers.se/~lada/MoF/flow_viz.html
11. Explain the flow physics in a channel bend (Fig. 3.6). Watch also the on-line lecture *Pressure field and acceleration*
    http://www.tfd.chalmers.se/~lada/MoF/flow_viz.html.
    (a) at 28 minutes into the movie the teacher discusses how the pressure varies in a fixed-body rotation flow
    (b) at 16 minutes into the movie the teacher discusses how the pressure varies for the flow in a bend.
12. Explain the flow physics in a channel bend (Fig. 3.6).
13. Derive the flow equations for fully developed flow between two parallel plates, i.e. fully developed channel flow (Eqs. 3.18, 3.22 and 3.26)
14. Show that the continuity equation is automatically satisfied in 2D when the velocity is expressed in the streamfunction, Ψ
15. Starting from Eq. 3.41, derive the equation for two-dimensional boundary-layer flow expressed in the streamfunction (Eq. 3.45).
16. Derive the Blasius equation, Eq. 3.53. Start from Eq. 3.45

TME225 Learning outcomes 2019: week 4

1. Explain (using words and a figure) why vorticity can be created only by an imbalance (i.e. a gradient) of shear stresses. Explain why pressure and the gravity force cannot create vorticity.

2. The incompressible Navier-Stokes equation can be re-written on the form

$$\frac{\partial v_i}{\partial t} + \underbrace{\frac{\partial k}{\partial x_i}}_{\text{no rotation}} - \underbrace{\varepsilon_{ijk} v_j \omega_k}_{\text{rotation}} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 v_i}{\partial x_j \partial x_j} + f_i$$

   Derive the transport equation (3D) for the vorticity vector, Eq. 4.20

3. Show that the divergence of the vorticity vector, ω_i, is zero

4. Explain vortex stretching and vortex tilting. The vortex stretching can be shown in experiments,

5. Watch the on-line lecture *Vorticity, part 2* (11 minutes into the movie) at
   http://www.tfd.chalmers.se/~lada/MoF/flow_viz.html

6. Show that the vortex stretching/tilting term is zero in two-dimensional flow

7. Derive the 2D equation transport equation (Eq. 4.22) for the vorticity vector from the 3D transport equation (Eq. 4.20)

8. Show the similarities between the vorticity and temperature transport equations in fully developed flow between two parallel plates

9. Use the diffusion of vorticity to show that $\frac{\delta}{\ell} \propto \sqrt{\frac{\nu}{U\ell}} = \sqrt{\frac{1}{Re}}$ (see Fig. 4.4 and Eq. 3.14).

10. Watch the on-line lecture *Boundary layers* at
    http://www.tfd.chalmers.se/~lada/MoF/flow_viz.html

    i. Consider the flow over the flat plate (after two minutes) . How does the boundary layer thickness change when we move downstream?

    ii. What value does the fluid velocity take at the surface? What is this boundary conditions called: slip or no-slip? How do they define the boundary layer thickness?

    iii. How is the wall shear stress defined? How does it change when we move downstream? (how does this compare with the channel/boundary layer flow in TME225 Assignment 1?)

    iv. How is the circulation, Γ, defined? (cf. with Eq. 1.23) How is it related to vorticity? How do they compute Γ for a unit length ($> \delta$) of the boundary layer? How large is it? How does it change when we move downstream on the plate?

v. Where is the circulation (i.e. the vorticity) created? Where is the vorticity created in “your” channel/boundary layer flow (TME225 Assignment 1)? The vorticity is created at different locations in the flat-plate boundary layer and in the channel flow: can you explain why? (hint: in the former case

$$\frac{\partial p}{\partial x_1} = \mu \frac{\partial^2 v_1}{\partial x_2^2}\bigg|_{wall} = 0,$$

but not in the latter; this has an implication for $\gamma_{2,wall}$ [see Section 4.3])

vi. How do they estimate the boundary layer thickness? (cf. Section. 4.3.1)

TME225 Learning outcomes 2019: week 5

1. Watch the on-line lecture *Boundary layers* 10 minutes into the movie
   http://www.tfd.chalmers.se/~lada/MoF/flow_viz.html

   i. How does the boundary layer thickness change at a given x when we increase the velocity? Explain why.

