FVM for Fluid-Structure Interaction with Large Structural Displacements

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Outline

Objective

- Present a self-contained FSI application in a single software: **OpenFOAM**
- Examine FSI solution techniques, especially coupling algorithms
- Present advantages of single-software implementation and coupling

Topics

- Solution techniques in FSI simulations
  - Monolithic approach
  - Partitioned approach
- Partitioned approach with weak and strong coupling
  - Arbitrary Lagrangian-Eulerian (ALE) FVM solver for fluid flow
  - Updated Lagrangian FVM solver for elastic solid with large deformation
  - Iterative algorithm with adaptive under-relaxation for strong coupling
- OpenFOAM: Object-Oriented Computational Continuum Mechanics in C++
- Test cases: solid oscillations with large deformation and flow-induced vibration
- Summary and Future Work

FSI Solution Techniques

Solution Techniques for Coupled Problems

- **Partitioned approach**: Picard Iterations
  - Optimal for weakly coupled FSI problems
  - Separate mathematical model for fluid and solid continua
  - Shared or different discretisation method: FVM and FEM
  - Coupling achieved by enforcing the kinematic and dynamic conditions on the fluid-solid interface
  - Strong coupling by additional iteration loop over partial solvers (need a convergence acceleration method)
- **Monolithic approach**: Simultaneous Solution
  - Appropriate when fluid-structure interaction is very strong
  - Good stability and convergence properties
  - In some cases may lead to ill-conditioned matrices or sub-optimal discretisation or solution procedure in fluid or solid region

Levels of Coupling

- Unified mathematical model: single equation set (Ivanković, UC Dublin)
- Unified discretisation method and boundary coupling consistency
- Unified solution procedure: fluid + structure matrix solved in a single solver

Partitioned Approach

Partitioned Approach: Weak Coupling

- Solvers for fluid and solid are applied sequentially only once per time step
- Time lag between fluid and solid solution (explicit coupling)
- Special attention is needed in order to preserve 2nd order time accuracy

Artificial Added Mass Effect: Instability

- Occurs when weak coupling algorithm is used on FSI problems with incompressible fluid and a light/slender structure
- Numerical experiments show that the instability grows with
  - Decreasing time step size
  - Decreasing solid-fluid density ratio
  - Decreasing fluid viscosity
  - Decreasing solid stiffness

Selected FSI Simulation Approach

- Partitioned approach with weak and strong coupling algorithm
- ALE FVM solver for fluid with automatic mesh motion
- Updated Lagrangian FVM solver for elastic solid
- Adaptive under-relaxation and a strong coupling algorithm: Aitken

ALE FVM Fluid Flow Solver

Mathematical Model

- Momentum equation for incompressible laminar flow in ALE formulation
- Continuity equation for incompressible flow in ALE formulation
- Space conservation law (SCL)
- Automatic mesh motion based on boundary deformation: Laplace equation

Discretisation

- Collocated 2nd order FVM method on a deforming mesh for the fluid flow
- 2nd order backward scheme for temporal discretisation
- 2nd order FEM for discretisation of automatic mesh motion

Solution Procedure

- Segregated solution procedure on a moving mesh
- PISO pressure-velocity coupling algorithm in fluid flows

Updated Lagrangian FVM Solver

Mathematical Model

- Incremental momentum equation in updated Lagrangian formulation

$$\int_{V_u} \rho_u \frac{\partial \delta \mathbf{v}}{\partial t}\, \mathrm{d}V_u = \int_{S_u} \mathbf{n}_u \bullet \left(\delta\mathbf{\Sigma}_u + \mathbf{\Sigma}_u \bullet \delta\mathbf{F}_u^{\mathrm{T}} + \delta\mathbf{\Sigma}_u \bullet \delta\mathbf{F}_u^{\mathrm{T}} \right) \mathrm{d}S_u$$

- Incremental constitutive equation for St. Venant-Kirchhoff elastic solid

$$\delta\mathbf{\Sigma}_u = 2\mu\delta\mathbf{E}_u + \lambda\, \mathrm{tr}\,(\delta\mathbf{E}_u)\,\mathbf{I}$$

- Increment of Green-Lagrangian strain tensor

$$\delta\mathbf{E}_u = \frac{1}{2}\left[\nabla\delta\mathbf{u} + (\nabla\delta\mathbf{u})^{\mathrm{T}} + \nabla\delta\mathbf{u}\bullet(\nabla\delta\mathbf{u})^{\mathrm{T}}\right]$$

- Final form of momentum equation ready for discretisation

$$\int_{V_u} \rho_u \frac{\partial \delta \mathbf{v}}{\partial t}\, \mathrm{d}V_u - \oint_{S_u} \mathbf{n}_u \bullet (2\mu + \lambda)\nabla\delta\mathbf{u}\, \mathrm{d}S_u = \oint_{S_u} \mathbf{n}_u \bullet \mathbf{q}\, \mathrm{d}S_u$$

