Coupling of VOF with LPT to improve cavitation modeling

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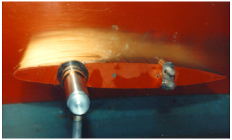
- Experiments



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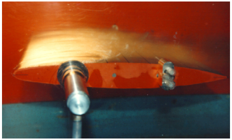

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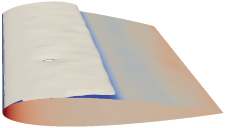

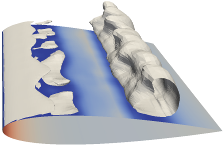

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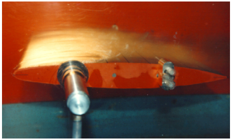

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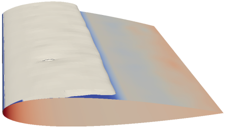
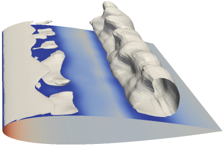
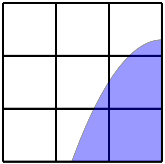




- Improvement
  Model the small bubbles which are present after the breakup of the attached cavity.

Mass transfer cavitation model
based on VOF (Volume Of Fluid)



- Bubbles larger than the grid size
- High vapor volume fraction
- Irregular structures: need to describe the interface

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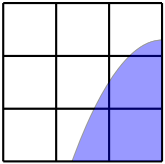

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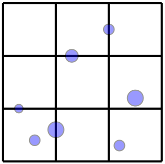

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- Shape can be considered spherical

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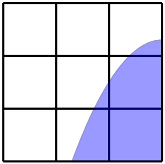

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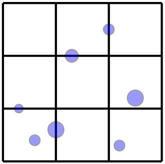

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Coupling of Eulerian and Lagrangian models : multi-scale model

Liquid volume fraction $\alpha \in [0, 1]$.

$$\rho = \alpha\rho_g + (1 - \alpha)\rho_l,$$
$$\mu = \alpha\mu_g + (1 - \alpha)\mu_l,$$

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Transport equation for the vapor volume fraction

$$\frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha \mathbf{U}) + \nabla \cdot (\alpha(1-\alpha)\mathbf{U_r}) = S_\alpha$$

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Mass and momentum equations for the mixture

$$\nabla \cdot \mathbf{U} = 0,$$

$$\frac{\partial \rho \mathbf{U}}{\partial t} + \nabla \cdot (\rho \mathbf{U} \otimes \mathbf{U}) = -\nabla p + \mu \nabla^2 \mathbf{U} + \rho \mathbf{g} - \mathbf{S_{st}} + \mathbf{S_P}.$$

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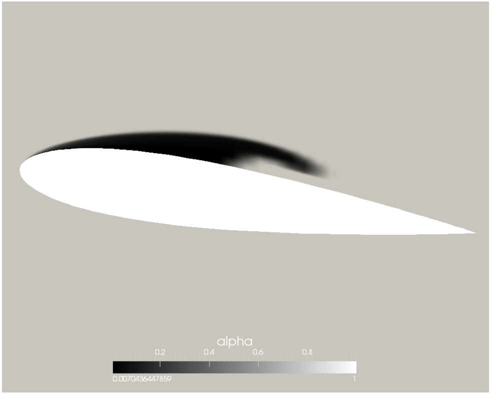
$$\mathbf{S_{st}} = \sigma_{st}\kappa\delta\mathbf{n}\ ,\ \mathbf{n} = \frac{\nabla\alpha}{|\nabla\alpha|}\ ,\ \kappa = \nabla\mathbf{n}.$$

Results with the mass transfer cavitation model



Particle P: position $\mathbf{x_P}$, diameter D_P, velocity $\mathbf{U_P}$ and density ρ_P.

$$\frac{d\mathbf{x_P}}{dt} = \mathbf{U_P},$$

$$m_P \frac{d\mathbf{U_P}}{dt} = \sum \mathbf{F}.$$

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Two-way coupling:

$$\mathbf{S_P} = \frac{-1}{V_{cell}\Delta t} \sum_P m_P((\mathbf{U_p})_{t_{out}} - (\mathbf{U_p})_{t_{in}})$$

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Four-way coupling: collision and coalescence.

