Electric welding arc modeling with the 3D solver OpenFOAM

- A comparison of different electromagnetic models -

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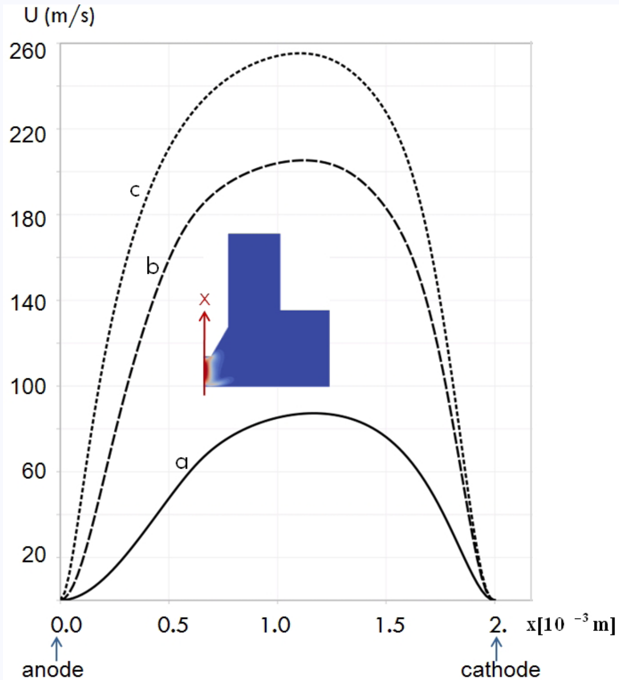
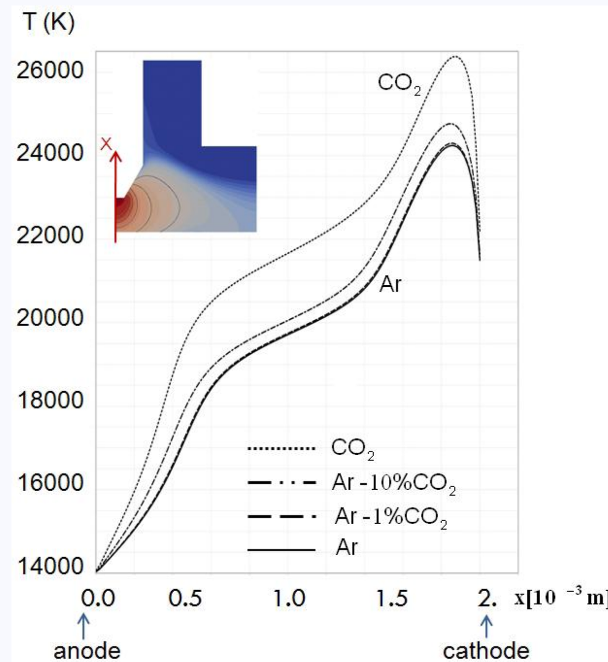
- Context / Motivation:
  better understand the heat source

- Software OpenFOAM-1.6.x
  - open source CFD software
  - C++ library of object-oriented classes for implementing solvers for continuum mechanics

IIW intermediate meeting, Trollhättan, Doc. XII-2017-11

" *Numerical simulation of Ar-x%CO_2 shielding gas and its effect on an electric welding arc*"





Influence of BC on anode an cathode *Influence of gas composition*

Here focus on a comparison of different electromagnetic models

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Model: thermal fluid part

Main assumptions (plasma core):

- one-fluid model
- local thermal equilibrium
- mechanically incompressible and thermally expansible
- *steady flow*
- *laminar flow(assuming laminar shielding gas inlet)*

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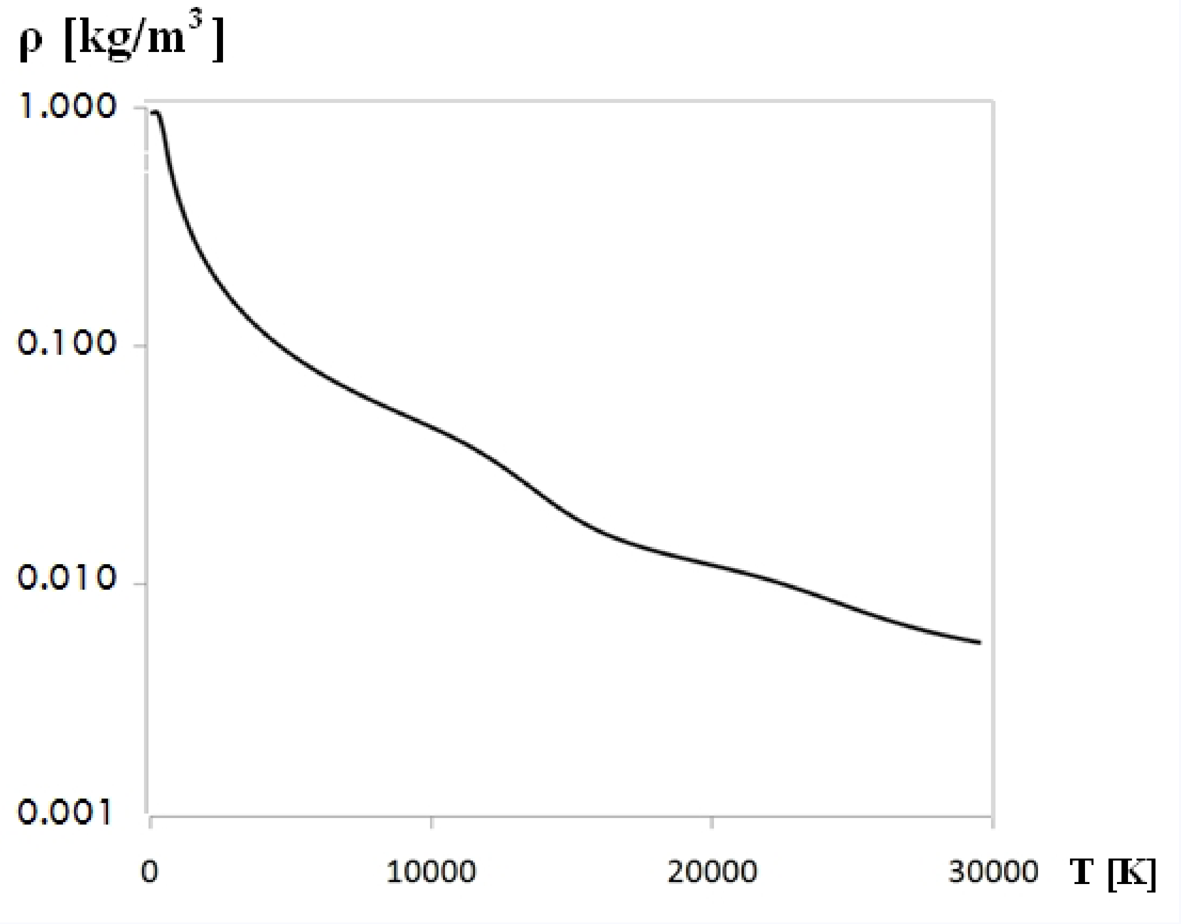

Argon plasma density as function of temperature.

Model: thermal fluid part

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Model: electromagnetic part

- 3D model with
- 2D axi-symmetric models:
  - ✓Electric potential formulation
  - ✓Magnetic field formulation

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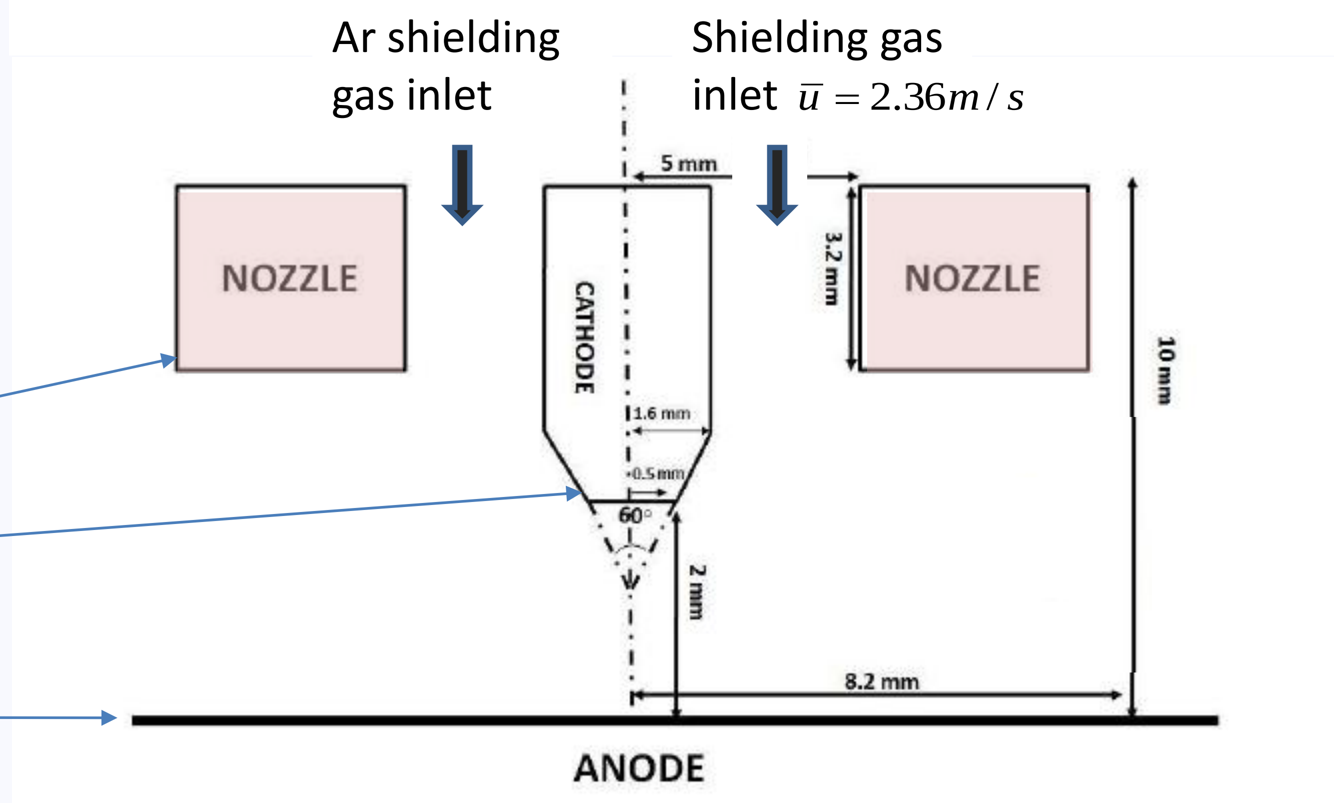
M.A. Ramírez , G. Trapaga and J. McKelliget (2003). A comparison between two different numerical formulations of welding arc simulation. *Modelling Simul. Mater. Sci. Eng*, **11**, pp. 675-695.