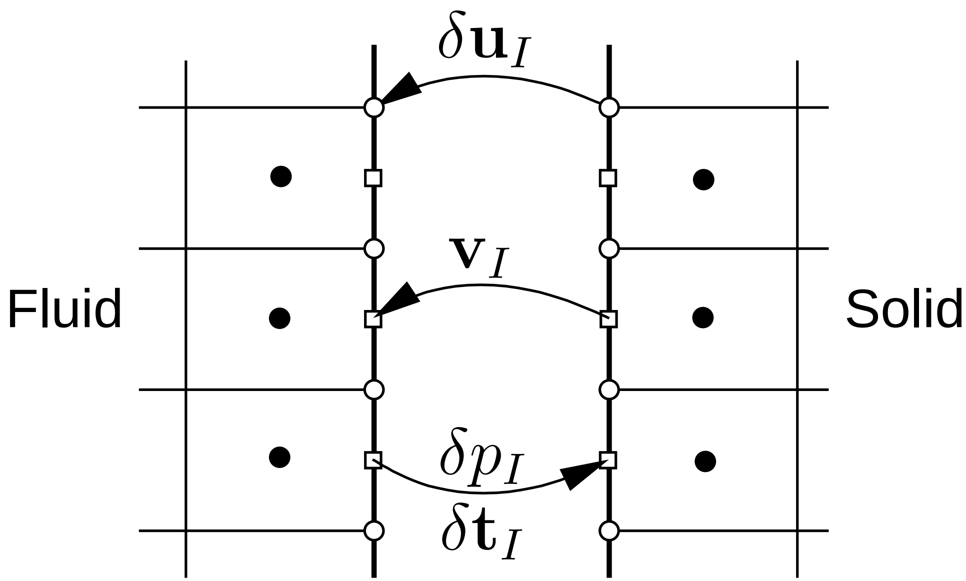
Updated Lagrangian FVM Solver

Solution Procedure

1. Update mesh according to displacement increment from the previous time step
2. Update second Piola-Kirchhoff stress tensor
3. Do until convergence
   - Calculated RHS of discretised momentum equation using last calculated displacement increment field
   - Solve momentum equation
4. Accumulate displacement vector and second Piola-Kirchhoff stress fields
5. On convergence, switch to the next time step