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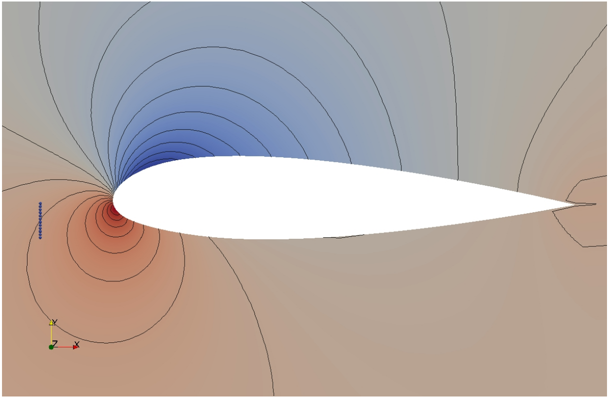
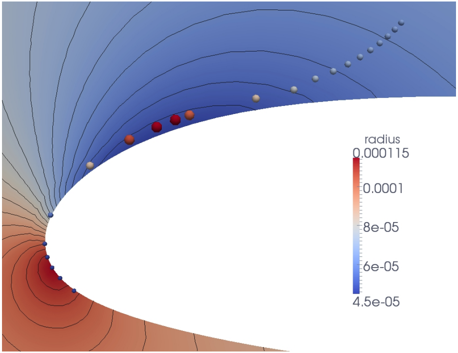
Four-way coupling: collision and coalescence.
Bubble dynamics (Rayleigh-Plesset equation):

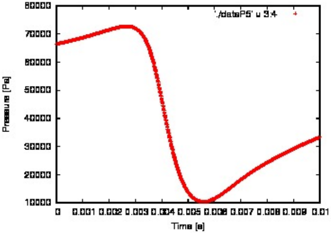
$$\frac{p_B - p_\infty}{\rho} = R\frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right)^2 + \frac{4\mu}{R}\frac{dR}{dt} + \frac{2\sigma_{st}}{\rho R}.$$

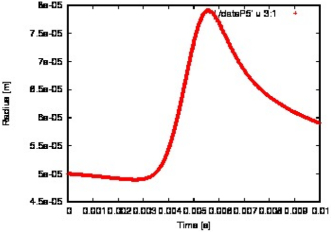
Injection of bubbles, initial radius 50 μm.



Results with the bubble cavitation model







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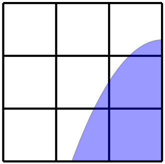

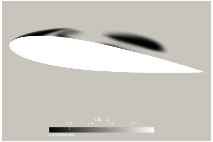

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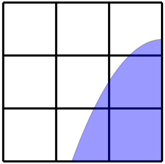
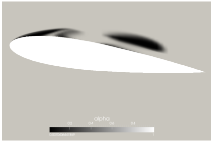
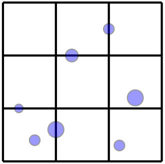
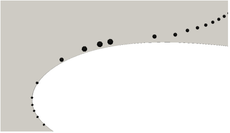




Bubble cavitation model based on LPT (Lagrangian Particle Tracking)





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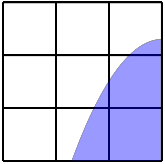
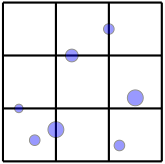
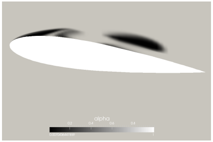
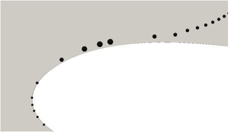


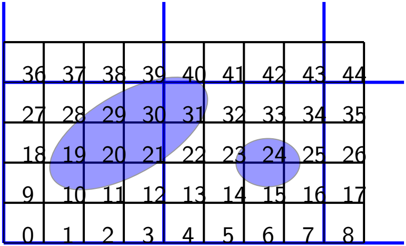






1. **Coupling of Eulerian and Lagrangian models : multi-scale model**
2. **Identify the small bubbles suitable for LPT**

- Eulerian grid, h
- Lagrangian grid, $h_{LAG} = 4h$



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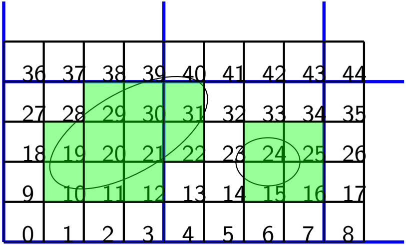
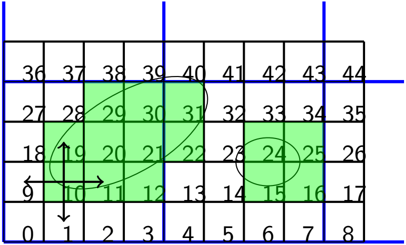


- Mark the cells with $\alpha < 0.95$ (i.e. at least 5% of gas.)