Test case: Tungsten Inert Gas welding

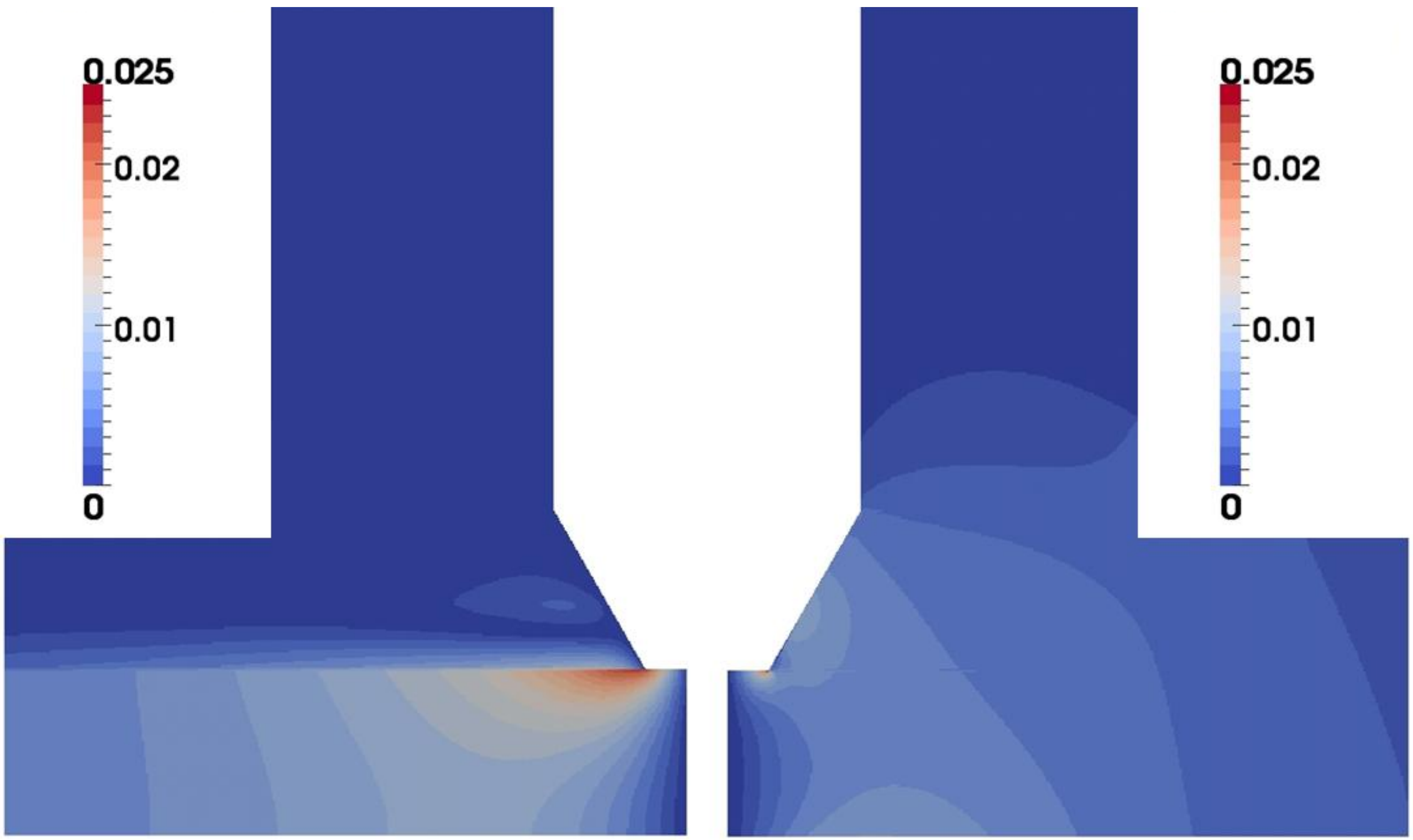
Applied current: I=200A

Picture of a TIG torch



Sketch of the cross section of a TIG torch

$$B_\theta = \frac{\mu_0}{r}\int_0^r J_{axial}(l)\; l\; dl \qquad\qquad B_\theta = (\nabla \times \vec{A})_\theta$$



Magnetic field magnitude calculated with the electric potential formulation (left) and the 3D

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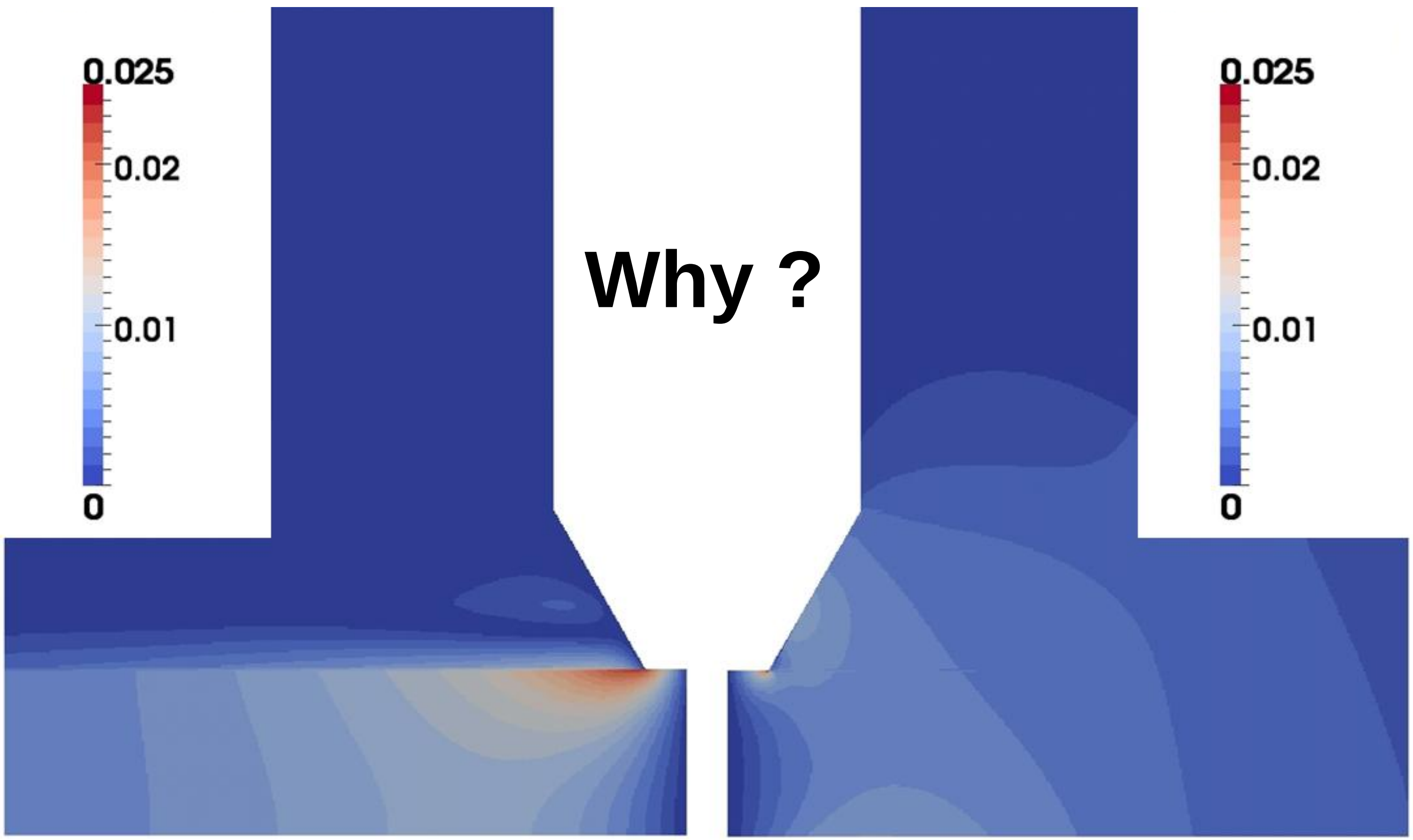

Magnetic field magnitude calculated with the electric potential formulation (left) and the 3D

Maxwell's equations

- Gauss' law for magnetism: $\nabla \cdot \vec{B} = 0$
- Ampère's law: $\varepsilon_0 \mu_0 \partial_t \vec{E} = \nabla \times \vec{B} - \mu_0 \vec{J}$
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Assumptions (plasma core)

- $\lambda_{Debye} \approx 10^{-8} m << L_c$ ⟹ local electro-neutrality
- quasi-steady electromagnetic phenomena

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  $\beta_{Lar} = \nu_{Lar} / \nu_{coll} << 1$ ⟹ $\vec{J}_{Hall} = \frac{-\sigma}{n_e} \vec{J} \times \vec{B} << \vec{J}_{drift} = \sigma \vec{E}$
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Electromagnetic model for arc plasma core

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(1) with Lorentz gauge : $\nabla \cdot \vec{A} = 0$

Electromagnetic model for arc plasma core (1)

$$\nabla \cdot \left(\sigma \nabla V\right) = 0$$

$$\Delta \vec{A} = \mu_0 \sigma \nabla V$$

with

$$\vec{J} = \sigma \vec{E} = -\sigma \nabla V$$

$$\vec{B} = \nabla \times \vec{A}$$

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Electromagnetic model for arc plasma core [1]

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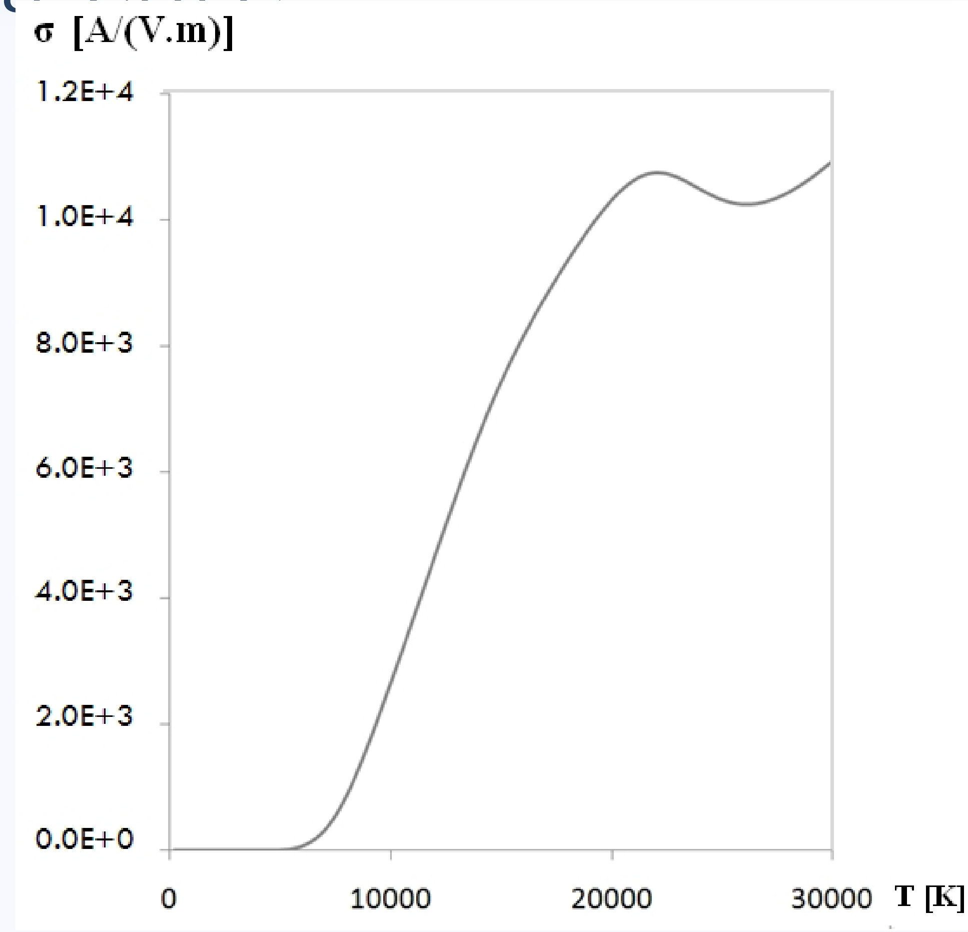

Argon plasma electric conductivity as function of temperature

(1) with Lorentz gauge : $\nabla \cdot \vec{A} = 0$

2D axi-symmetric case

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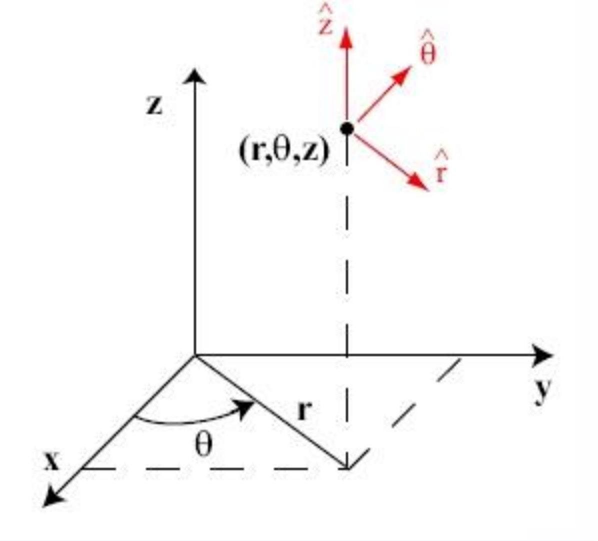

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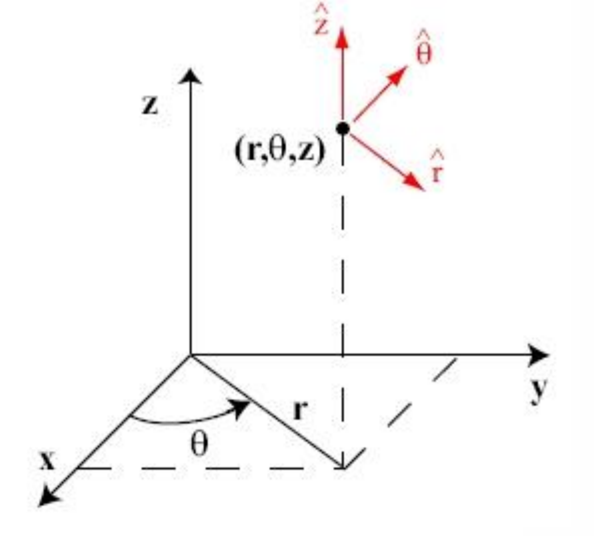