   ii. Consider the flow in a contraction: what happens with the boundary layer thickness after the contraction?

   iii. Why is the vorticity level higher after the contraction?

   iv. Is the wall shear stress lower or higher after the contraction? Why?

   v. Consider the flow in a divergent channel (a diffusor): what happens with the boundary layer thickness and the wall shear stress?

   vi. What happens when the angle of the diffusor increases?

   vii. What do we mean by a "separated boundary layer"? How large is the wall shear stress at the separation point?

   viii. The second part of the movie deals with turbulent flow: we'll talk about that in the next lecture (and the remaining ones).

2. Derive the Bernoulli equation (Eq. 4.32)
   Hint: The gravitation term is first expressed as a potential $g_i = -\partial\hat{\Phi}/\partial x_i$.

3. Consider the derivative of the complex function $(f(z+z_0)-f(z))/z_0$ where $z = x + iy$ and $f = u + iv$. The derivative of f must be independent in which coordinate direction the derivative is taken (either along the real or the imaginary axis). Show that this leads to the *Cauchy-Riemann* equations

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

4. Show that in inviscid flow, both the velocity potential and the streamfunction satisfy the Laplace equation.

5. Consider the complex potential $f = \Phi + i\Psi$. Show that $f = C_1 z^n$ satisfies the Laplace equation. Derive the velocity polar components for $n = 1$ and $n = 2$. What physical flow do these two cases correspond to?

6. Look at *Potential flow* at Wikipedia

   (a) Figure 1 shows the streamfunction around an airfoil. Recall that the Bernoulli equation applies for a streamline.

   (b) Figure 2 shows the streamfunction for plane flow ($n = 1$) and a doublet ($n = -1$, separation ε, see Section 4.4.6)

   (c) Figure 3 shows the streamfunction for cylinder flow

   (d) Figure 4 shows the flow around a blunt corner, $n = 1/2$ (streamlines in blue and potential lines in cerise)

   (e) Figure 5 shows the flow around a sharp corner, $n = 2/3$ (streamlines in blue and potential lines in cerise)

(f) Figure 6 shows parallel flow, $n = 1$ (streamlines in blue and potential lines in cerise)

(g) Figure 7 shows flow along a corner (angle larger than 90^o), $n = 3/2$ (streamlines in blue and potential lines in cerise)

(h) Figure 8 shows flow in a 90^o corner, i.e. stagnation flow, see Section 4.4.3.2, $n = 2$ (streamlines in blue and potential lines in cerise)

(i) Figure 9 shows flow in a corner (angle smaller than 90^o), $n = 3$ (streamlines in blue and potential lines in cerise)

(j) Figure 10 shows flow for a doublet (separation $\varepsilon = 0$, see Section 4.4.6, $n = -1$ (streamlines in blue and potential lines in cerise)

7. Derive the polar velocity components for the complex potential $f = \dot{m}\ln z/(2\pi)$ and $f = -i\Gamma \ln z/(2\pi)$ (Γ denotes circulation). What does the physical flow look like? Show that they satisfy the Laplace equation.

8. A doublet is a combination of a radial sink and source and its complex potential reads $f = \mu/(\pi z)$. Combine it with the potential for parallel flow ($f = V_\infty z$). Derive the resulting velocity field around a cylinder (in polar components). Hint: the radius is defined as $r_0^2 = \mu/(\pi V_\infty)$.

9. Consider the potential flow around a cylinder. Show that the radial velocity is zero at the surface. Use the Bernoulli equation to get the surface pressure. Show that the drag and lift forces are zero. Where are the stagnation points located?

10. Add the complex potential of a vortex line, $f = -i\Gamma \ln z/(2\pi)$ (Γ denotes circulation) to the complex potential for cylinder flow. Compute the polar velocity components. Where are the stagnation points located? What happens with the location of the stagnation point(s) when the circulation becomes very large? How is the lift of the cylinder computed (which applies for any body).