- Eulerian grid, h
- Lagrangian grid, $h_{LAG} = 4h$



- Mark the cells with $\alpha < 0.95$ (i.e. at least 5% of gas.)

Cell Number	Bubble ID
10	0

- Eulerian grid, h
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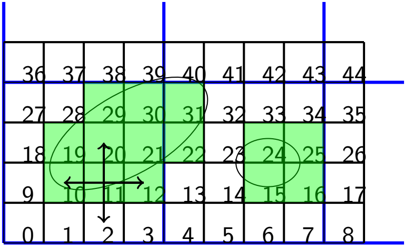
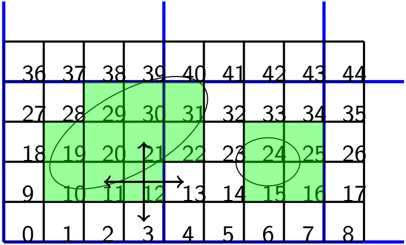


- Mark the cells with $\alpha < 0.95$ (i.e. at least 5% of gas.)

Cell Number	Bubble ID
10	0
11	0

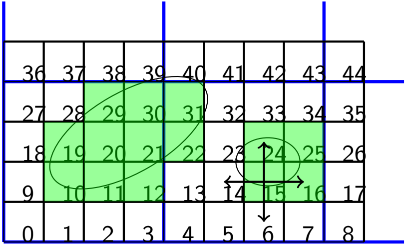
- Eulerian grid, h
- Lagrangian grid, $h_{LAG} = 4h$



- Mark the cells with $\alpha < 0.95$ (i.e. at least 5% of gas.)

Cell Number	Bubble ID
10	0
11	0
12	0

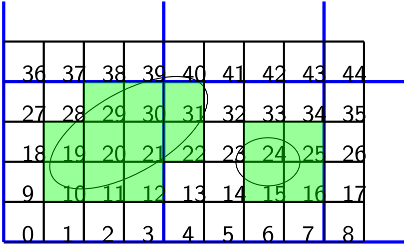
- Eulerian grid, h
- Lagrangian grid, $h_{LAG} = 4h$



- Mark the cells with $\alpha < 0.95$ (i.e. at least 5% of gas.)

Cell Number	Bubble ID
10	0
11	0
12	0
15	1

- Eulerian grid, h
- Lagrangian grid, $h_{LAG} = 4h$



- Mark the cells with $\alpha < 0.95$ (i.e. at least 5% of gas.)

Cell Number	Bubble ID
10	0
11	0
12	0
15	1
16	1
19	0
20	0
21	0
22	0
24	1
25	1
29	0
30	0
31	0

- Eulerian grid, h
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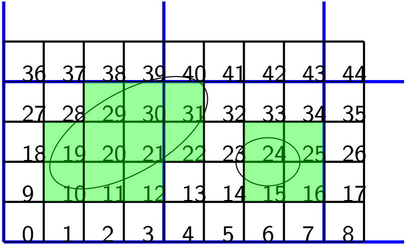

$$V_P = \int_{\Omega} (1-\alpha)\, d\Omega = \sum_{cell@ID} (1-\alpha)\, V_{cell@ID}$$

- Mark the cells with $\alpha < 0.95$ (i.e. at least 5% of gas.)

Cell Number	Bubble ID
10	0
11	0
12	0
15	1
16	1
19	0
20	0
21	0
22	0
24	1
25	1
29	0
30	0
31	0

- Eulerian grid, h
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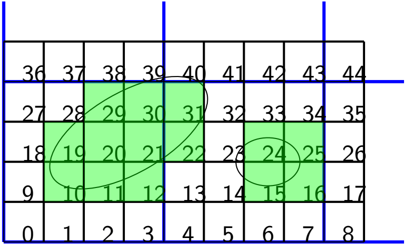

$$V_P = \int_\Omega (1-\alpha)\, d\Omega = \sum_{cell@ID} (1-\alpha)\, V_{cell@ID}$$

$$\mathbf{x_P} = \frac{1}{V_P} \sum_{cell@ID} (1-\alpha)\, \mathbf{x}_{cell@ID}$$

- Mark the cells with $\alpha < 0.95$ (i.e. at least 5% of gas.)

