Example: Implementation of the rodFoam solver

(This can be viewed as en example of the 'modification' part of a student project - download files from course homepage)

`rodFoam` solves the Maxwell's equations. The code is inherently steady state, requiring an initial condition and boundary conditions.

Governing equations

- Maxwell's equations

$$\nabla \times E = 0 \tag{1}$$

where E is the electric field strength.

$$\nabla \cdot B = 0 \tag{2}$$

where B is the magnetic flux density.

$$\nabla \times H = J \tag{3}$$

where H is the magnetic field strength and J is current density.

Governing equations

- Charge continuity

$$\nabla \cdot J = 0 \tag{4}$$

- Ohm's law

$$J = \sigma E \tag{5}$$

where σ is the electric conductivity

- Constitutive law

$$B = \mu_0 H \tag{6}$$

where μ_0 is the magnetic permeability of vacum

Combining Equations (1)-(6) and assuming Coulomb gauge condition ($\nabla \cdot A = 0$) leads to Poissons's equation for the magnetostatic fields and Laplace's equation for the electric potential.

Governing equations in OpenFoam

- Equation for the electric potential:

$$\nabla \cdot [\sigma(\nabla\phi)] = 0 \quad (7)$$

- OpenFOAM representation:

```
solve ( fvm::laplacian(sigma,ElPot) );
```

- Equation for the magnetic potential:

$$\nabla^2 A = \mu_0 \sigma(\nabla\phi) \quad (8)$$

- OpenFOAM representation:

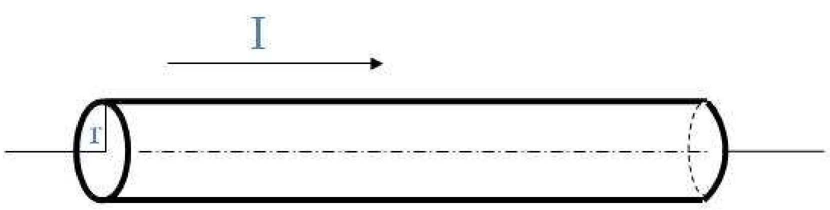
```
solve ( fvm::laplacian(A) == sigma*muMag*(fvc::grad(ElPot)) );
```

A description of the rodFoam solver

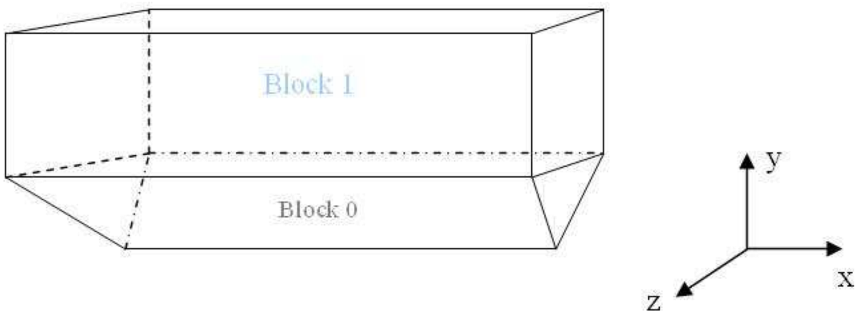
Important files:

- `files` and `options` files
- `createFields.H`
- `rodFoam.C`
- `createFieldsGeometry.H`
- `IeEqn.H`

Mesh generation, ”rodFoamCase” case



Electric rod.



Computational domain

Boundary and initial conditions

- Boundary conditions:

	block 0, sides	block 1, sides	block1, top
A	$\nabla A = 0$	$\nabla A = 0$	$A = 0$
ϕ	$\phi_{left} = 707, \phi_{right} = 0$	$\nabla \phi = 0$	$\nabla \phi = 0$

- The internal field and boundary conditions of σ are nonuniform:

$$\sigma = \begin{cases} 2700 & \text{if } x < R \text{ where R -radius of the block 1} \\ 1e-5 & \text{otherwise} \end{cases}$$

- Use `setFields` to set the internal field

Setting and running the case

Set up the case using the following files:

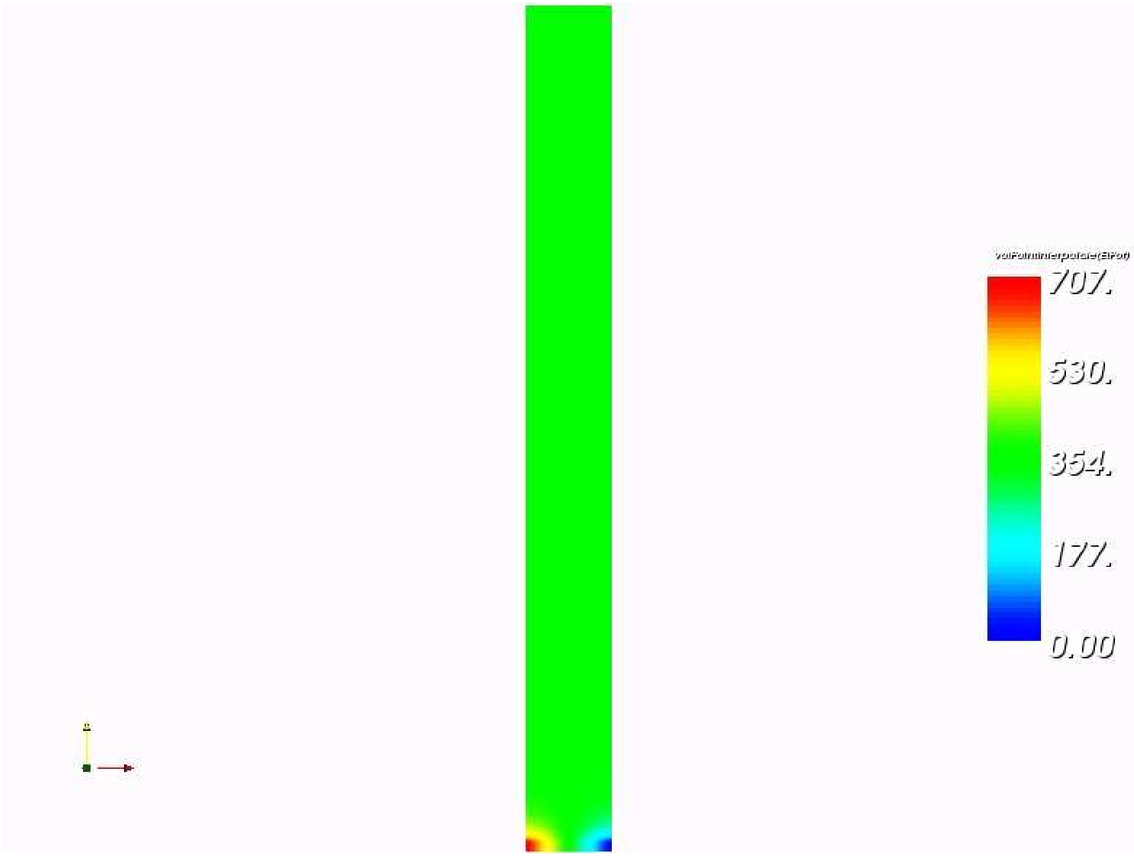
- `constant/geometryProperties`
- `constant/transportProperties`
- `system/controlDict`
- `system/fvSchemes`
- `system/fvSolution`

Run the case by:

- `./Allrun 2>&1 | tee log_Allrun`

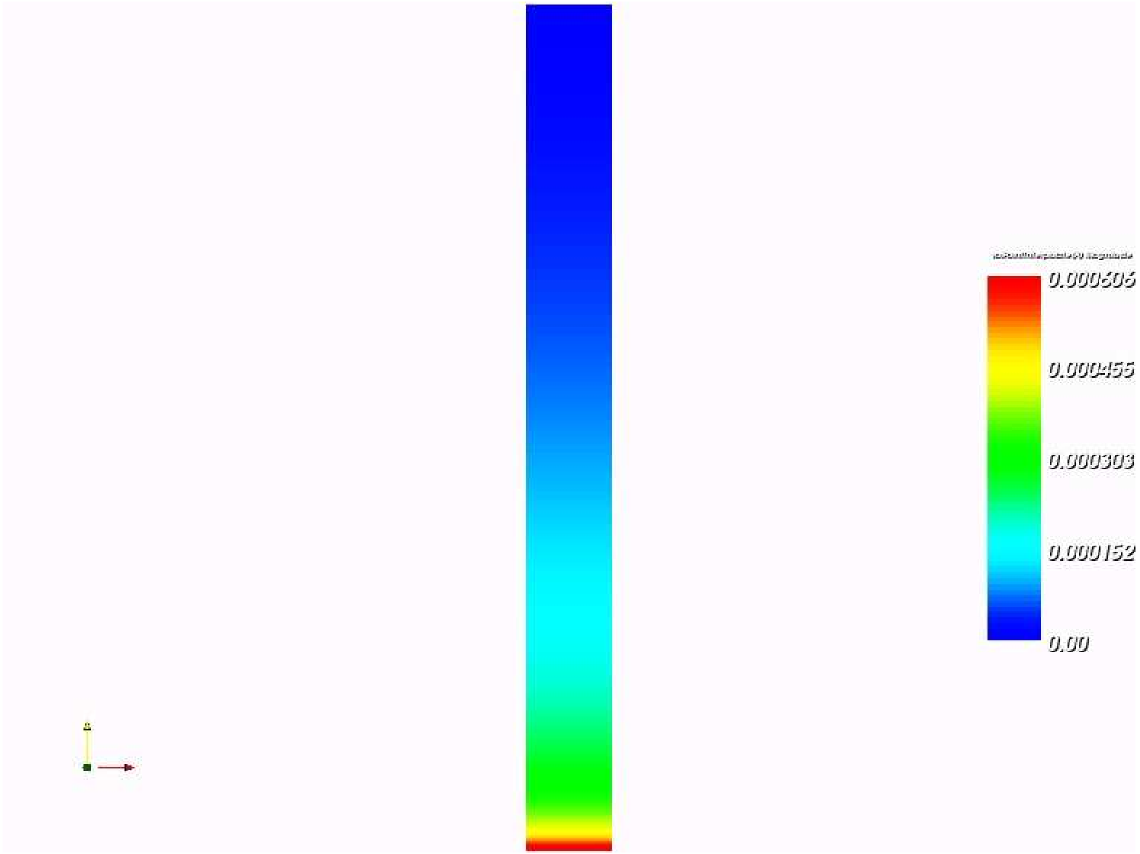
paraFoam plot.

- `paraFoam`



Electric potential ϕ. .

paraFoam plot.



Magnitude of magnetic potential vector A.

Gnuplot. Validation

- Run `sample` using dictionary `system/sampleDict`
- For this we need to extract the components:
  ```
  foamCalc components A
  foamCalc components B
  ```
- Run `sample`
- Run `gnuplot rodComparisonAxBz.plt`
- Visualize using:
  ```
  ggv rodAxVSy.ps rodBzVSy.ps
  ```

Analytic solution

- Analytic solution for x component of magnetic potential vector A

$$A_x = \begin{cases} A_x(0) - \frac{\mu_0 J x^2}{4} & \text{if } r < R, \\ A_x(0) - \frac{\mu_0 J R^2}{2}[0.5 + ln(r/R)] & \text{otherwise} \end{cases}$$

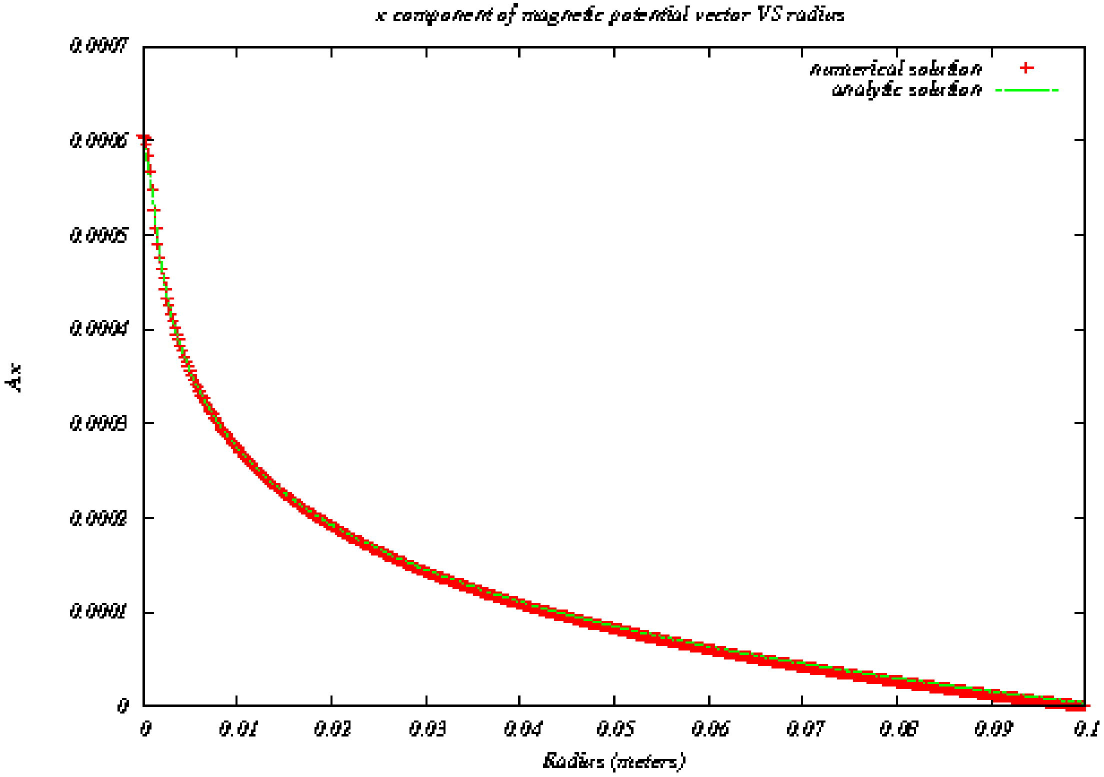
where $A_x(0) = 0.000606129$, $J = 19.086e + 7$ is the current density and R is the radius of the electric rod.