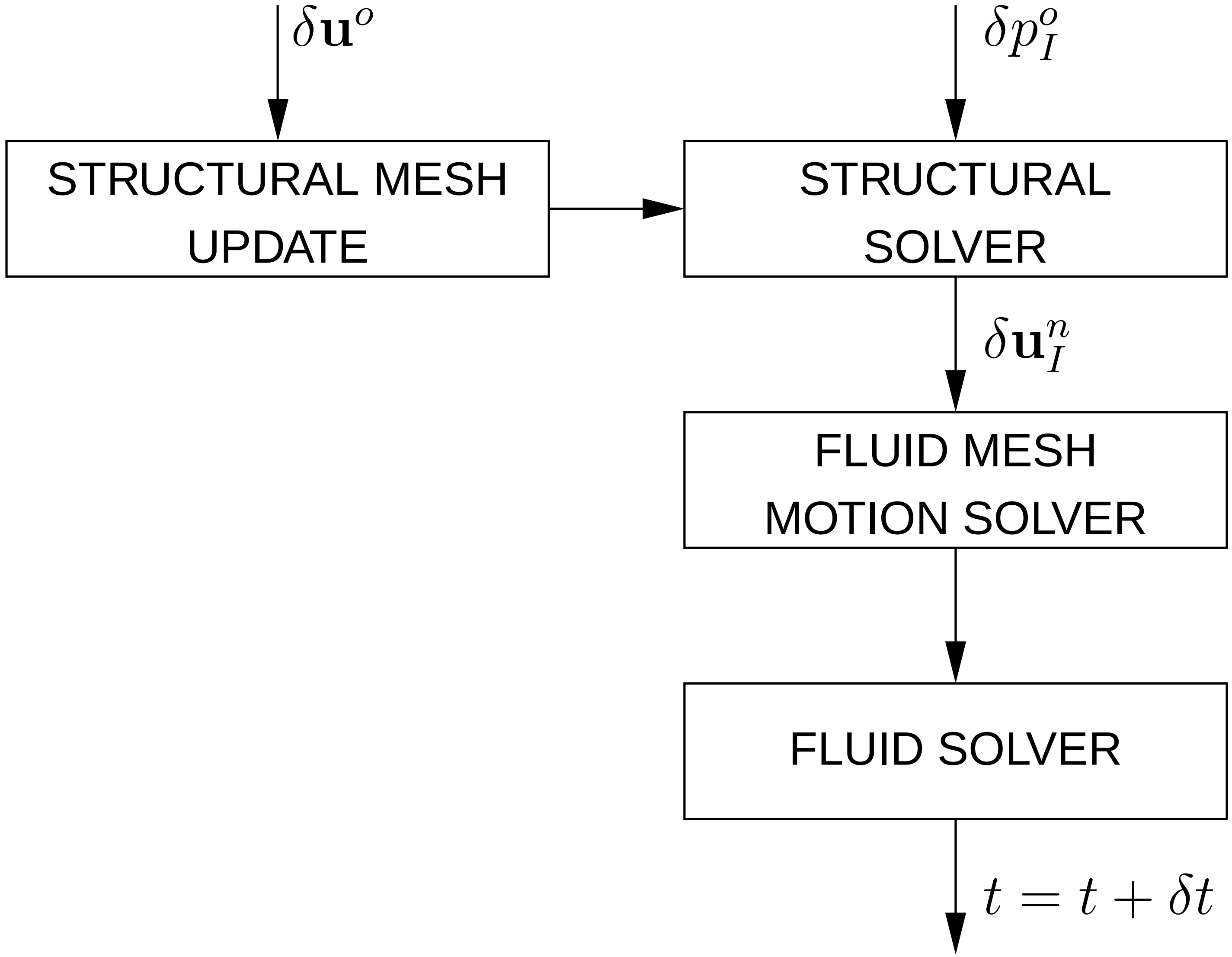
Fluid-Structure Coupling

Transfer of Coupling Data

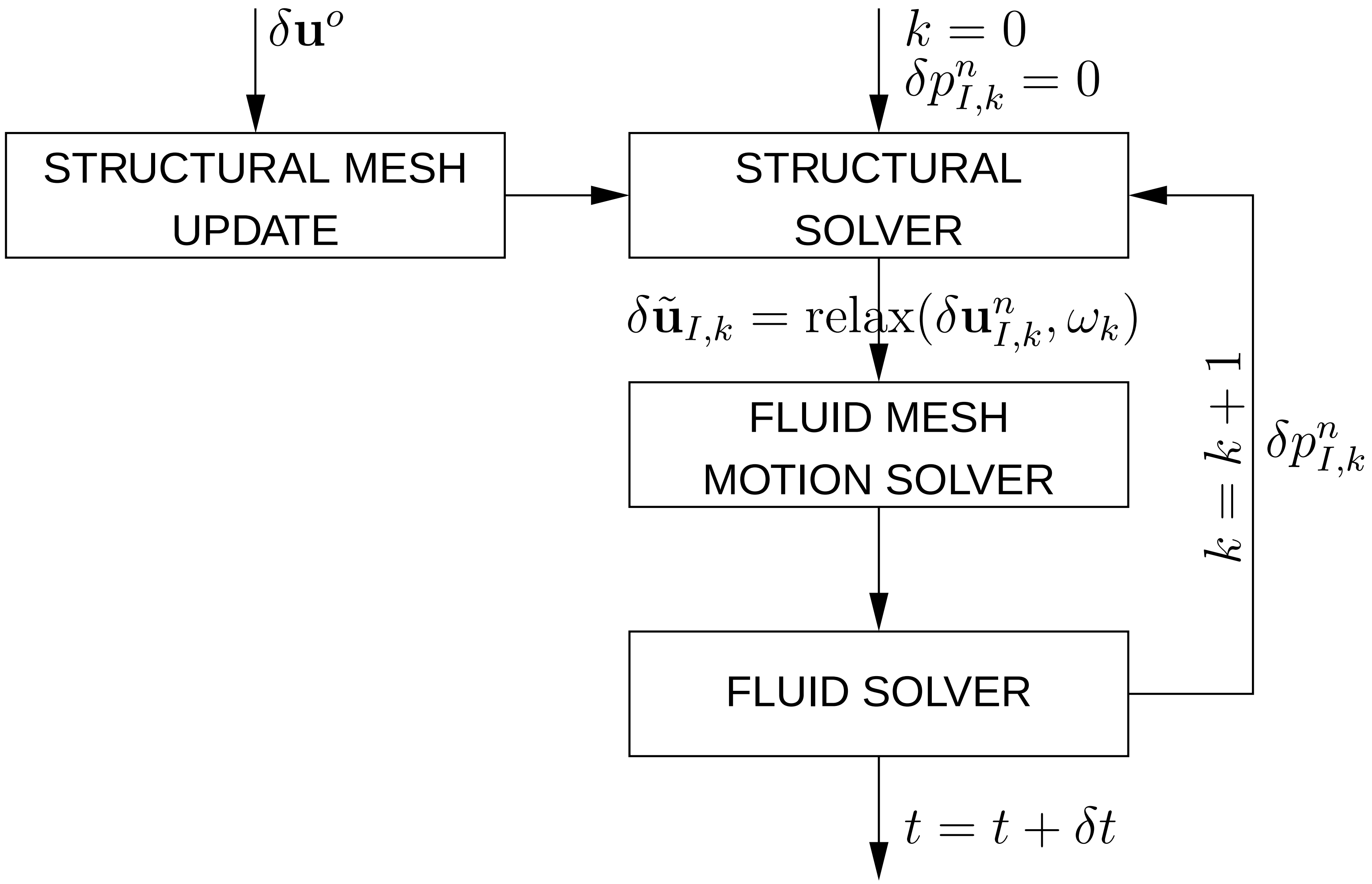


- Variables are transfered between the fluid and solid surface using patch-to-patch interpolation
- Pressure and viscous force increment (δp_I and $\delta \mathbf{t}_I$) at the fluid side of the interface is transfered to the solid side of the interface
- Displacement increment ($\delta \mathbf{u}_I$) and velocity ($\mathbf{v}_I$) at the solid side of the interface is transfered to the fluid side of the interface

Weak FSI Coupling



Strong FSI Coupling



WIKKI OpenFOAM

Adaptive Under-Relaxation

Adaptive Under-Relaxation Algorithm by Aitken

$$\delta\tilde{\mathbf{u}}_{I,k} = \omega_k \delta\mathbf{u}_{I,k} + (1 - \omega_k)\delta\tilde{\mathbf{u}}_{I,k-1}$$

$$\omega_k = 1 - \gamma_k$$

$$\gamma_k = \gamma_{k-1} + (\gamma_{k-1} - 1)\frac{(\Delta\delta\mathbf{u}_{k-1} - \Delta\delta\mathbf{u}_k)\bullet\Delta\delta\mathbf{u}_k}{(\Delta\delta\mathbf{u}_{k-1} - \Delta\delta\mathbf{u}_k)^2}$$

where

$$\Delta\delta\mathbf{u}_k = \delta\tilde{\mathbf{u}}_{k-1} - \delta\mathbf{u}_k$$

$$\Delta\delta\mathbf{u}_{k-1} = \delta\tilde{\mathbf{u}}_{k-2} - \delta\mathbf{u}_{k-1}$$

OpenFOAM: Open Source CCM

OpenFOAM: Object-Oriented Computational Continuum Mechanics in C++

- Natural language of continuum mechanics: partial differential equations
- Example: turbulence kinetic energy equation

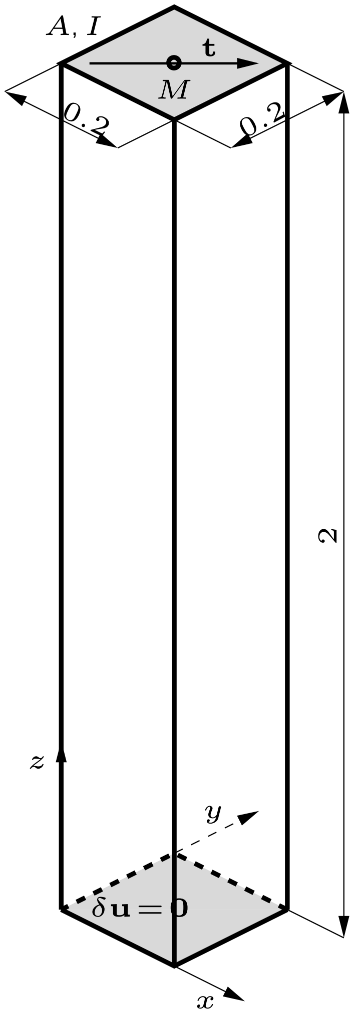
$$\frac{\partial k}{\partial t} + \nabla\bullet(\mathbf{u}k) - \nabla\bullet[(\nu + \nu_t)\nabla k] = \nu_t \left[\frac{1}{2}(\nabla\mathbf{u} + \nabla\mathbf{u}^T)\right]^2 - \frac{\epsilon_o}{k_o}\,k$$

- Objective: **represent differential equations in their natural language**

```
solve
(
   fvm::ddt(k)
 + fvm::div(phi, k)
 - fvm::laplacian(nu() + nut, k)
== nut*magSqr(symm(fvc::grad(U)))
 - fvm::Sp(epsilon/k, k)
);
```