11. What is the Magnus effect? Explain the three applications in the text: why is it efficient to use loops in table tennis? Why does the Magnus effect help a football player get the ball around the wall (of players) when making a free-kick? How does the Magnus effect help propulsing a ship using Flettner rotors. To look at old and new installations of Flettner rotors, see Wikipedia.

12. Consider the inviscid flow around an airfoil, see Fig. 4.19. In inviscid theory it would look like Fig. 4.20. What has been done to achieve the flow in Fig. 4.21? How is the lift computed?

13. Watch the on-line lecture *Boundary layers (17 minutes into the movie* at `http://www.tfd.chalmers.se/~lada/MoF/flow_viz.html`

   i. The flow is “tripped” into turbulence. How?

   ii. When the flow along the lower wall of the diffusor is tripped into turbulent flow, the separation region is suppressed. Try to explain why.

   iii. Two boundary layers – one on each side of the plate – are shown. The upper one is turbulent and the lower one is laminar. What is the difference in the two velocity profiles? (cf. my figures in the ’summary of lectures’) Explain the differences.

iv. Why is the turbulent wall shear stress larger for the turbulent boundary layer? What about the amount of circulation (and vorticity) in the laminar and turbulent boundary layer? How are they distributed?

v. Consider the airfoil: when the boundary layer on the upper (suction) side is turbulent, stall occurs at a higher angle of incidence compared when the boundary layer is laminar. Why?

vi. Vortex generator are place on the suction side in order prevent or delay separation. Try to explain why separation is delayed.

14. What characterizes turbulence? Explain the characteristics. What is a turbulent eddy?

15. Explain the cascade process. How large are the largest scales? What is dissipation? What dimensions does it have? Which eddies extract energy from the mean flow? Why are these these eddies "best" at extracting energy from the mean flow?

16. What are the Kolmogorov scales? Use dimensional analysis to derive the expression for the velocity scale, v_η, the length scale, ℓ_η and the time scale, τ_η.

17. Make a figure of the energy spectrum. The energy spectrum consists of three subregions: which? Describe their characteristics. Show the flow of turbulent kinetic energy in the energy spectrum. Given the energy spectrum, $E(\kappa)$, how is the turbulent kinetic energy, k, computed? Use dimensional analysis to derive the $-5/3$ Kolmogorov law.

18. What does isotropic turbulence mean? What about the shear stresses?

19. Show how the ratio of the large eddies to the dissipative eddies depends on the Reynolds number (see Eq. 5.16). Using these estimations, you can show how the number of cells in DNS (Direct Numerical Simulations) depends on Reynolds number (see eBook).

20. Watch the on-line lecture *Turbulence* at
`http://www.tfd.chalmers.se/~lada/MoF/flow_viz.html`

i. Why does the irregular motion of wave on the sea not qualify as turbulence?

ii. How is the turbulence syndrome defined?

iii. The movie shows laminar flow in a pipe. The viscosity is decreased, and the pressure drop (i.e. the resistance, the drag, the loss) decreases. Why? The viscosity is further decreased, and the pressure drop increases. Why? How does the characteristics of the water flow coming out of the pipe change due to the second decrease of viscosity?

iv. It is usually said that the flow in a pipe gets turbulent at a Reynolds number of 2300. In the movie they show that the flow *can* remain laminar up to 8 000. How do they achieve that?

v. Dye is introduced into the pipe. For laminar flow, the dye does not mix with the water; in turbulent flow it does. When the mixing occurs, what happens with the pressure drop?

21. Watch the on-line lecture *Turbulence* (10 minutes into the movie) at
`http://www.tfd.chalmers.se/~lada/MoF/flow_viz.html`

i. Draw a laminar and turbulent velocity profile for pipe flow. What is the main difference? In which flow is the wall shear stress $\tau_w = \mu \frac{\partial \bar{v}_1}{\partial x_2}$ largest, laminar or turbulent?

ii. In turbulent flow, the velocity near the wall is larger than in laminar flow. Why?

iii. Discuss the connection between mixing and the cross-stream (i.e. v_2') fluctuations.

iv. Try to explain the increased pressure drop in turbulent flow with increased mixing.