Then

$\partial_\theta \rightarrow \mathbf{0}$

with

with

with

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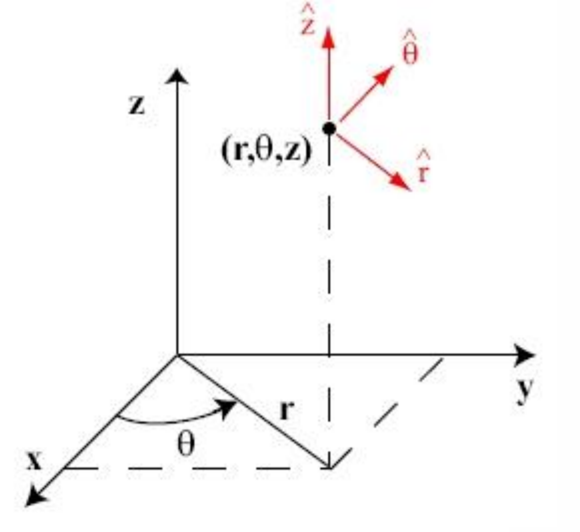

Then

$\partial_\theta \rightarrow$ **0**

$V(r,z)$

with

with

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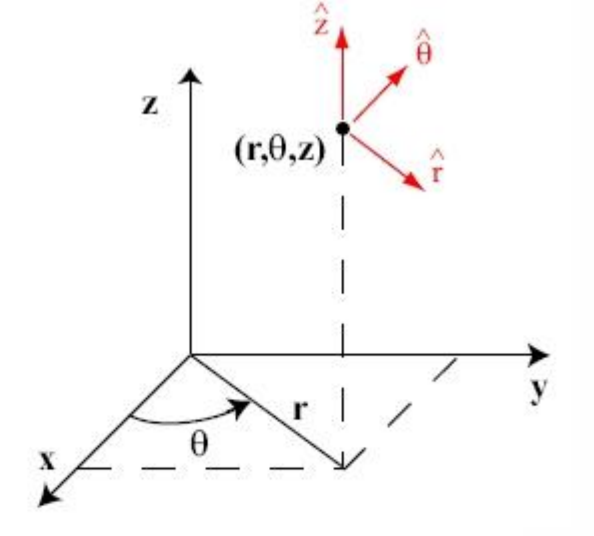
2D axi-symmetric case (1)



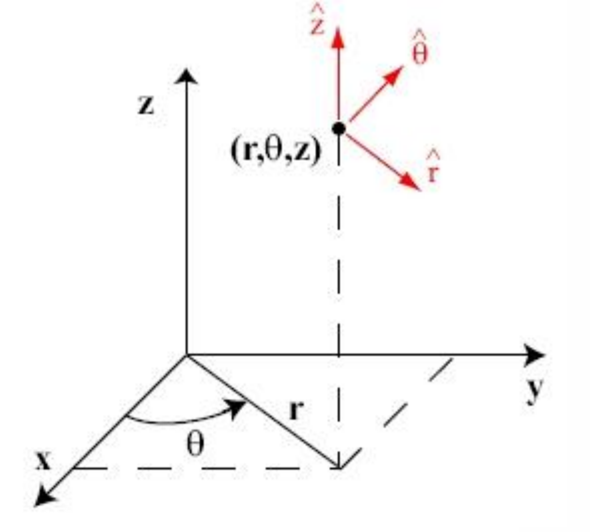
Then

$$\partial_\theta \rightarrow 0$$

$$V(r,z)$$

$$\vec{A} = (A_r\,, 0\,, A_z)$$ with $$A_r(r,z),\ A_z(r,z)$$

2D axi-symmetric case (1)



Then

$$\partial_\theta \rightarrow 0$$

$$V(r,z)$$

$$\vec{A}=(A_r\,,0\,,A_z)$$ with $$A_r(r,z),\ A_z(r,z)$$

$$\vec{J}=(J_r,0,\,J_z)$$ with $$J_r(r,z),\ J_z(r,z)$$

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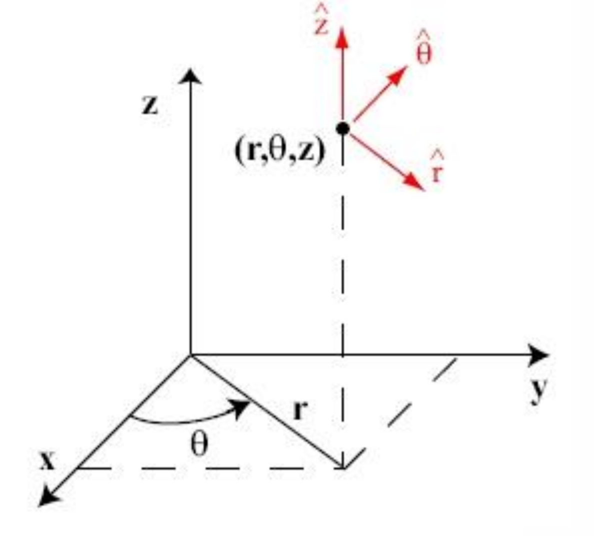


Then

$$\partial_\theta \to 0$$

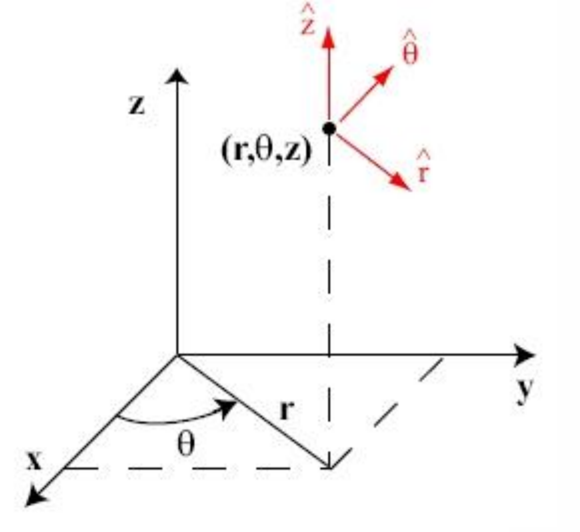
$$V(r,z)$$

$$\vec{A} = (A_r, 0, A_z) \quad \text{with} \quad A_r(r,z),\ A_z(r,z)$$

$$\vec{J} = (J_r, 0, J_z) \quad \text{with} \quad J_r(r,z),\ J_z(r,z)$$

$$\vec{B} = (0, B_\theta, 0) \quad \text{with} \quad B_\theta(r,z)$$

2D axi-symmetric case [1]



$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\sigma\frac{\partial V}{\partial r}\right)+\frac{\partial}{\partial z}\left(\sigma\frac{\partial V}{\partial z}\right)=0$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_r}{\partial r}\right)+\frac{\partial^2 A_r}{\partial z^2}-\frac{A_r}{r^2}=\mu_0\sigma\frac{\partial V}{\partial r}$$

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$\vec{J}$: $J_r=\sigma E_r=-\sigma\frac{\partial V}{\partial r}$ $\quad J_\theta=0$ $\quad J_z=\sigma E_z=-\sigma\frac{\partial V}{\partial z}$

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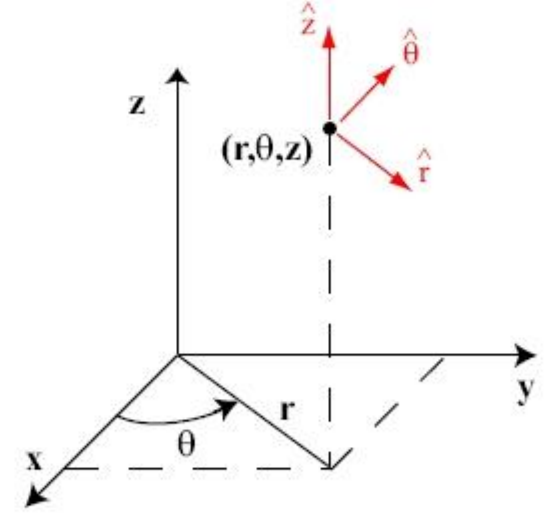

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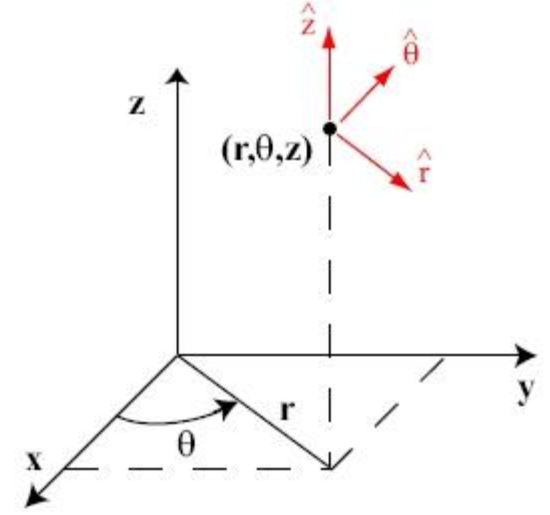

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2D axi-symmetric case [(1)]

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_r}{\partial r}\right)+\frac{\partial^2 A_r}{\partial z^2}-\frac{A_r}{r^2}=\mu_0\sigma\frac{\partial V}{\partial r}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_z}{\partial r}\right)+\frac{\partial^2 A_z}{\partial z^2}=\mu_0\sigma\frac{\partial V}{\partial z}$$

$\overset{(1)}{\Longleftrightarrow}$

$$\frac{\partial B_\theta}{\partial z}=\mu_0\sigma\frac{\partial V}{\partial r}=-\mu_0 J_r$$

$$\frac{1}{r}\frac{\partial (rB_\theta)}{\partial r}=-\mu_0\sigma\frac{\partial V}{\partial z}=\mu_0 J_z$$

(1) with Lorentz gauge : $\nabla\cdot\vec{A}=0$

2D axi-symmetric case (1)

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_r}{\partial r}\right)+\frac{\partial^2 A_r}{\partial z^2}-\frac{A_r}{r^2}=\mu_0\sigma\frac{\partial V}{\partial r}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_z}{\partial r}\right)+\frac{\partial^2 A_z}{\partial z^2}=\mu_0\sigma\frac{\partial V}{\partial z}$$

(1) ⇔

$$\frac{\partial B_\theta}{\partial z}=\mu_0\sigma\frac{\partial V}{\partial r}=-\mu_0 J_r$$

$$\frac{1}{r}\frac{\partial(rB_\theta)}{\partial r}=-\mu_0\sigma\frac{\partial V}{\partial z}=\mu_0 J_z$$

(2)

Induction diffusion equation

$$\frac{\partial}{\partial r}\left(\frac{1}{\sigma r}\frac{\partial(rB_\theta)}{\partial r}\right)+\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\frac{\partial B_\theta}{\partial z}\right)=0$$

(1) with Lorentz gauge : $\nabla\cdot\vec{A}=0$

(2) as $\partial^2_{r,z}V=\partial^2_{z,r}V$

2D axi-symmetric case (1) – equivalent formulations

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_r}{\partial r}\right)+\frac{\partial^2 A_r}{\partial z^2}-\frac{A_r}{r^2}=\mu_0\sigma\frac{\partial V}{\partial r}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_z}{\partial r}\right)+\frac{\partial^2 A_z}{\partial z^2}=\mu_0\sigma\frac{\partial V}{\partial z}$$

(1) ⟺

$$\frac{\partial B_\theta}{\partial z}=\mu_0\sigma\frac{\partial V}{\partial r}=-\mu_0 J_r$$

$$\frac{1}{r}\frac{\partial(rB_\theta)}{\partial r}=-\mu_0\sigma\frac{\partial V}{\partial z}=\mu_0 J_z$$

(2)

Induction diffusion equation

$$\frac{\partial}{\partial r}\left(\frac{1}{\sigma r}\frac{\partial(rB_\theta)}{\partial r}\right)+\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\frac{\partial B_\theta}{\partial z}\right)=0$$

(2)

$$B_\theta(r,z)=\int_{l=z_0}^{l=z} J_z(r,l)\,dl + B_\theta(r,z_0)$$

(1) with Lorentz gauge : $\nabla\cdot\vec{A}=0$

(2) as $\partial^2_{r,z}V=\partial^2_{z,r}V$

2D axi-symmetric (1) case – equivalent formulations

F1

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_r}{\partial r}\right)+\frac{\partial^2 A_r}{\partial z^2}-\frac{A_r}{r^2}=\mu_0\sigma\frac{\partial V}{\partial r}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_z}{\partial r}\right)+\frac{\partial^2 A_z}{\partial z^2}=\mu_0\sigma\frac{\partial V}{\partial z}$$

(1)

$$\frac{\partial B_\theta}{\partial z}=\mu_0\sigma\frac{\partial V}{\partial r}=-\mu_0 J_r$$

$$\frac{1}{r}\frac{\partial(rB_\theta)}{\partial r}=-\mu_0\sigma\frac{\partial V}{\partial z}=\mu_0 J_z$$

(2)