- Correspondence between the implementation and the original equation is clear
- Implementation of complex physics is easy, with polyhedral mesh support, shared matrix format, efficient solvers, parallelisation and coupling tools in library form

Vibration of a 3-D Beam



Mechanical properties:

$$\rho = 1000\ \frac{\mathrm{kg}}{\mathrm{m}^3}$$

$$E = 15.293 \times 10^6\ \frac{\mathrm{N}}{\mathrm{m}^2}$$

$$\nu = 0.3$$

Geometric properties:

$$A = 0.04\ \mathrm{m}^2$$

$$I = 0.0001333\ \mathrm{m}^4$$

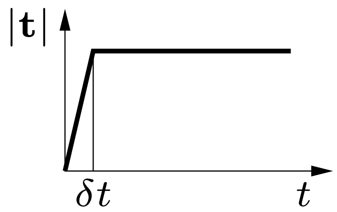
First natural frequency:

$$f_1 = 1\ \mathrm{Hz}$$

Load:

$$\mathbf{t} = \left(\frac{|\mathbf{t}|}{\sqrt{2}}, \frac{|\mathbf{t}|}{\sqrt{2}}, 0 \right)\ \frac{\mathrm{N}}{\mathrm{m}^2}$$

$$\mathcal{F} = \frac{|\mathbf{t}| A L^2}{E I}$$

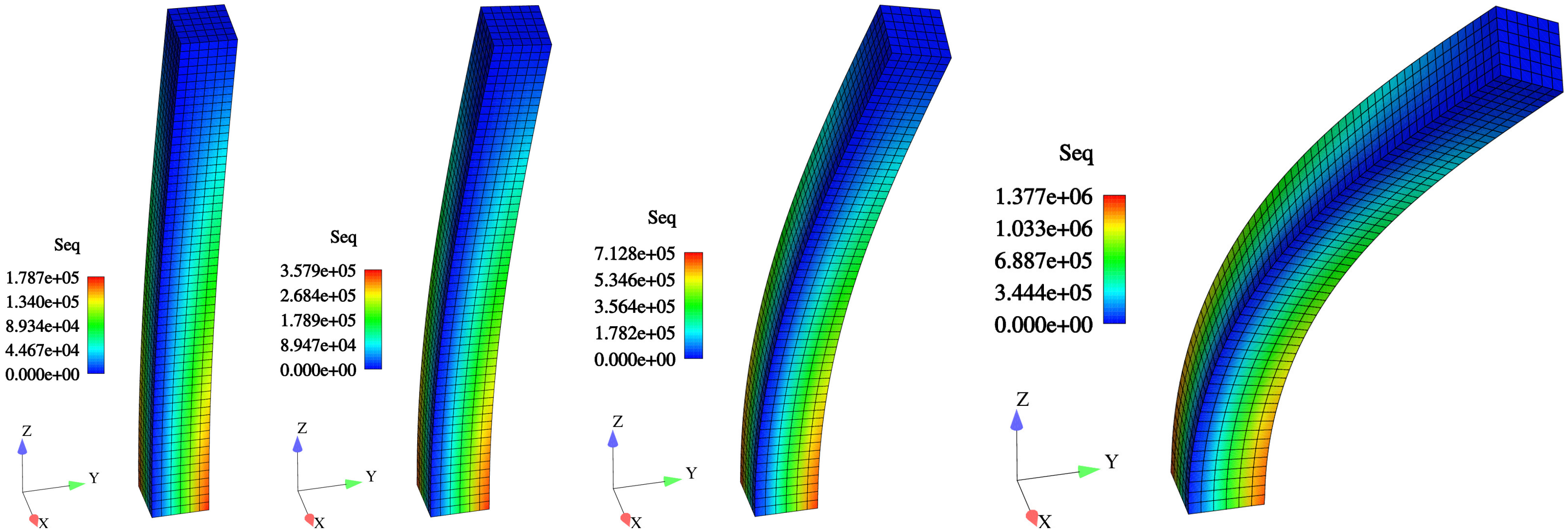


Vibration of a 3-D Beam

Static Equilibrium Solution

- Deflection (d_M) for various loadings ($\mathcal{F}$) and two meshes (6×60, 10×100)