Cell Number	Bubble ID
10	0
11	0
12	0
15	1
16	1
19	0
20	0
21	0
22	0
24	1
25	1
29	0
30	0
31	0

- Eulerian grid, h
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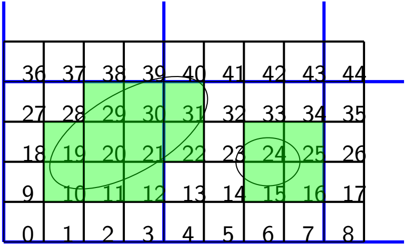

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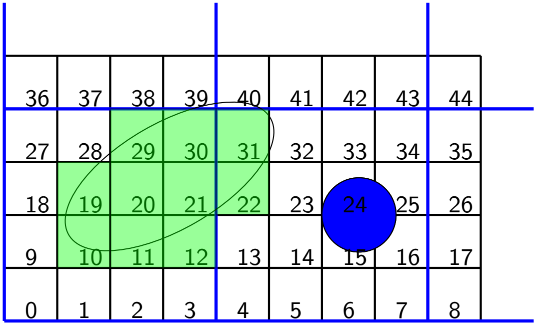
$$\mathbf{x_P} = \frac{1}{V_P} \sum_{cell@ID} (1-\alpha)\, \mathbf{x}_{cell@ID}$$

$$V_P < 10\% V_{LAG}?$$

- Mark the cells with $\alpha < 0.95$ (i.e. at least 5% of gas.)

Cell Number	Bubble ID
10	0
11	0
12	0
15	1
16	1
19	0
20	0
21	0
22	0
24	1
25	1
29	0
30	0
31	0

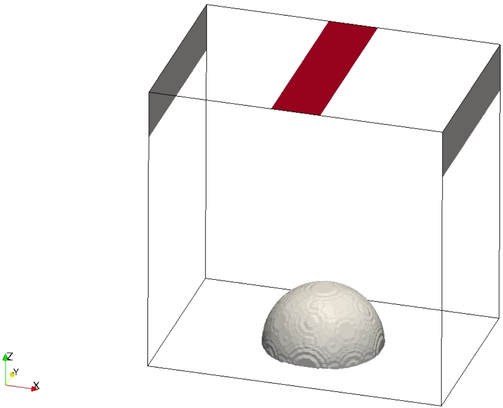


- Equivalent spherical bubble determined from the irregular vapor structure

$$D_P = (\frac{6}{\pi} V_P)^{1/3}$$

- Create the point particle $P(D_P, \mathbf{x_P}, \mathbf{U_P})$ for the LPT simulation
- Remove the bubble from the VOF simulation : α set to 1