v. The center part of the pipe is colored with blue dye and the wall region is colored with red dye: by looking at this flow, try to explain how turbulence creates a *Reynolds shear stress*.

vi. The red and blue dye nicely show the turbulent eddies (fluctuations)

vii. After 16 minutes, the flow in jets is consider. Two turbulent jet flows are shown, one at low Reynolds number and one at high Reynolds number. They look very similar in one way and very different in another way. Which scales are similar and which are different?

viii. he two turbulent jet flows have the same energy input and hence the same dissipation. Use this fact to explain why the smallest scales in the high Reynolds number jet must be smaller that those in the low Reynolds number jet.

ix. At the end of the presentation of the jet flow, they explain the *cascade process*.

x. Explain the analogy of a water fall (cascade of water, the water passes down the cascade) and the turbulent cascade process.

TME225 Learning outcomes 2019: week 6

1. Use the decomposition $v_i = \bar{v}_i + v_i'$ to derive the time-averaged Navier-Stokes equation. A new terms appears: what is it called? Simplify the time-averaged Navier-Stokes equation for boundary layer flow. What is the total shear stress?

2. How is the friction velocity, u_τ, defined? Define x_2^+ and $\bar{v}_1^+$.

3. The wall region is divided into an inner and outer region. The inner region is furthermore divided into a viscous sublayer, buffer layer and log-layer. Make a figure and show where these regions are valid (Fig. 6.2)

4. Look at *Turbulent flow around a wing* at You Tube

   (a) At 1:07 minutes into the movie, they present the flow using the λ_2 criterion. This is the second eigenvalue of $S_{ij}^2 + \Omega_{ij}^2$ where $\lambda_1 > \lambda_2 > \lambda_3$. This criterion finds vortex cores i.e. the center of turbulent eddies

   (b) At 1:18 minutes they show the prescribed transition on the upper side (suction side)

   (c) At 1:42 minutes, for example, you can see turbulent vortex core in the outer part of the boundary layer

   (d) At 1:56 minutes, the incipient (the start of) separation near the trailing edge is shown

5. What are the relevant velocity and length scales in the viscous-dominated region ($x_2^+ \lesssim 5$)? Derive the linear velocity law in this region (Eq. 6.22). What are the suitable velocity and length scales in the inertial region (the fully turbulent region)? Derive the log-law for this region.

6. Consider fully developed turbulent channel flow. In which region (viscous sublayer, buffer layer or log-layer) does the viscous stress dominate? In which region is the turbulent shear stress large? Integrate the channel flow equations and show that the total shear stress varies as $1 - x_2/\delta$ (Eq. 6.20).

7. In fully developed turbulent channel flow, the time-averaged Navier-Stokes consists only of three terms. Make a figure and show how the velocity and Reynolds shear stress vary across the channel. After that, show how the three terms vary across the channel (Fig. 6.6). Which two terms balance each other in the outer region? Which term drives ("pushes") the flow in the x_1 direction? Which two terms are large in the inner region? Which term drives the flow?

8. Derive the exact transport equation for turbulent kinetic energy, k. Discuss the physical meaning of the different terms in the k equation. Which terms are transport terms? Which is the main source term? Main sink (i.e. negative source) term?

9. Watch the on-line lecture *Turbulence* (20 minutes into the movie) at
   `http://www.tfd.chalmers.se/~lada/MoF/flow_viz.html`

   i. The movie says that there is a similarity of the small scales in a channel flow and in a jet flow. What do they mean?

ii. What happens with the small scales when the Reynolds number is increased? What happens with the large scales? Hence, how does the ratio of the large scales to the small scales change when the Reynolds number increases (see Eq. 5.16)

iii. In decaying turbulence, which scales die first? The scenes of the clouds show this in a nice way.

iv. Even though the Reynolds number may be large, there are a couple of physical phenomena which may inhibit turbulence and keep the flow laminar: mention three.

v. Consider the flow in the channel where the fluid on the top (red) and the bottom (yellow) are separated by a horizontal partition. The two fluids are identical. Study how the two fluids mix downstream of the partition. In the next example, the fluid on the top is hot (yellow) and light, and the one at the bottom (dark blue) is cold (heavy); how do the fluids mix downstream of the partition, better or worse than in the previous example? This flow situation is called *stable stratification*. In the last example, the situation is reversed: cold, heavy fluid (dark blue) is moving on top of hot, light fluid (yellow). How is the mixing affected? This flow situation is called *unstable stratification*. Compare in meteorology where heating of the ground may cause unstable stratification or when *inversion* causes stable stratification. You can read about stable/unstable stratification in Section 12.1 at p. 157.