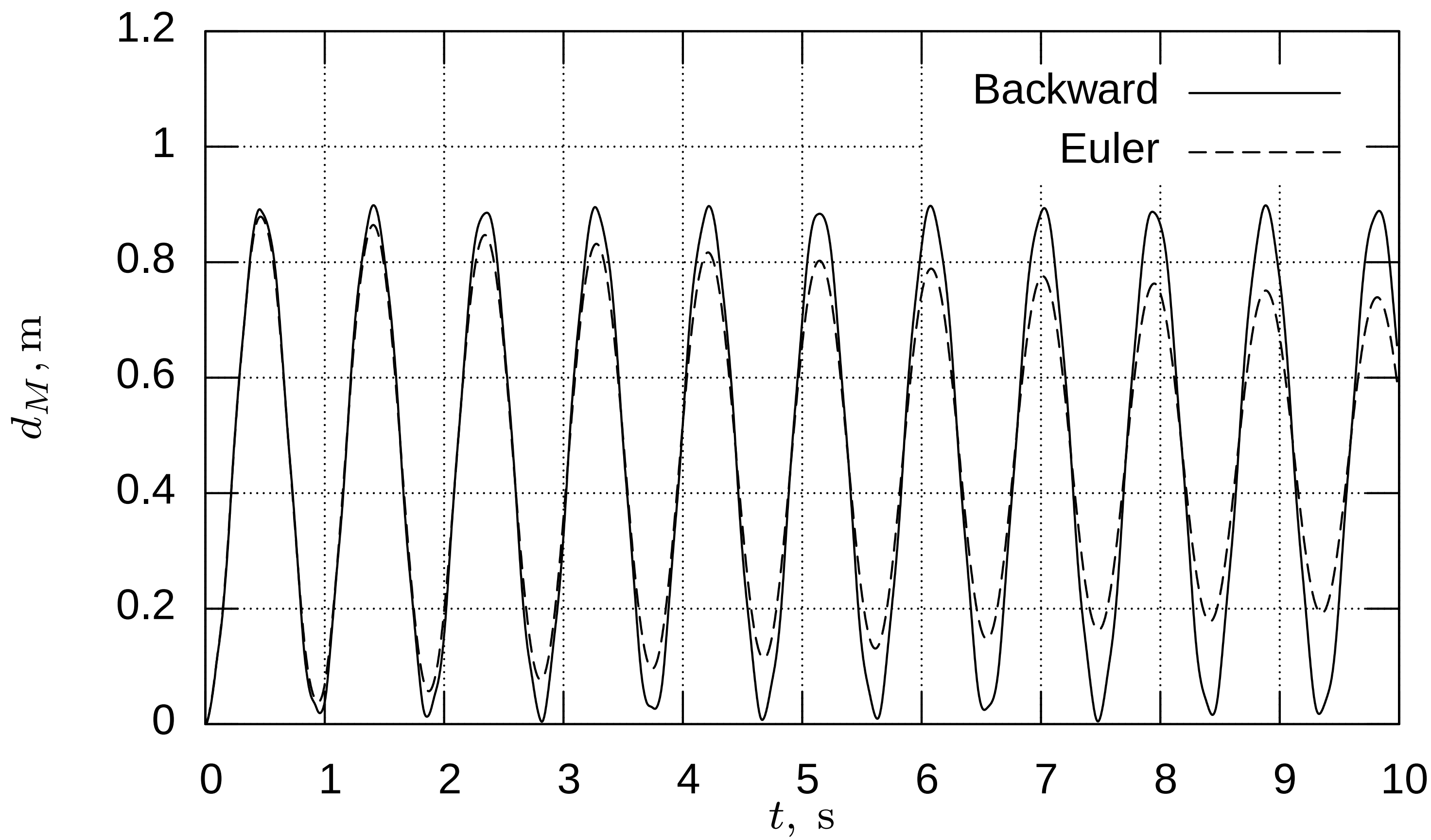
$\mathcal{F}$	d_{Ma}	$d_{M,6\times60}$	$E_{6\times60}$	$d_{M,10\times100}$	$E_{10\times100}$	Accuracy
0.2	0.13272	0.1255	5.44 %	0.1309	1.37 %	2.68
0.4	0.26196	0.2481	5.29 %	0.2581	1.47 %	2.50
0.8	0.49890	0.4797	3.85 %	0.4909	1.60 %	1.71
1.6	0.85882	0.8290	3.47 %	0.8507	0.95 %	1.50



Vibration of a 3-D Beam

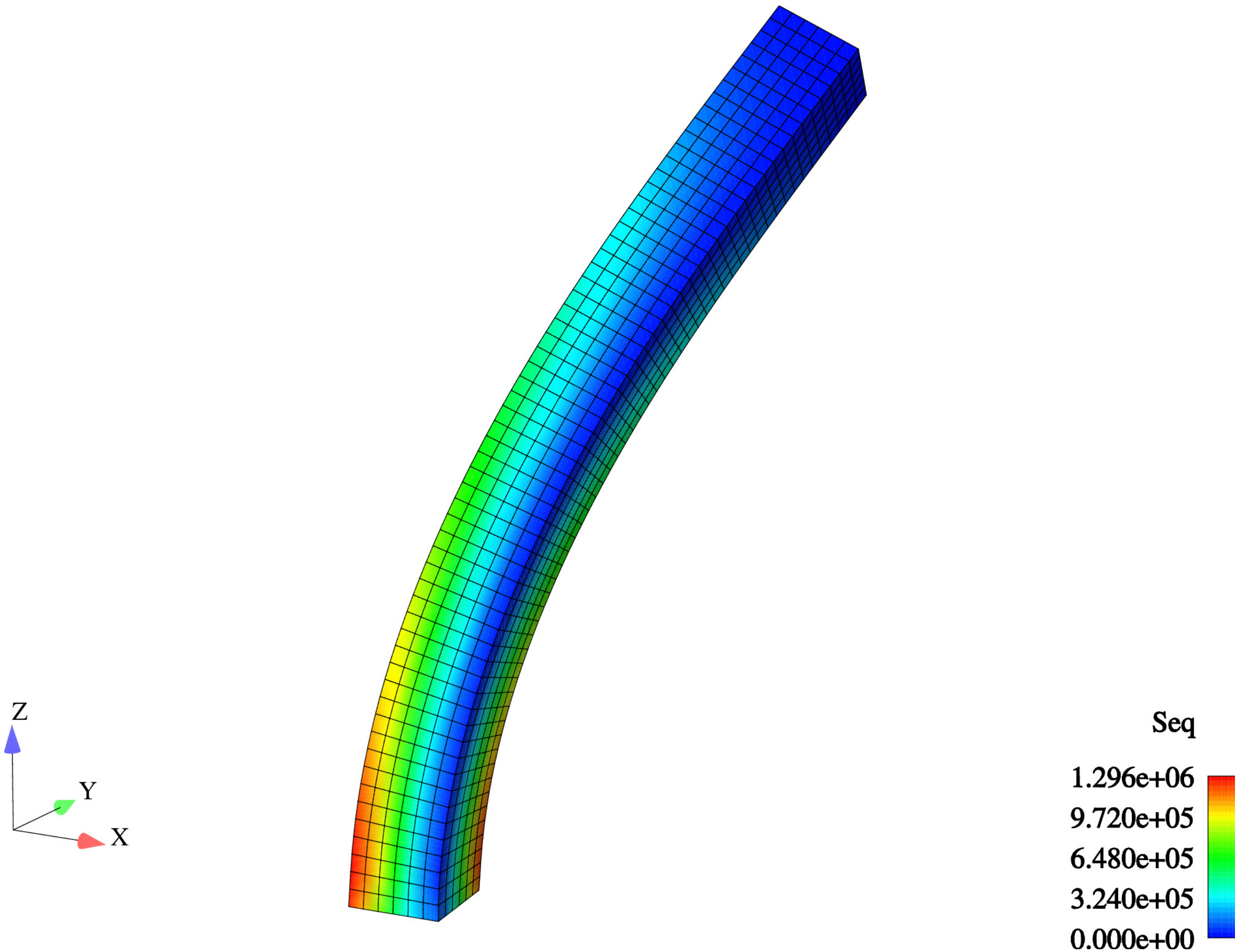
Dynamic Solution

- Tip deflection as a function of time for the Backward and Euler temporal discretisation scheme
- Time step size $\delta t = 0.002$ s corresponds to the wave $Co = 6$