Induction diffusion equation

F2

$$\frac{\partial}{\partial r}\left(\frac{1}{\sigma r}\frac{\partial(rB_\theta)}{\partial r}\right)+\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\frac{\partial B_\theta}{\partial z}\right)=0$$

(2)

F3

$$B_\theta(r,z)=\int_{l=z_0}^{l=z} J_z(r,l)\ dl\ +\ B_\theta(r,z_0)$$

(1) with Lorentz gauge : $\nabla\cdot\vec{A}=0$

(2) as $\partial^2_{r,z}V=\partial^2_{z,r}V$

2D axi-symmetric case – equivalent formulations

F1

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_r}{\partial r}\right)+\frac{\partial^2 A_r}{\partial z^2}-\frac{A_r}{r^2}=\mu_0\sigma\frac{\partial V}{\partial r}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_z}{\partial r}\right)+\frac{\partial^2 A_z}{\partial z^2}=\mu_0\sigma\frac{\partial V}{\partial z}$$

(1)

$$\frac{\partial B_\theta}{\partial z}=\mu_0\sigma\frac{\partial V}{\partial r}=-\mu_0 J_r$$

$$\frac{1}{r}\frac{\partial(rB_\theta)}{\partial r}=-\mu_0\sigma\frac{\partial V}{\partial z}=\mu_0 J_z$$

(2)

F2

Induction diffusion equation

$$\frac{\partial}{\partial r}\left(\frac{1}{\sigma r}\frac{\partial(rB_\theta)}{\partial r}\right)+\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\frac{\partial B_\theta}{\partial z}\right)=0$$

(2)

F3

$$B_\theta(r,z)=\int_{l=z_0}^{l=z} J_z(r,l)\;dl\;+\;B_\theta(r,z_0)$$

(1) with Lorentz gauge : $\nabla\cdot\vec{A}=0$

(2) as $\partial^2_{r,z}V=\partial^2_{z,r}V$

2D axi-symmetric case (1) – equivalent formulations

F1

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_r}{\partial r}\right)+\frac{\partial^2 A_r}{\partial z^2}-\frac{A_r}{r^2}=\mu_0\sigma\frac{\partial V}{\partial r}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_z}{\partial r}\right)+\frac{\partial^2 A_z}{\partial z^2}=\mu_0\sigma\frac{\partial V}{\partial z}$$

(1) ⇔

$$\frac{\partial B_\theta}{\partial z}=\mu_0\sigma\frac{\partial V}{\partial r}=-\mu_0 J_r$$

$$\frac{1}{r}\frac{\partial(rB_\theta)}{\partial r}=-\mu_0\sigma\frac{\partial V}{\partial z}=\mu_0 J_z$$

(2) ↓

F2

Induction diffusion equation

$$\frac{\partial}{\partial r}\left(\frac{1}{\sigma r}\frac{\partial(rB_\theta)}{\partial r}\right)+\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\frac{\partial B_\theta}{\partial z}\right)=0$$

(2) ↓

F3

$$B_\theta(r,z)=\int_{l=z_0}^{l=z} J_z(r,l)\ dl\ +\ B_\theta(r,z_0)$$

(1) with Lorentz gauge : $\nabla\cdot\vec{A}=0$

(2) as $\partial^2_{r,z}V=\partial^2_{z,r}V$

2D axi-symmetric case – equivalent formulations

F1

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_r}{\partial r}\right)+\frac{\partial^2 A_r}{\partial z^2}-\frac{A_r}{r^2}=\mu_0\sigma\frac{\partial V}{\partial r}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_z}{\partial r}\right)+\frac{\partial^2 A_z}{\partial z^2}=\mu_0\sigma\frac{\partial V}{\partial z}$$

(1)

$$\frac{\partial B_\theta}{\partial z}=\mu_0\sigma\frac{\partial V}{\partial r}=-\mu_0 J_r$$

$$\frac{1}{r}\frac{\partial(rB_\theta)}{\partial r}=-\mu_0\sigma\frac{\partial V}{\partial z}=\mu_0 J_z$$

(2)

F2

Induction diffusion equation

$$\frac{\partial}{\partial r}\left(\frac{1}{\sigma r}\frac{\partial(rB_\theta)}{\partial r}\right)+\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\frac{\partial B_\theta}{\partial z}\right)=0$$

(2)

F3

$$B_\theta(r,z)=\int_{l=z_0}^{l=z} J_z(r,l)\,dl + B_\theta(r,z_0)$$

(1) with Lorentz gauge : $\nabla\cdot\vec{A}=0$

(2) as $\partial^2_{r,z}V=\partial^2_{z,r}V$

Magnetic field formulation

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_r}{\partial r}\right)+\frac{\partial^2 A_r}{\partial z^2}-\frac{A_r}{r^2}=\mu_0\sigma\frac{\partial V}{\partial r}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_z}{\partial r}\right)+\frac{\partial^2 A_z}{\partial z^2}=\mu_0\sigma\frac{\partial V}{\partial z}$$

(1)

$$\frac{\partial B_\theta}{\partial z}=\mu_0\sigma\frac{\partial V}{\partial r}=-\mu_0 J_r$$

$$\frac{1}{r}\frac{\partial (rB_\theta)}{\partial r}=-\mu_0\sigma\frac{\partial V}{\partial z}=\mu_0 J_z$$

(2)

Induction diffusion equation

$$\frac{\partial}{\partial r}\left(\frac{1}{\sigma r}\frac{\partial (rB_\theta)}{\partial r}\right)+\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\frac{\partial B_\theta}{\partial z}\right)=0$$

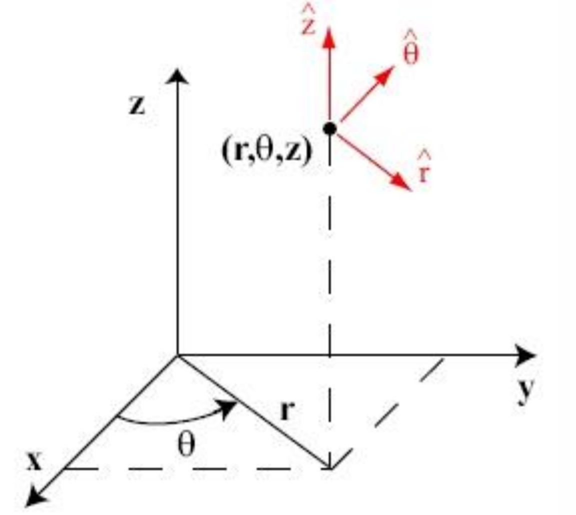
(2)

$$B_\theta(r,z)=\int_{l=z_0}^{l=z} J_z(r,l)\ dl\ +\ B_\theta(r,z_0)$$

(1) with Lorentz gauge : $\nabla\cdot\vec{A}=0$

(2) as $\partial^2_{r,z}V=\partial^2_{z,r}V$

2D axi-symmetric case (1)



$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\sigma\frac{\partial V}{\partial r}\right)+\frac{\partial}{\partial z}\left(\sigma\frac{\partial V}{\partial z}\right)=0$$

F1

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_r}{\partial r}\right)+\frac{\partial^2 A_r}{\partial z^2}-\frac{A_r}{r^2}=\mu_0\sigma\frac{\partial V}{\partial r}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_z}{\partial r}\right)+\frac{\partial^2 A_z}{\partial z^2}=\mu_0\sigma\frac{\partial V}{\partial z}$$

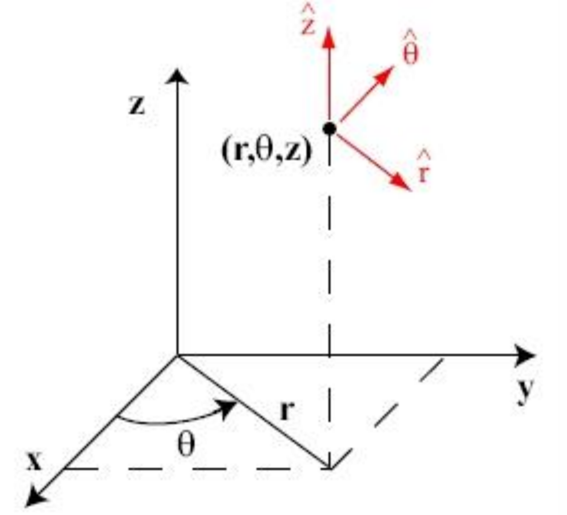
with

$\vec{J}$: $J_r=\sigma E_r=-\sigma\frac{\partial V}{\partial r}$ and $J_z=\sigma E_z=-\sigma\frac{\partial V}{\partial z}$

$\vec{B}$: $B_\theta=\frac{\partial A_r}{\partial z}-\frac{\partial A_z}{\partial r}$

(1) with Lorentz gauge : $\nabla\cdot\vec{A}=0$

2D axi-symmetric case (1)



$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\sigma\frac{\partial V}{\partial r}\right)+\frac{\partial}{\partial z}\left(\sigma\frac{\partial V}{\partial z}\right)=0$$

F2

Induction diffusion equation

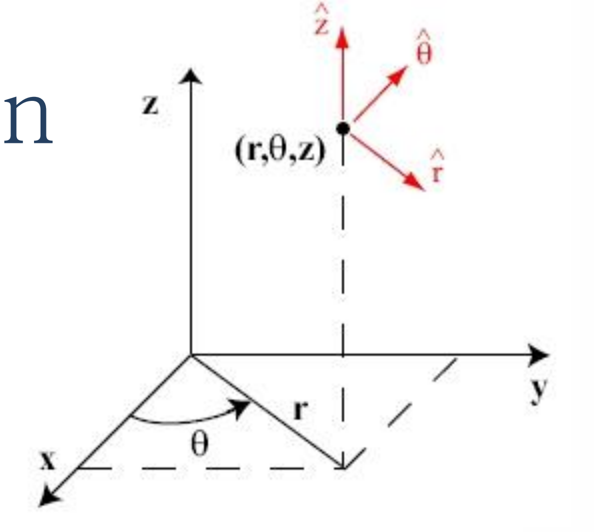
$$\frac{\partial}{\partial r}\left(\frac{1}{\sigma r}\frac{\partial(rB_\theta)}{\partial r}\right)+\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\frac{\partial B_\theta}{\partial z}\right)=0$$

with

$\vec{J}$: $J_r=\sigma E_r=-\sigma\frac{\partial V}{\partial r}$ and $J_z=\sigma E_z=-\sigma\frac{\partial V}{\partial z}$

(1) with Lorentz gauge : $\nabla\cdot\vec{A}=0$

Electric potential formulation



$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\sigma\frac{\partial V}{\partial r}\right)+\frac{\partial}{\partial z}\left(\sigma\frac{\partial V}{\partial z}\right)=0$$

Induction diffusion equation

F2

$$\frac{\partial}{\partial r}\left(\frac{1}{\sigma r}\frac{\partial(rB_\theta)}{\partial r}\right)+\frac{\partial}{\partial z}\left(\frac{1}{\sigma}\frac{\partial B_\theta}{\partial z}\right)=0$$

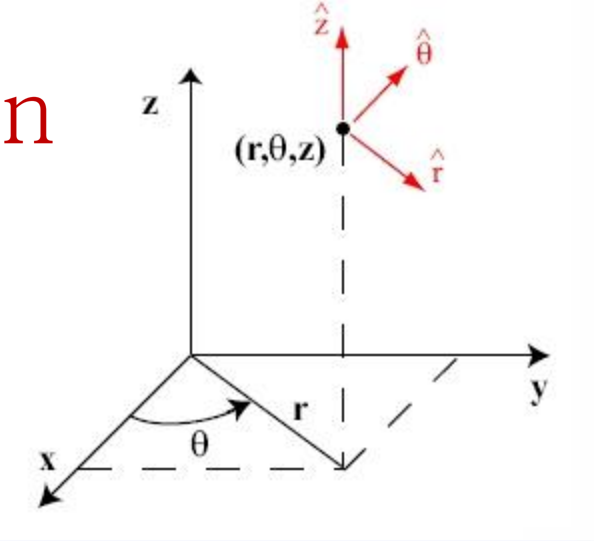
0

with

$\vec{J}$: $J_r=\sigma E_r=-\sigma\frac{\partial V}{\partial r}$ and $J_z=\sigma E_z=-\sigma\frac{\partial V}{\partial z}$

(1) with Lorentz gauge : $\nabla\cdot\vec{A}=0$

Electric potential formulation



$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\sigma\frac{\partial V}{\partial r}\right)+\frac{\partial}{\partial z}\left(\sigma\frac{\partial V}{\partial z}\right)=0$$

$$B_\theta(r)=\frac{\mu_0}{r}\int_{l=r_0}^{l=r} l\ J_z(l)\ dl\ +\ B_\theta(r_0)$$