10. Given the exact k equation, give the equation for 2D boundary-layer flow (Eq. 8.24). All spatial derivatives are kept in the dissipation term: why? In the turbulent region of the boundary layer, the k equation is dominated by two terms. Which ones? Which terms are non-zero at the wall?

11. Where is the production term, $P^k = -\overline{v_1' v_2'} \partial \bar{v}_1 / \partial x_2$, largest? In order to explain this, show how $-\overline{v_1' v_2'}$ and $\partial \bar{v}_1 / \partial x_2$ vary near the wall.

12. Derive the exact transport equation for mean kinetic energy, K. Discuss the physical meaning of the different terms. One term appears in both the k and the K equations: which one? Consider the dissipation terms in the k and the K equations: which is largest near the wall and away from the wall, respectively?

13. Which terms in the k equation need to be modeled? Explain the physical meaning of the different terms in the k equation.

14. Show how the modeled production term in the $k - \varepsilon$ model is derived. Show how it can be expressed in $\bar{s}_{ij}$

15. Show how the turbulent diffusion (i.e. the term which includes the triple correlation) in the k equation is modeled.

16. Given the modeled k equation, derive the modeled ε equation.

17. How are the Reynolds stress tensor, $\overline{v_i' v_j'}$, and the turbulent heat flux vector, $\overline{v_i' \theta'}$, modeled in the Boussinesq approach?

18. Watch the on-line lecture *Pressure field and acceleration* at `http://www.tfd.chalmers.se/~lada/MoF/flow_viz.html`

i. The water flow goes through the contraction. What happens with the velocity and pressure. Try to explain.

ii. Fluid particles become thinner and elongated in the contraction. Explain why.

iii. In the movie they show that the acceleration along s, i.e. $\dfrac{dV_s^2/2}{ds}$, is related to the pressure gradient $\dfrac{dp}{ds}$. Compare this relation with the Bernoullii equation (Eq. 4.34)

19. Watch the on-line lecture *Pressure field and acceleration* (6 minutes into the movie) at
`http://www.tfd.chalmers.se/˜lada/MoF/flow_viz.html`

i. Water flow in a manifold (a pipe with many outlets) is presented. The pressure decreases slowly downstream. Why?

ii. The bleeders (outlets) are opened. The pressure now increases in the downstream direction. Why?

iii. What is the stagnation pressure? How large is the velocity at a stagnation point?

iv. What is the static pressure? How can it be measured? What is the difference between the stagnation and the static pressures?

v. A venturi meter is a pipe that consists of a contraction and an expansion (i.e. a diffusor). The bulk velocities at the inlet and outlet are equal, but still the pressure at the outlet is lower than that at the inlet. There is a pressure drop. Why?

vi. What happens with the pressure drop when there is a separation in the diffusor?

vii. They increase the speed in the venturi meter. The pressure difference in the contraction region and the outlet increases. Since there is atmospheric pressure at the outlet, this means that the pressure in the contraction region must decrease as we increase the velocity of the water. Finally the water starts to boil, although the water temperature may be around 10^oC. This is called cavitation (this causes large damages in water turbines).

viii. Explain how suction can be created by blowing in a pipe.

20. Watch the on-line lecture *Pressure field and acceleration* (19 minutes into the movie) at
`http://www.tfd.chalmers.se/˜lada/MoF/flow_viz.html`

i. What is the Coanda effect?

ii. The water from the tap which impinges on the horizontal pipe attaches to the surface of the pipe because of the Coanda effect. How large is the pressure at the surface of the pipe relative to the surrounding pressure?

iii. Explain the relation between streamline curvature and pressure (cf. Section 3.2.1).

iv. At the end of the contraction, there is an adverse pressure gradient ($\partial p/\partial x > 0$). Explain why.