Vibration of a 3-D Beam

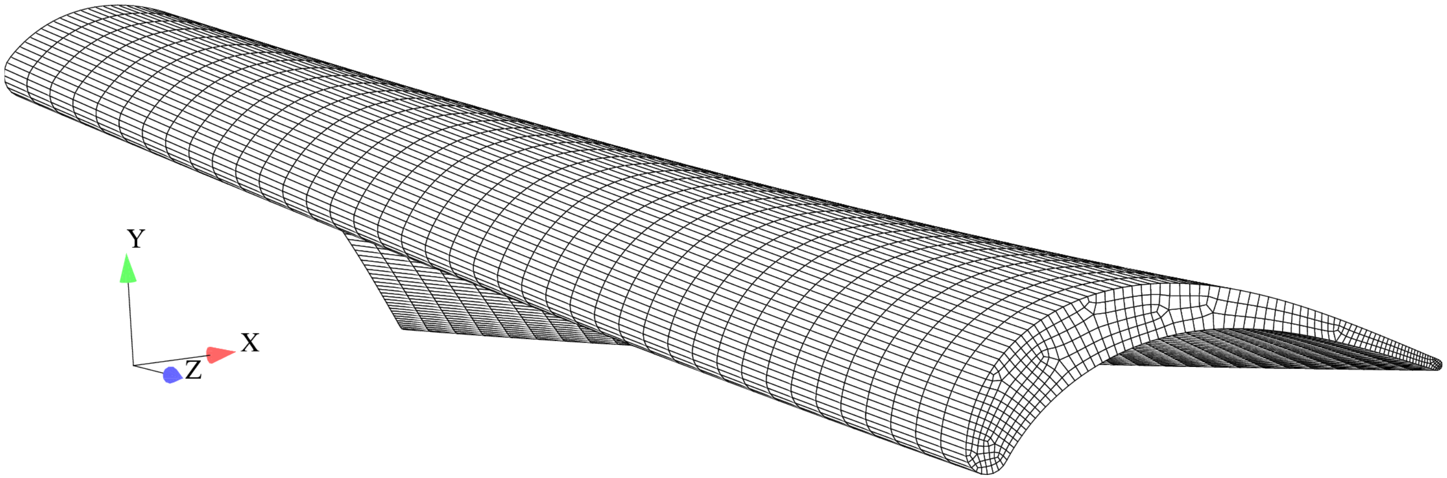
Beam Vibration: Linear Elastic Material with Large Deformations



Vibration of a Turbine Blade

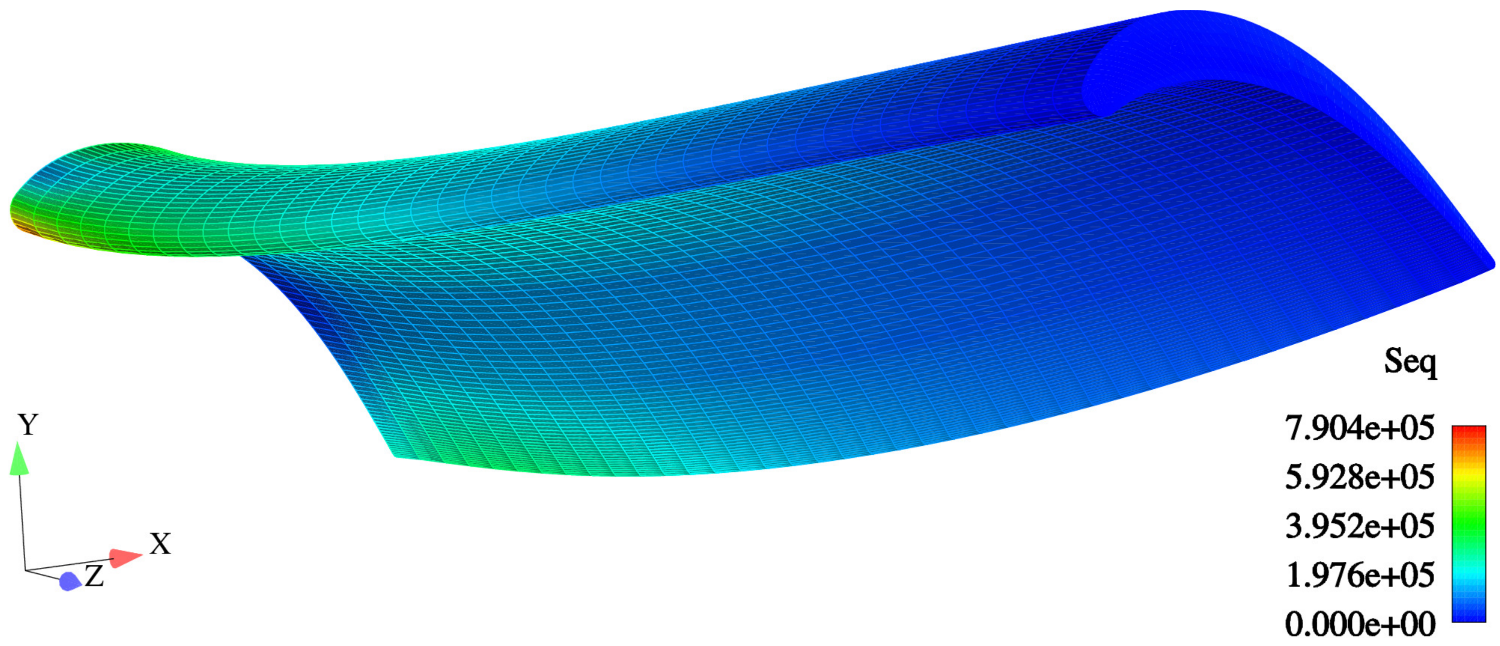
Vibration on an Unstructured Mesh

- Twisted axial turbine blade, $0.8\ \mathrm{m}$ in height, $0.2\ \mathrm{m}$ cord
- Mechanical properties are the same as of the 3-D square beam
- The unstructured hexahedral mesh consists of 50000 cells
- The blade is constrained at one end and subjected to a suddenly applied traction force at the other end