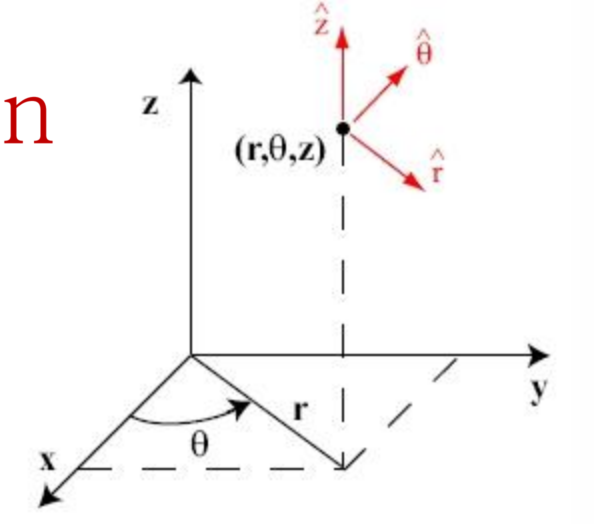
with

$\vec{J}$:

$$J_r=\sigma E_r=-\sigma\frac{\partial V}{\partial r}\quad\text{and}\quad J_z=\sigma E_z=-\sigma\frac{\partial V}{\partial z}$$

(1) with Lorentz gauge : $\nabla\cdot\vec{A}=0$

Electric potential formulation



$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\,\sigma\frac{\partial V}{\partial r}\right)+\frac{\partial}{\partial z}\left(\sigma\frac{\partial V}{\partial z}\right)=0$$

$$B_\theta(r)=\frac{\mu_0}{r}\int_{l=r_0}^{l=r} l\ J_z(l)\ dl\ +\ B_\theta(r_0)$$

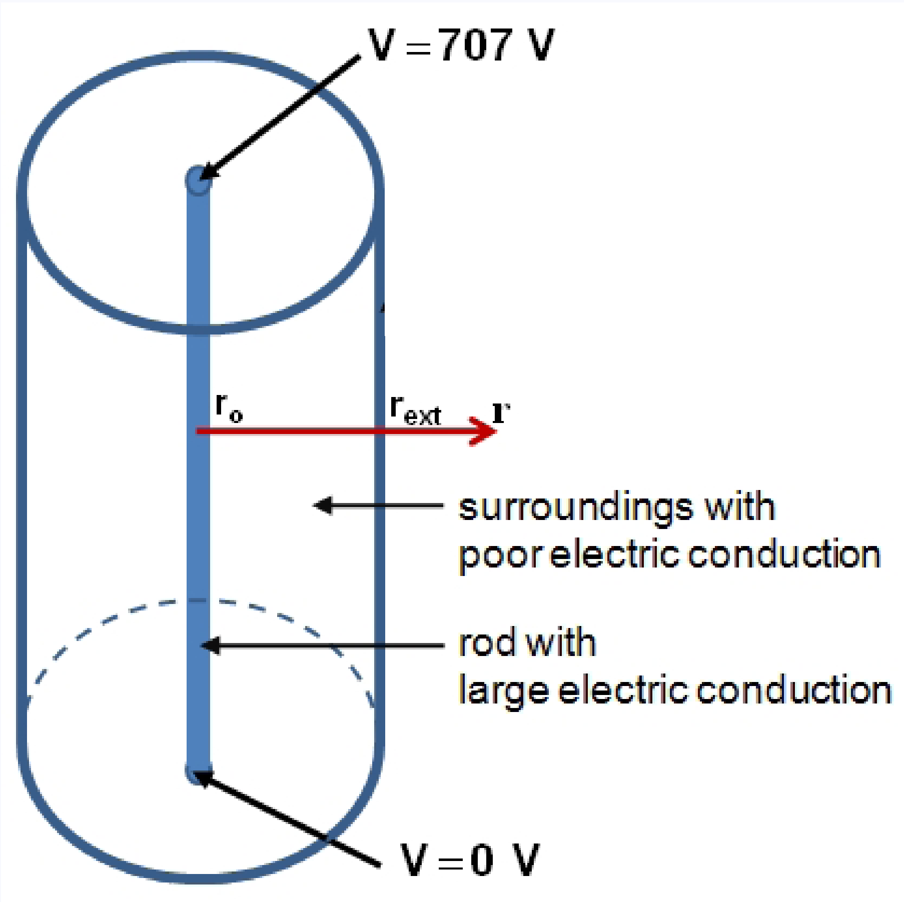
with

$\vec{J}$:

$$J_r=\sigma E_r=-\sigma\frac{\partial V}{\partial r} \quad \ll \quad J_z=\sigma E_z=-\sigma\frac{\partial V}{\partial z}$$