- Analytic solution for z component of magnetic field B

$$B_z = \begin{cases} \frac{\mu_0 J x}{2} & \text{if } r < R, \\ \frac{\mu_0 J R^2}{2r} & \text{otherwise} \end{cases}$$

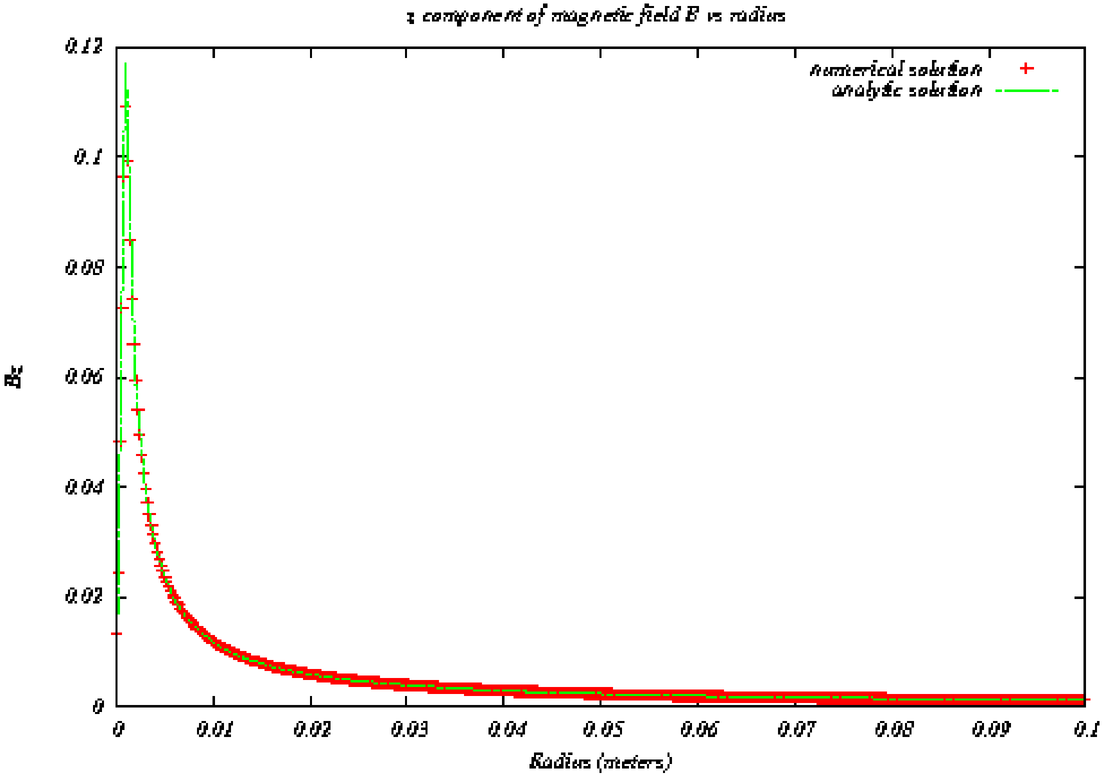
where $J = 19.086e+7$ is the current density and R is the radius of the electric rod.

Gnuplot.Validation



x-component of magnetic potential vector A vs radius of the domain.

Gnuplot. Validation



z-component of the magnetic field B vs radius of the domain