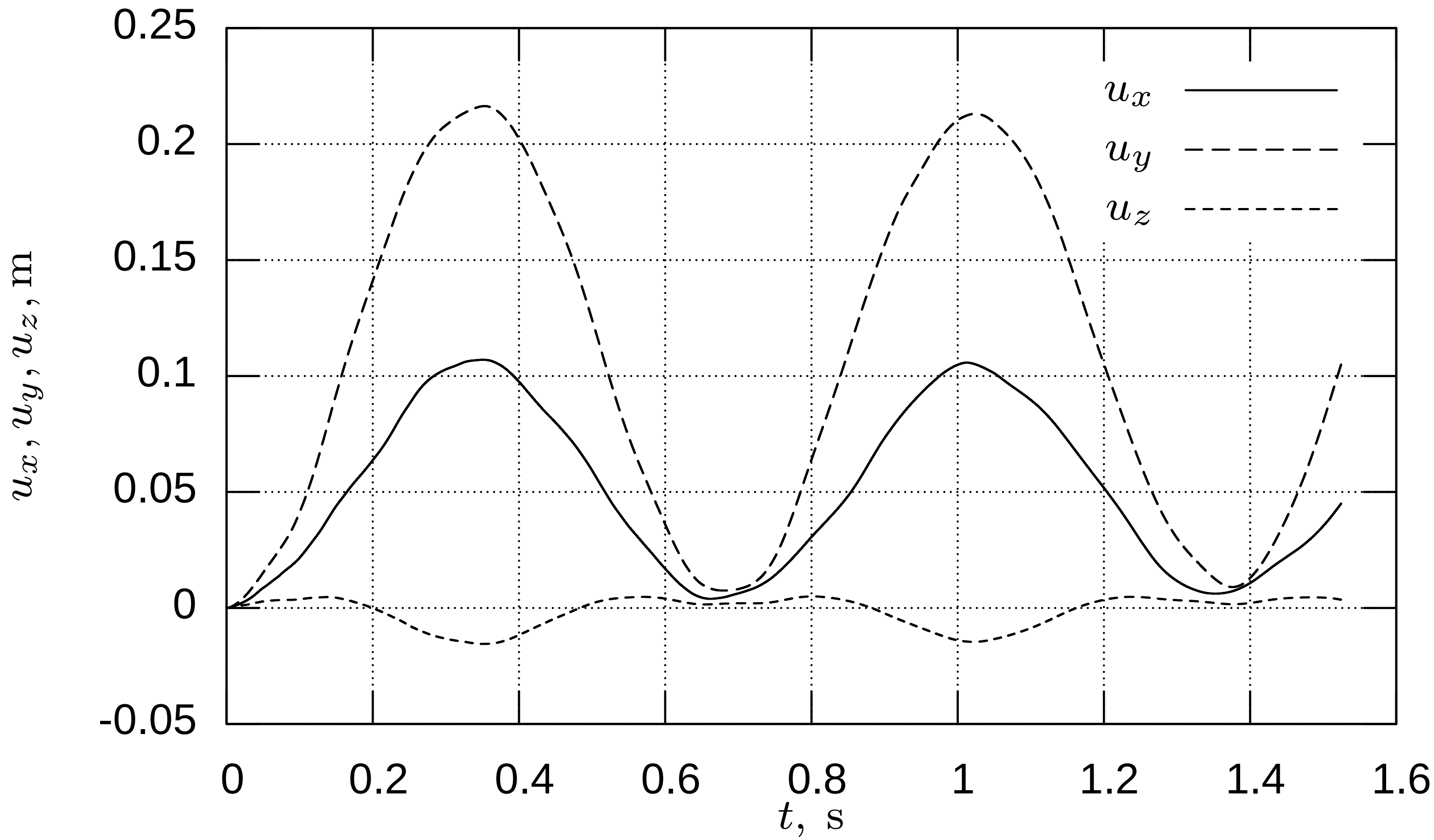
Vibration of a Turbine Blade

Mean Equilibrium State



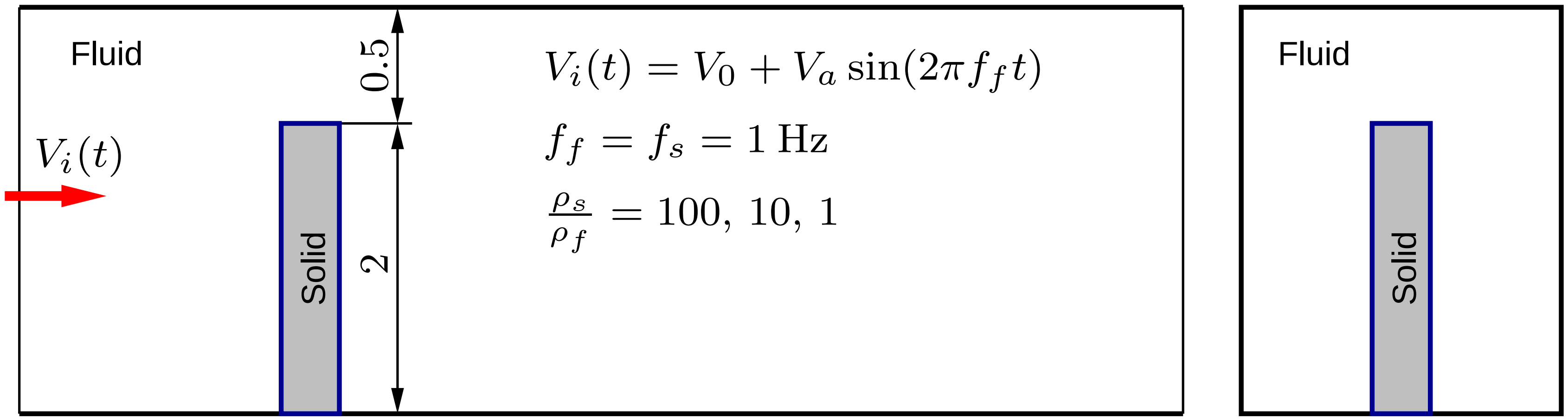
Vibration of a Turbine Blade

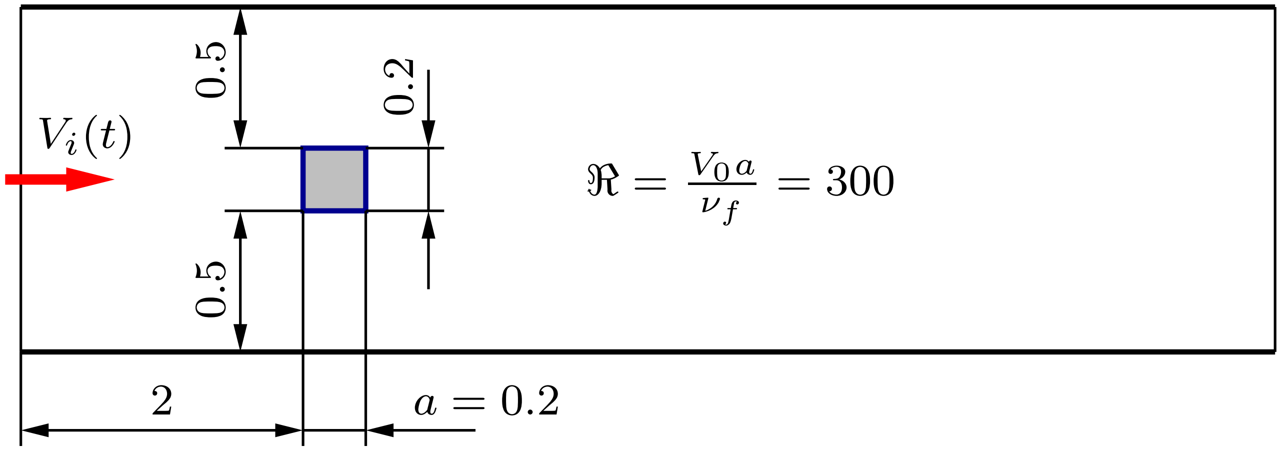