(1) with Lorentz gauge : $\nabla\cdot\vec{A}=0$

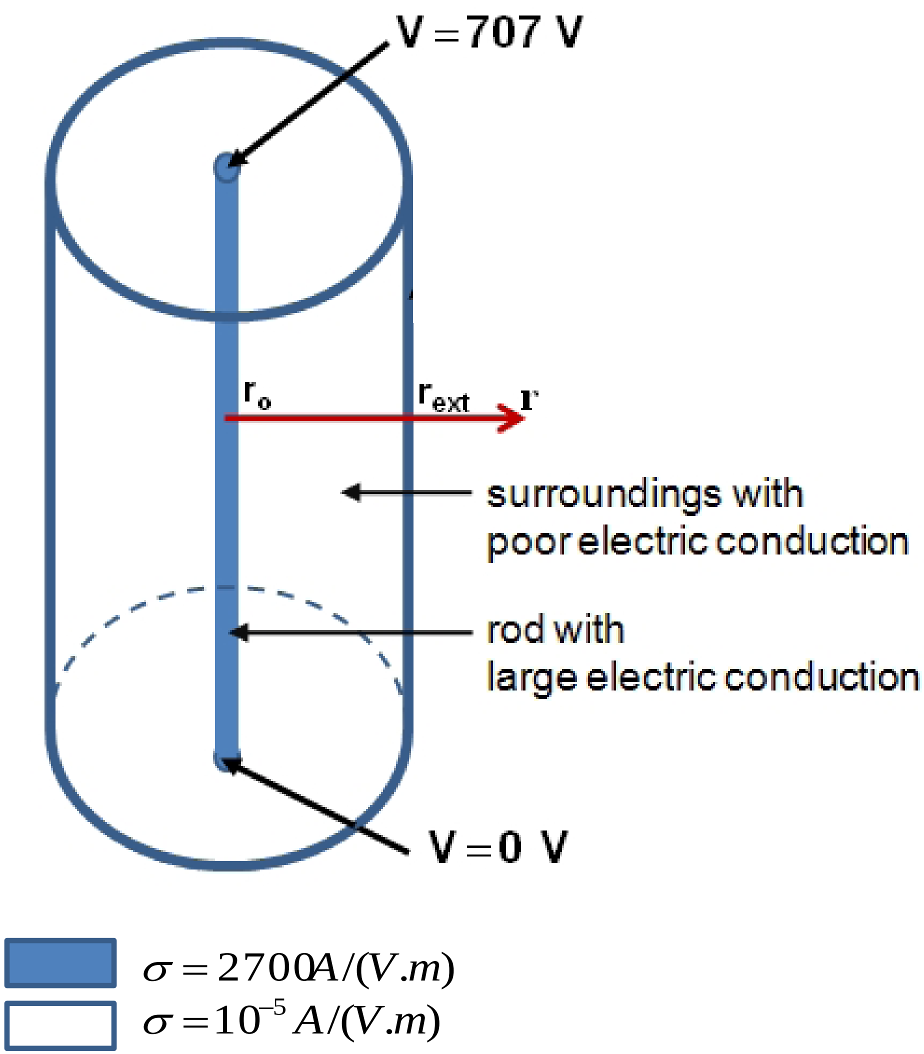
Test case: infinite conducting rod

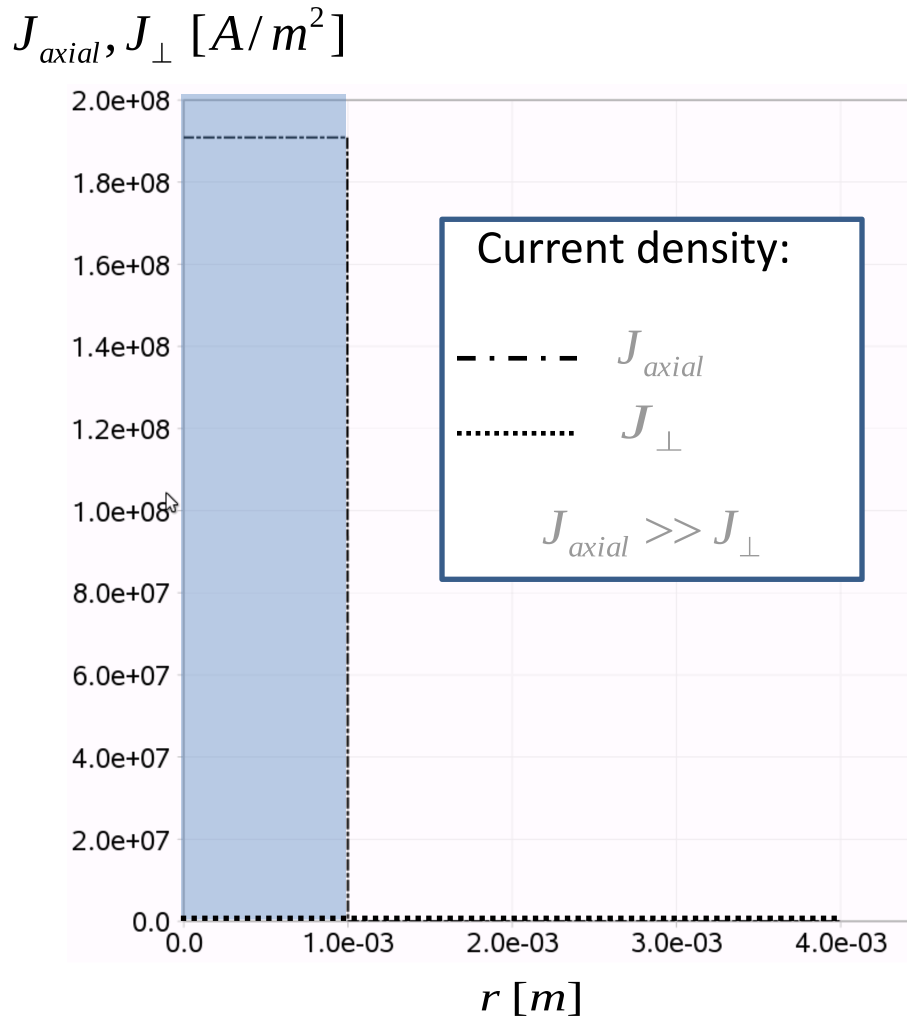


$\sigma = 2700 A/(V.m)$

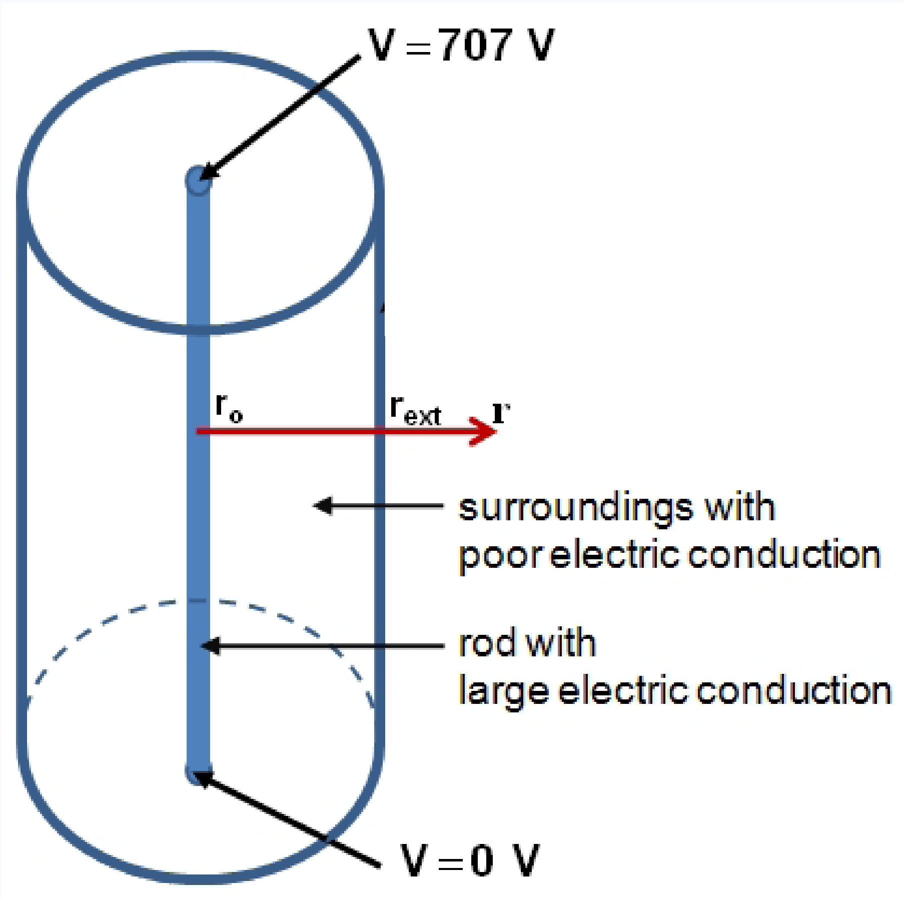
$\sigma = 10^{-5} A/(V.m)$

Test case: infinite conducting rod





Test case: infinite conducting rod



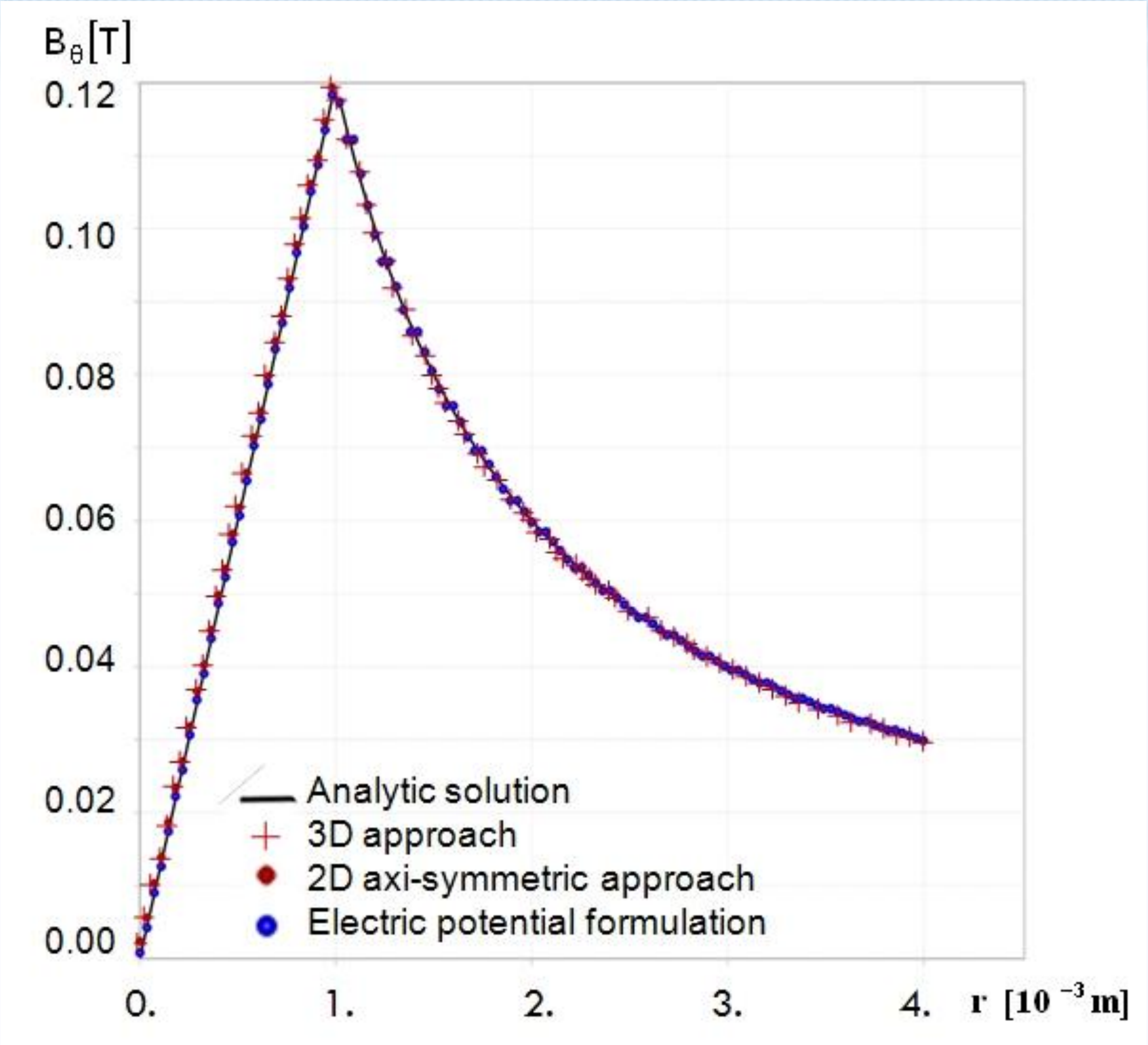
Analytic solution:

$$B_\theta(r) = \frac{\mu_o J_{axial}\, r}{2} \quad if \quad r < r_o$$

$$B_\theta(r) = \frac{\mu_o J_{axial}\, r_o^2}{2\, r} \quad if \quad r \geq r_o$$

$\sigma = 2700 A/(V.m)$

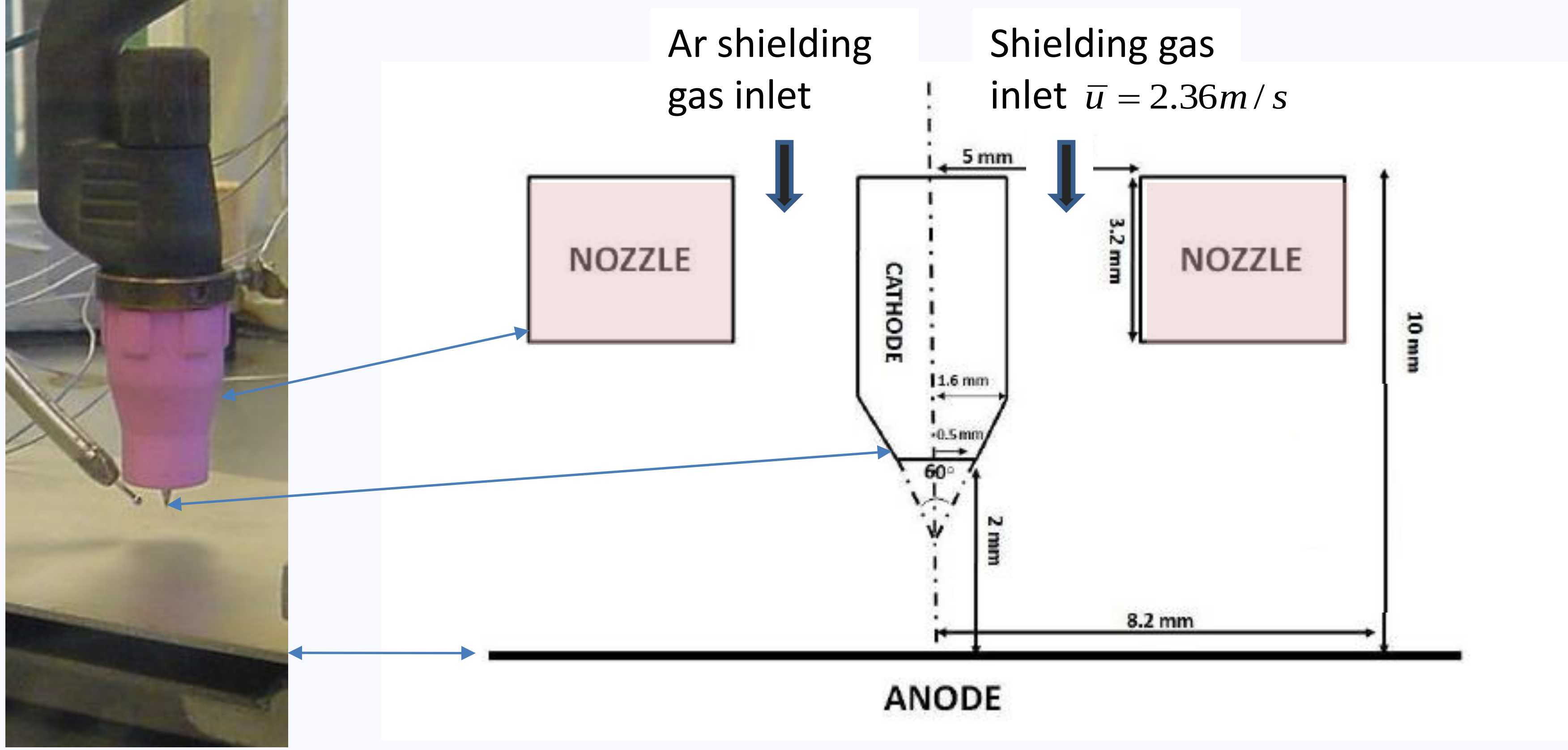
$\sigma = 10^{-5} A/(V.m)$



Azimuthal component of the magnetic field along the radial direction (r_0= 10^{-3}m)