Test case 1



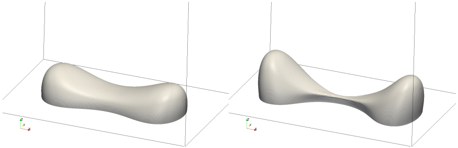


Figure: VOF simulation

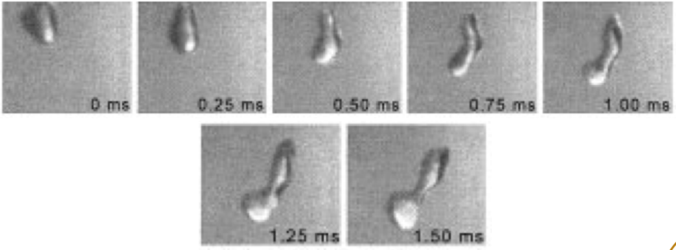
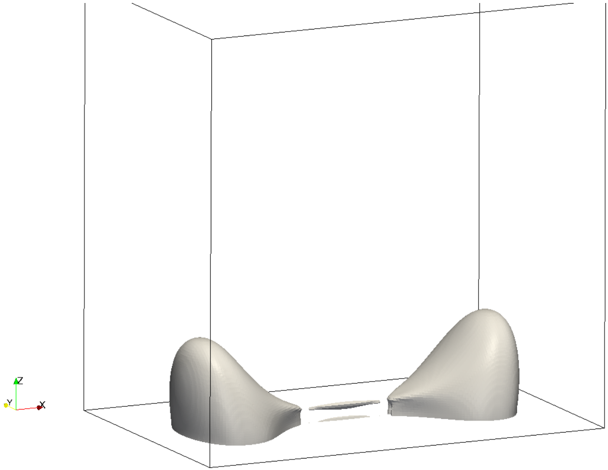
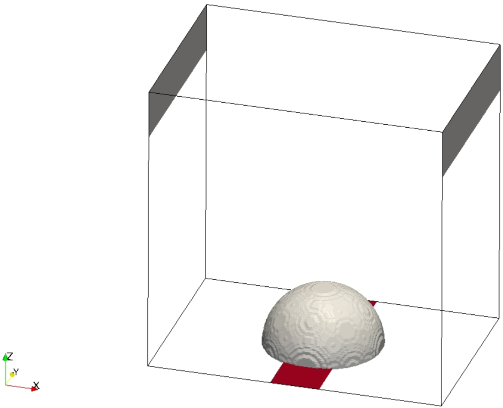


Figure: Andersson R. and Andersson B., *Modeling the breakup of particles in turbulent flows, AIChE Journal 2006.*

Test case 1



Test case 2



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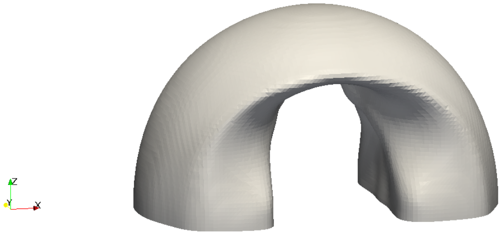
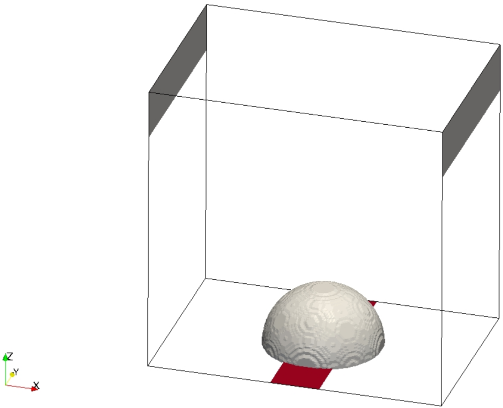


Test case 3



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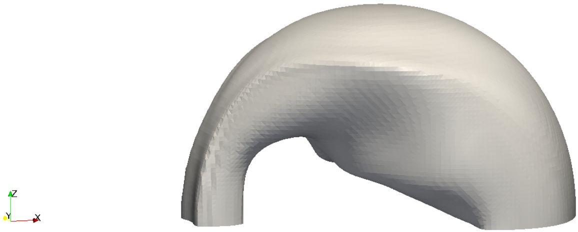
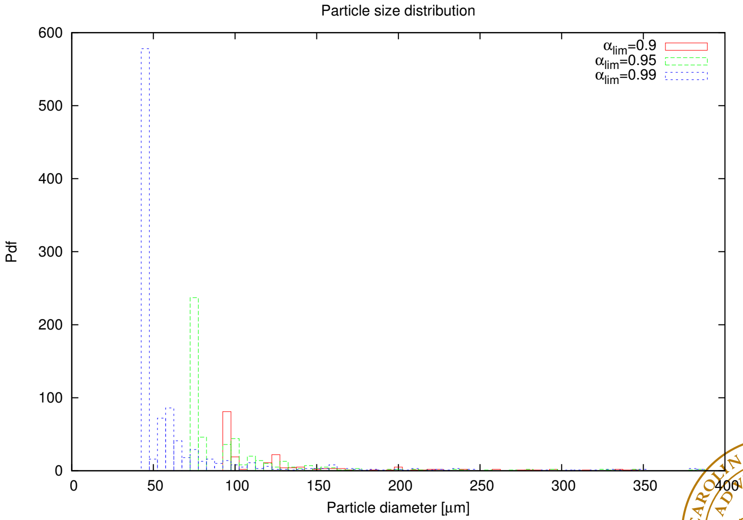
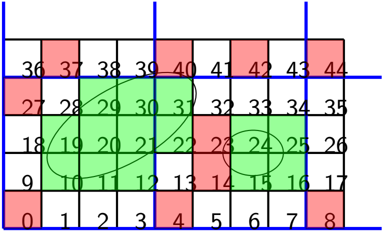
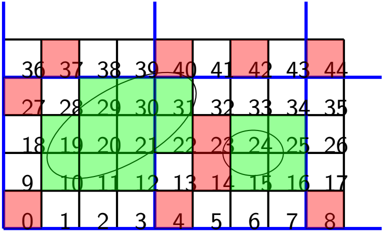