TME225 Learning outcomes 2019: week 7

1. Two options are used for treating the wall boundary conditions: which ones? Explain the main features.

2. Consider wall functions. Show how the expression

$$u_\tau = \frac{\kappa \bar{v}_{1,P}}{\ln(E u_\tau \delta x_2/\nu)}$$

is obtained. What is the wall boundary condition for the velocity equation?

3. How is the k equation simplified in the log-law region? Show how the boundary condition

$$k_P = C_\mu^{-1/2} u_\tau^2$$

for k is derived (wall functions).

4. Show how the boundary condition for ε

$$\varepsilon_P = P^k = \frac{u_\tau^3}{\kappa \delta x_2}$$

is derived (wall functions).

5. How fine should the grid be near the wall when using a low-Reynolds number model? Why must the turbulence model be modified?

6. Use Taylor expansion (including boundary conditions and the continuity equation) to show how the three velocity components vary near the wall. Show then how $\bar{v}$, $\bar{v}$, $\overline{v_1'^2}$, $\overline{v_1'v_2'}$, ε, $\partial \bar{v}_1/\partial x_2$ and k vary near the wall.

7. The exact k equation reads

$$\rho \bar{v}_1 \frac{\partial k}{\partial x_1} + \rho \bar{v}_2 \frac{\partial \rho k}{\partial x_2} = -\rho \overline{v_1' v_2'} \frac{\partial \bar{v}_1}{\partial x_2} - \frac{\partial \overline{p' v_2'}}{\partial x_2} - \frac{\partial}{\partial x_2}\left(\frac{1}{2}\rho \overline{v_2' v_i' v_i'}\right) + \mu \frac{\partial^2 k}{\partial x_2^2} - \mu \overline{\frac{\partial v_i'}{\partial x_j}\frac{\partial v_i'}{\partial x_j}}$$

Show how the production term, the viscous and turbulent diffusion terms and the dissipation vary near the wall.

8. The modeled k eq. reads

$$\rho \bar{v}_1 \frac{\partial k}{\partial x_1} + \rho \bar{v}_2 \frac{\partial \rho k}{\partial x_2} = \mu_t \left(\frac{\partial \bar{v}_1}{\partial x_2}\right)^2 + \frac{\partial}{\partial x_2}\left(\frac{\mu_t}{\sigma_k}\frac{\partial k}{\partial x_2}\right) + \mu \frac{\partial^2 k}{\partial x_2^2} - \rho \varepsilon$$

Show how the production term, the turbulent diffusion term and the dissipation vary near the wall.

9. Looking at how the exact and the modelled terms in the k behave near walls, which terms need to modified? How?

10. The modeled ε eq. reads

$$\rho\bar{v}_1\frac{\partial\varepsilon}{\partial x_1}+\rho\bar{v}_2\frac{\partial\varepsilon}{\partial x_2}=C_{\varepsilon 1}\frac{\varepsilon}{k}P^k+\frac{\partial}{\partial x_2}\left(\frac{\mu_t}{\sigma_\varepsilon}\frac{\partial\varepsilon}{\partial x_2}\right)$$
$$+\mu\frac{\partial^2\varepsilon}{\partial x_2^2}-C_{\varepsilon 2}\rho\frac{\varepsilon^2}{k}$$

Show how all terms behave as the wall is approached. Which terms do not go to zero. Do they balance each other? If not, what modification is needed?

11. In low-Reynolds number models, what is the boundary condition for k?

12. A boundary condition for ε can be derived (Eq. 11.165) by looking at the two terms in the k eq. that do not go to zero. Show this boundary condition.

13. Another boundary condition for ε

$$\varepsilon_{wall}=2\nu\left(\frac{\partial\sqrt{k}}{\partial x_2}\right)^2$$

can be derived by using Taylor expansion. Derive the boundary condition above. A third b.c. for ε reads

$$\varepsilon_{wall}=2\nu\frac{k}{x_2^2}$$

Show that this agrees with Taylor expansion.