Test case: Tungsten Inert Gas welding

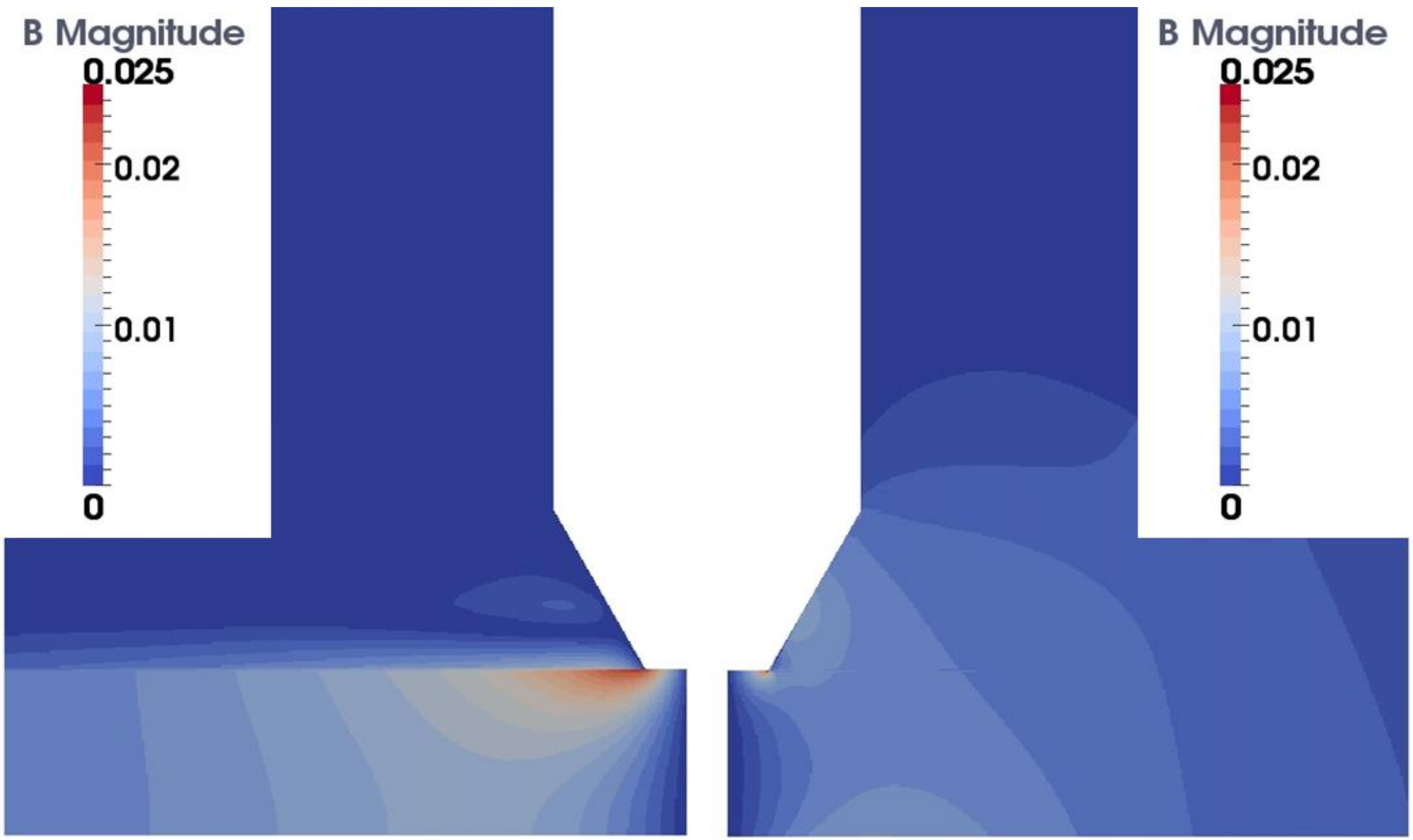
Applied current: I=200A



Picture of a TIG torch

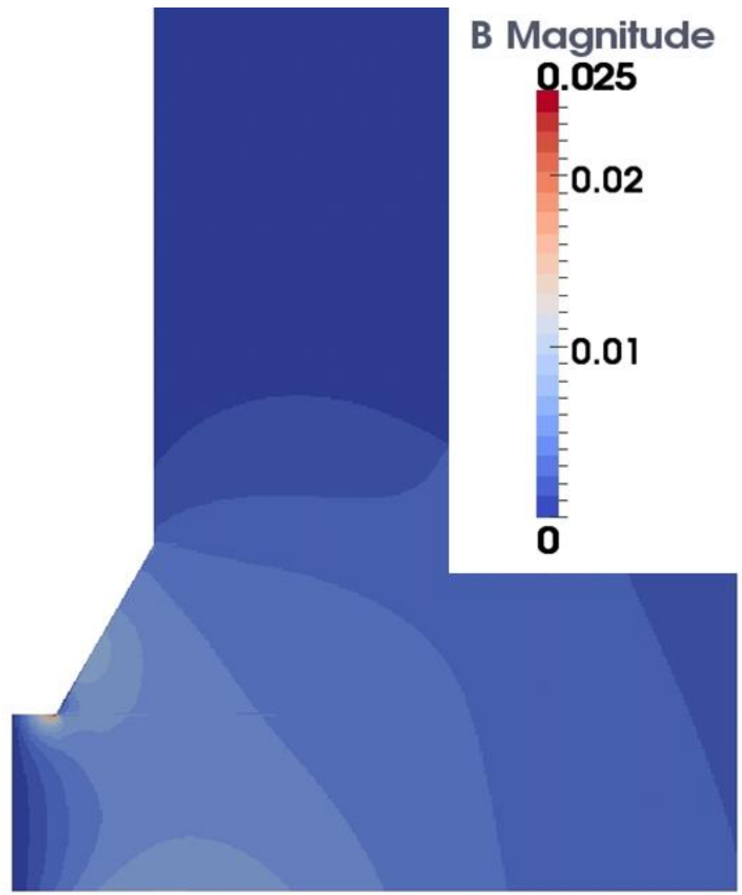
Sketch of the cross section of a TIG torch

$$B_\theta(r) = \frac{\mu_0}{r}\int_0^r J_{axial}(l)\, l\, dl \qquad\qquad B_\theta = (\nabla \times \vec{A})_\theta$$

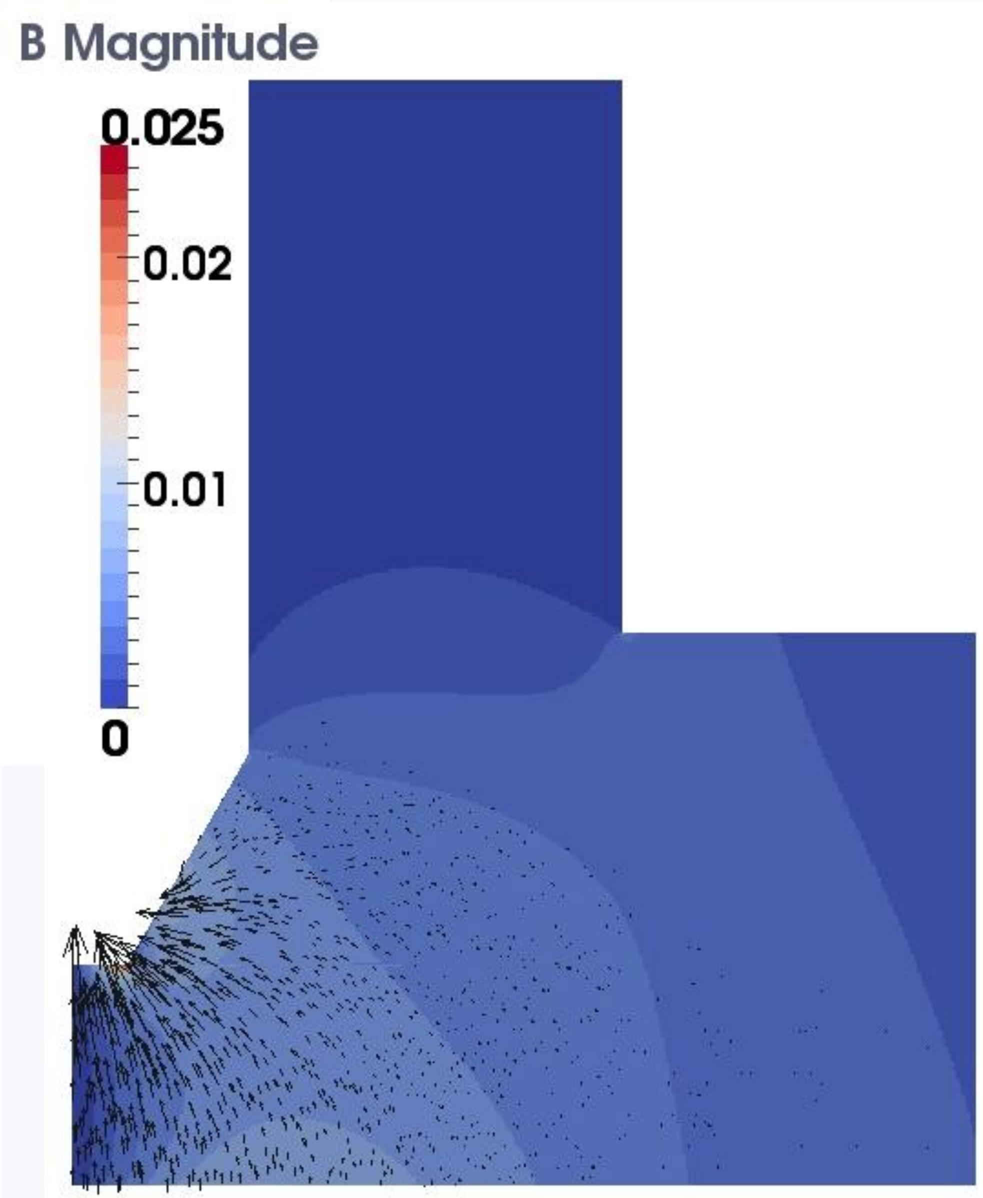


Magnetic field magnitude calculated with the electric potential formulation (left) and the axi-symmetric

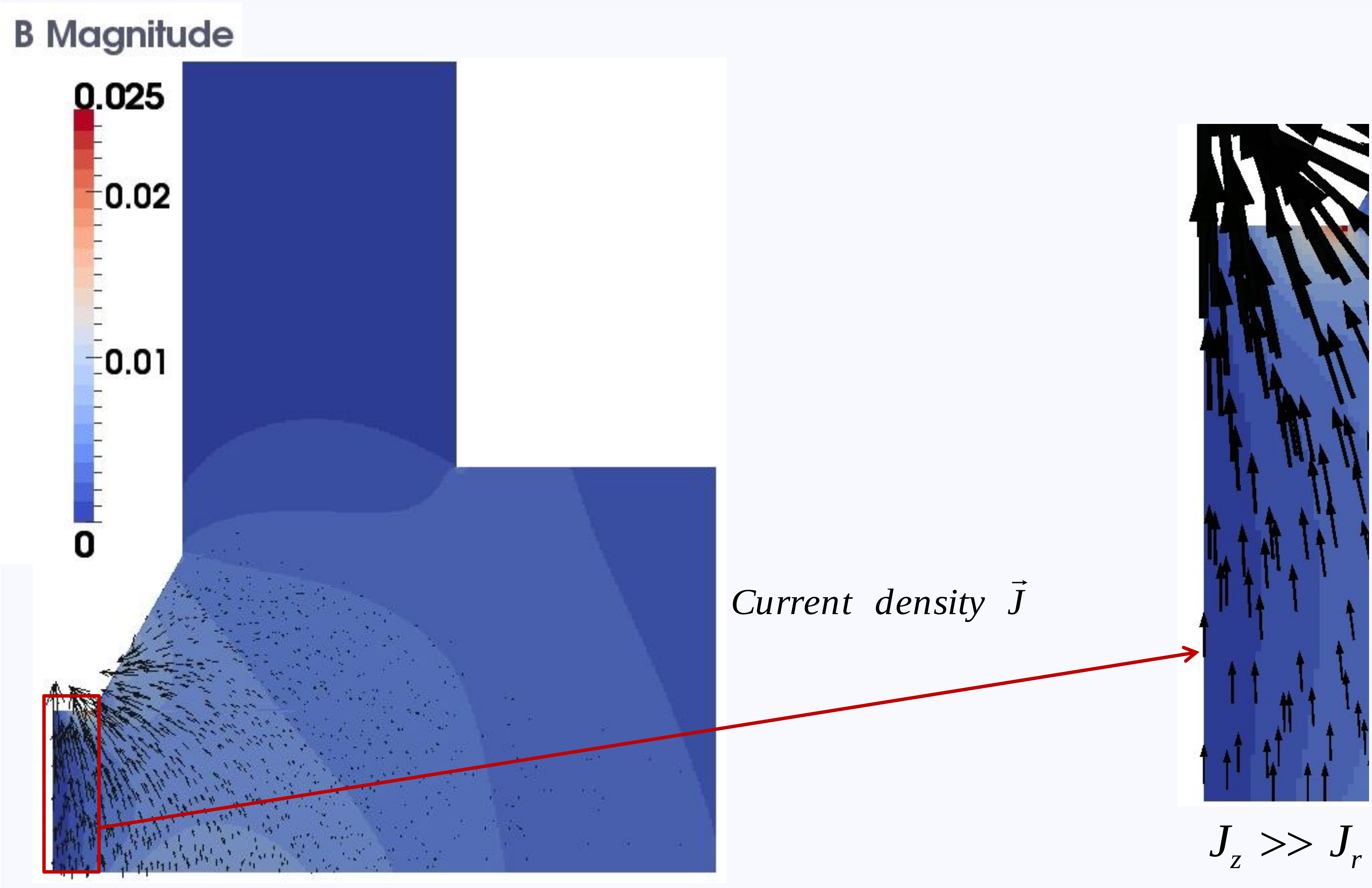
$$B_\theta = (\nabla \times \vec{A})_\theta$$