Particle size distribution
Pdf
Particle diameter [µm]
αlim=0.9
αlim=0.95
αlim=0.99
0
100
200
300
400
500
600
0
50
100
150
200
250
300
350
400

Mark the cells with $\alpha <$ ~~0.95~~ 0.99



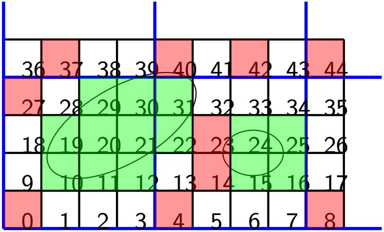
Mark the cells with $\alpha <$ ~~0.95~~ 0.99



Cell Number	Bubble ID
0	0
4	1
8	2

- More small bubbles are captured (The "isolated" cells with $\alpha \in [0.95, 0.99[$ are not neglected anymore)

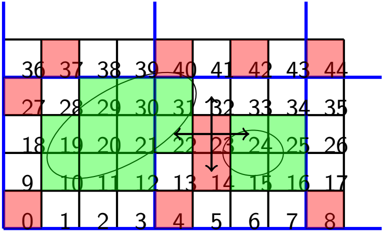
Mark the cells with $\alpha <$ ~~0.95~~ 0.99



Cell Number	Bubble ID
0	0
4	1
8	2
10	3
11	3
12	3
14	4
15	4
16	4
19	3
20	3
21	3
22	3
23	

- More small bubbles are captured (The "isolated" cells with $\alpha \in [0.95, 0.99[$ are not neglected anymore)

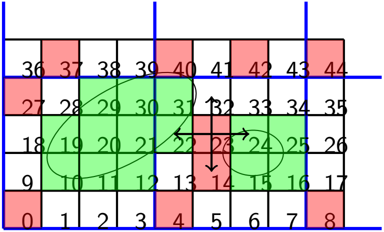
Mark the cells with $\alpha <$ ~~0.95~~ 0.99



Cell Number	Bubble ID
0	0
4	1
8	2
10	3
11	3
12	3
14	~~4~~ 3
15	~~4~~ 3
16	~~4~~ 3
19	3
20	3
21	3
22	3
23	3

- More small bubbles are captured (The "isolated" cells with $\alpha \in [0.95, 0.99[$ are not neglected anymore)

Mark the cells with $\alpha <$ ~~0.95~~ 0.99



Cell Number	Bubble ID
0	0
4	1
8	2
10	3
11	3
12	3
14	~~4~~ 3
15	~~4~~ 3
16	~~4~~ 3
19	3
20	3
21	3
22	3
23	3

- More small bubbles are captured (The "isolated" cells with $\alpha \in [0.95, 0.99[$ are not neglected anymore)
- Less medium and large bubbles are captured (The "connecting" cells with $\alpha \in [0.95, 0.99[$ merge bubbles and the condition $V_P < 10\% V_{LAG}$ isn't satisfied anymore)

- Implementation of a multi-scale model.
  - VOF model complemented with a four-way coupling LPT model.
  - Identification of the small bubbles suitable for LPT approach.
- Model applied to simple cases of air bubbles breaking up under the impact of a water jet.
  - Ability to capture and track the small bubbles which a pure VOF model fails to describe.
  - Particles size and distribution depend on the parameters introduced.

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Onging work : test on a cavitating case of the complete model.

- Include mass transfer with the VOF approach.
- Include the description of the small bubbles dynamics with the LPT approach.
- Convert a particle back to the VOF approach when
  - it comes near an isosurface $\alpha = 0.5$ or
  - it is not suitable for LPT anymore (coalescence)