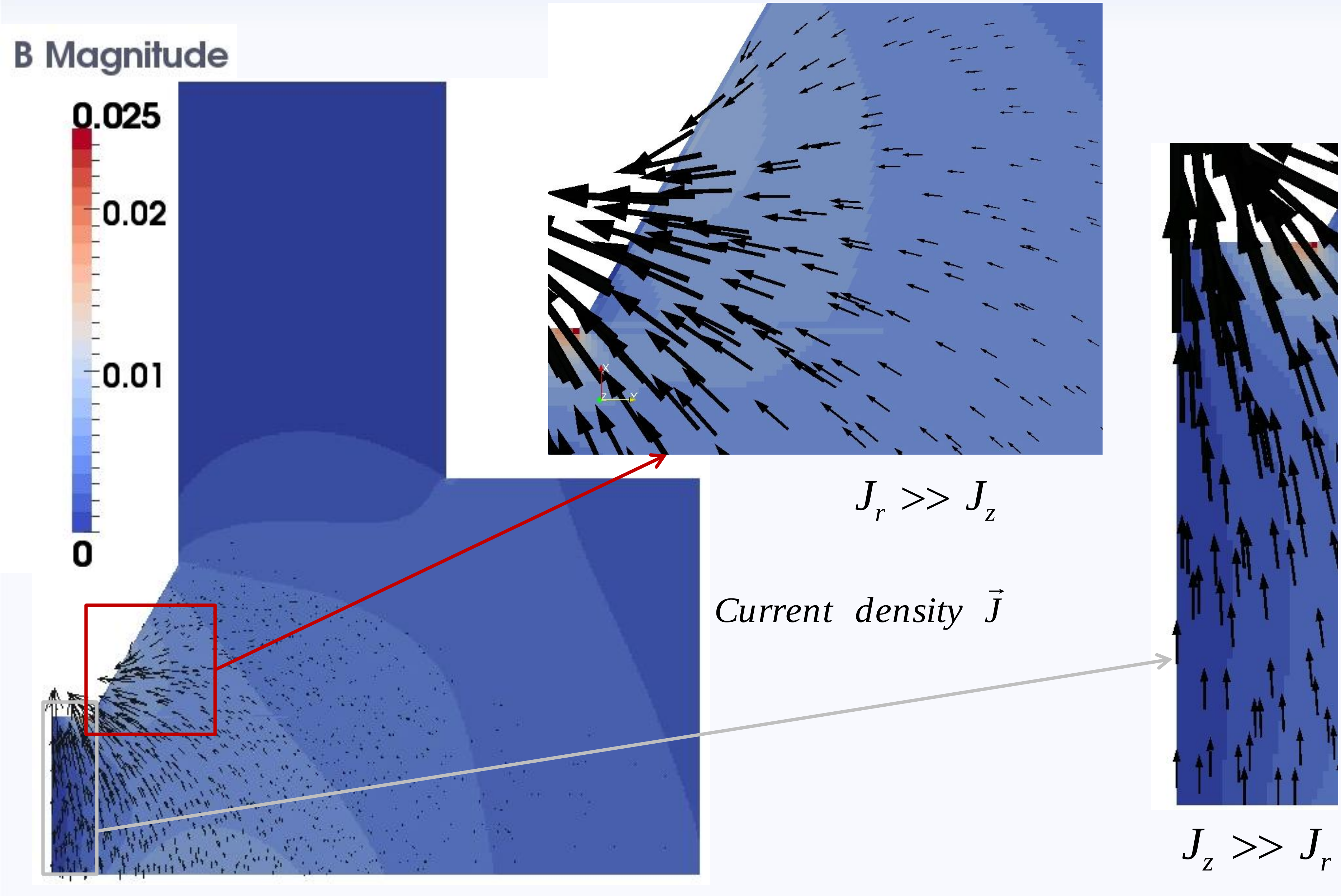


Magnetic field magnitude calculated with the axi-

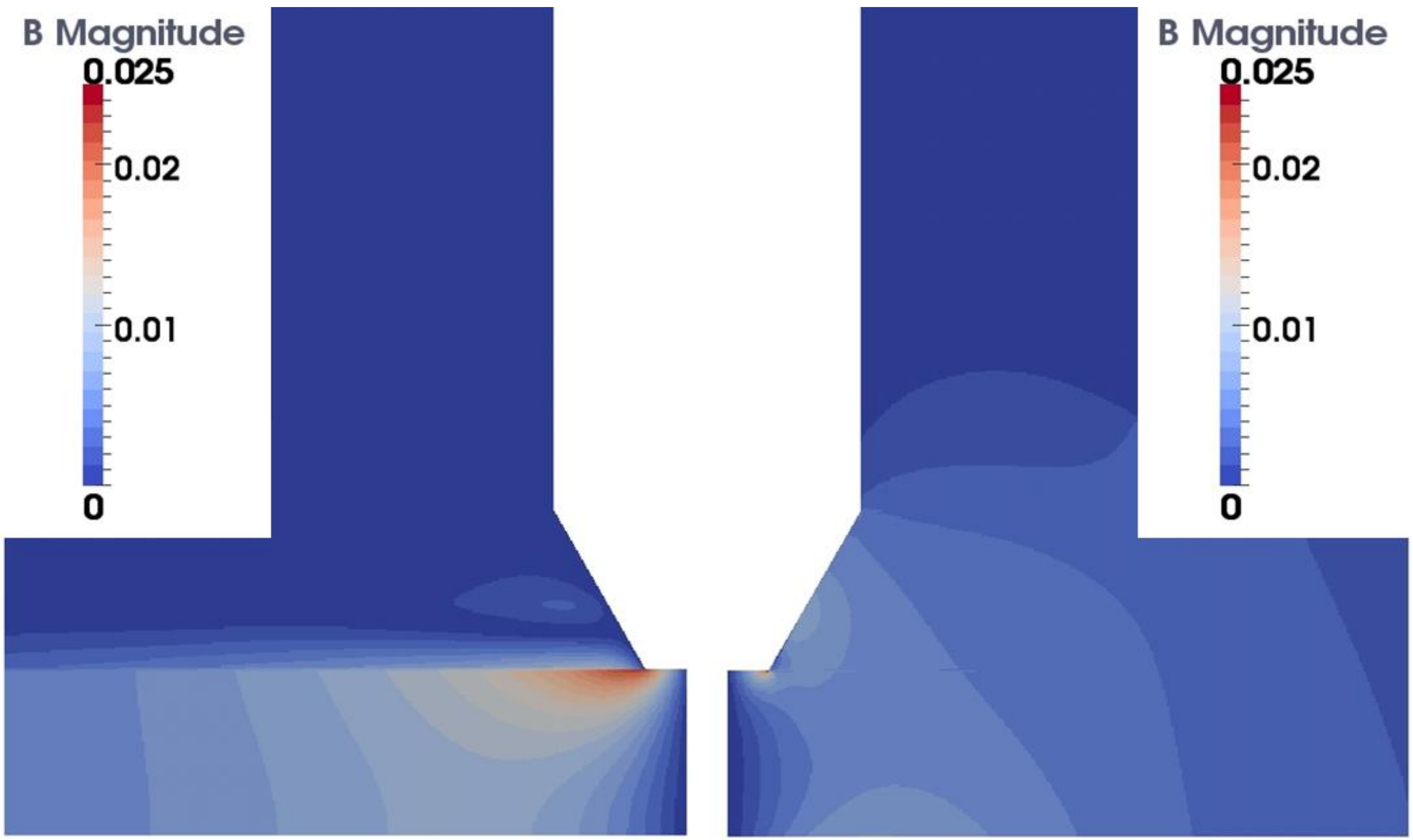


Current density $\vec{J}$


B Magnitude
0.025
0.02
0.01
0
Current density $\vec{J}$
$J_z >> J_r$


B Magnitude
0.025
0.02
0.01
0
$J_r >> J_z$
Current density $\vec{J}$
$J_z >> J_r$

$$B_{\theta}(r) = \frac{\mu_0}{r}\int_0^r J_{axial}(l)\, l\, dl \qquad B_{\theta} = (\nabla \times \vec{A})_{\theta}$$



Magnetic field magnitude calculated with the electric potential formulation (left) and the axi-symmetric

electromagnetic model / conclusion

- 3D model with $V, \vec{A}$ ⟹ $\vec{E}, \vec{J}, \vec{B}$
- 2D axi-symmetric models:
  - ✓Electric potential formulation V ⟹ $\vec{E}, \vec{J}$ ⟹ B_θ
  - ✓Magnetic field formulation B_θ ⟹ $\vec{E}, \vec{J}$

Conclusions

- 3D model with $V, \vec{A} \Longrightarrow \vec{E}, \vec{J}, \vec{B}$
- 2D axi-symmetric models:

✓

✓

✓Electric potential formulation if

✓Magnetic field formulation has an "additional degree of freedom"

Conclusions

- 3D model with $V, \vec{A} \Longrightarrow \vec{E}, \vec{J}, \vec{B}$

- 2D axi-symmetric models:

✓ $V, A_r, A_z \Longrightarrow \vec{E}, \vec{J}, B_\theta$

✓

✓Electric potential if

✓Magnetic field formulation has an "additional degree of freedom"

Conclusions

- 3D model with $V, \vec{A} \Longrightarrow \vec{E}, \vec{J}, \vec{B}$

- 2D axi-symmetric models:

✓ $V, A_r, A_z \Longrightarrow \vec{E}, \vec{J}, B_\theta$

✓ $V, B_\theta \Longrightarrow \vec{E}, \vec{J}$

✓Electric potential formulation if

✓Magnetic field formulation has an "additional degree of freedom"

Conclusions

- 3D model with $V, \vec{A} \longrightarrow \vec{E}, \vec{J}, \vec{B}$
- 2D axi-symmetric models:

✓ $V, A_r, A_z \longrightarrow \vec{E}, \vec{J}, B_\theta$

✓ $V, B_\theta \longrightarrow \vec{E}, \vec{J}$

✓Electric potential formulation $V \longrightarrow \vec{E}, \vec{J} \longrightarrow B_\theta$ if

✓Magnetic field formulation has an "additional degree of freedom"

Conclusions

- 3D model with $V, \vec{A} \Longrightarrow \vec{E}, \vec{J}, \vec{B}$

- 2D axi-symmetric models:

✓ $V, A_r, A_z \Longrightarrow \vec{E}, \vec{J}, B_\theta$

✓ $V, B_\theta \Longrightarrow \vec{E}, \vec{J}$

✓Electric potential formulation $V \Longrightarrow \vec{E}, \vec{J} \Longrightarrow B_\theta$ if $J_r << J_z$

✓Magnetic field formulation has an "additional degree of freedom"

Conclusions

- 3D model with $V, \vec{A} \Longrightarrow \vec{E}, \vec{J}, \vec{B}$

- 2D axi-symmetric models:

✓ $V, A_r, A_z \Longrightarrow \vec{E}, \vec{J}, B_\theta$

✓ $V, B_\theta \Longrightarrow \vec{E}, \vec{J}$

✓Electric potential formulation $V \Longrightarrow \vec{E}, \vec{J} \Longrightarrow B_\theta$ if $J_r << J_z$

✓Magnetic field formulation has an "additional degree of freedom"

Conclusions

- 3D model with $V, \vec{A}$ ➡ $\vec{E}, \vec{J}, \vec{B}$

- 2D axi-symmetric models:

✓ V, A_r, A_z ➡ $\vec{E}, \vec{J}, B_\theta$

✓ V, B_θ ➡ $\vec{E}, \vec{J}$

✓Electric potential formulation V ➡ $\vec{E}, \vec{J}$ ➡ B_θ if $J_r << J_z$

✓Magnetic field formulation has an ”additional degree of freedom”

- Acknowledgements

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To **KK-foundation**,
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the **Sustainable Production Initiative at Chalmers**
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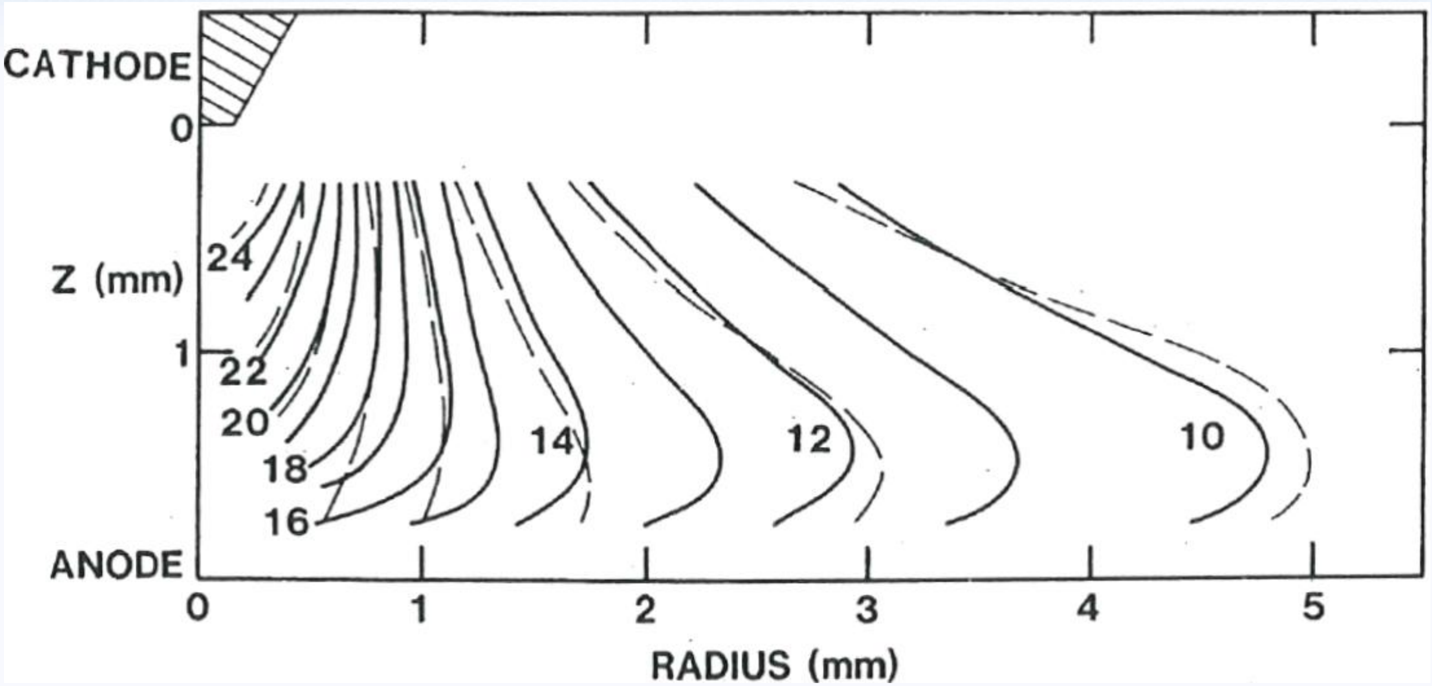
Thank you for your attention !



Measured temperature profile for a current intensity 200 A and 2 mm long arc.

Figure from*: G.N. Haddad and A.J.D. Farmer (1985) .Temperature measurements in gas tungsten arcs, Welding J, **64**, pp. 339-342.*

Boundary conditions:

*M.C. Tsai, and Sindo Kou (1990). Heat transfer and fluid flow in welding arcs produced by sharpened and flat electrodes, Int. J. Heat Mass Transfer, **33**, pp. 2089-2098.*