Dynamic Response: displacement components of the top of leading edge



Flow-Induced Vibration

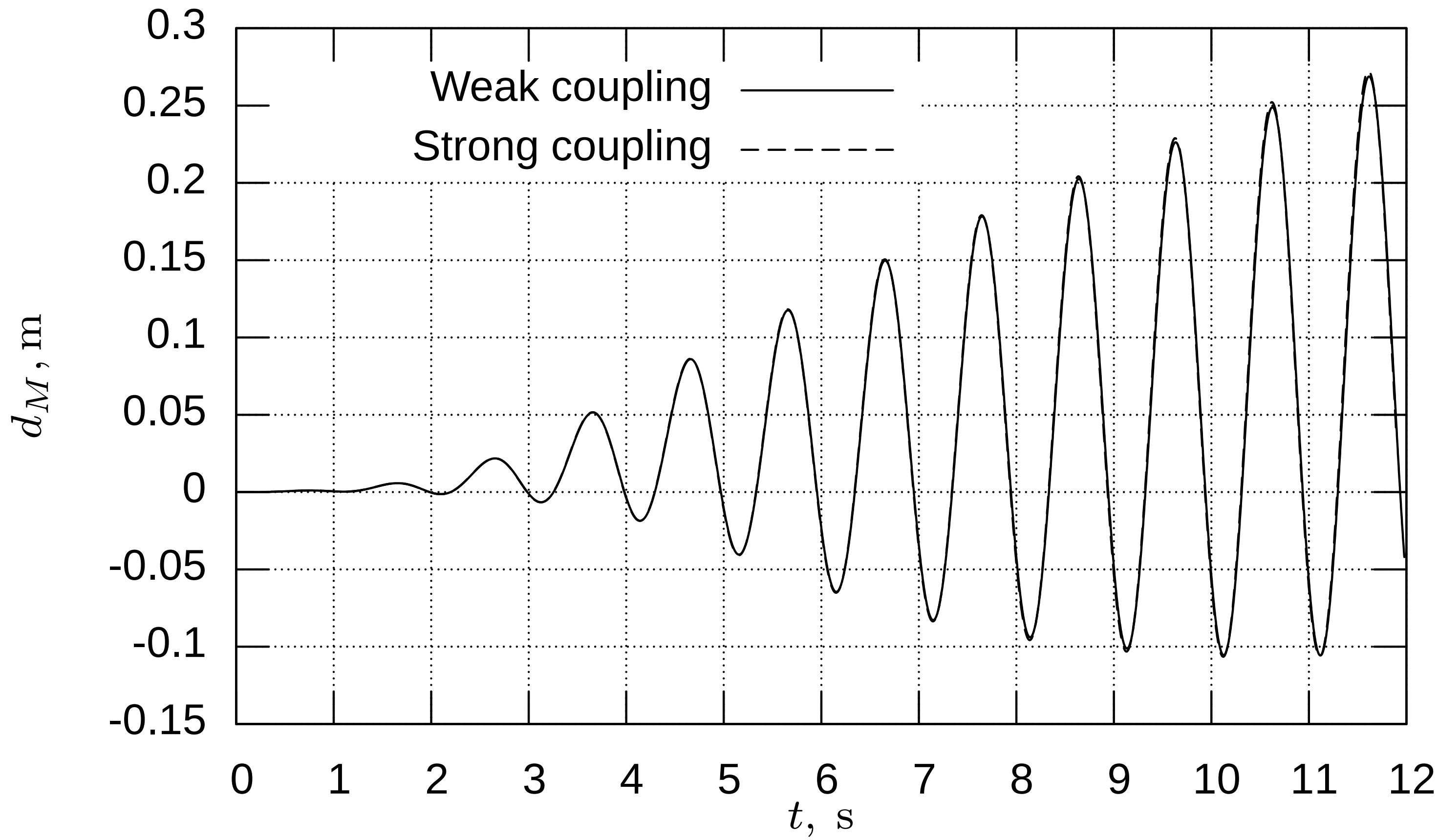
Flow-Induced Vibration: Problem Description





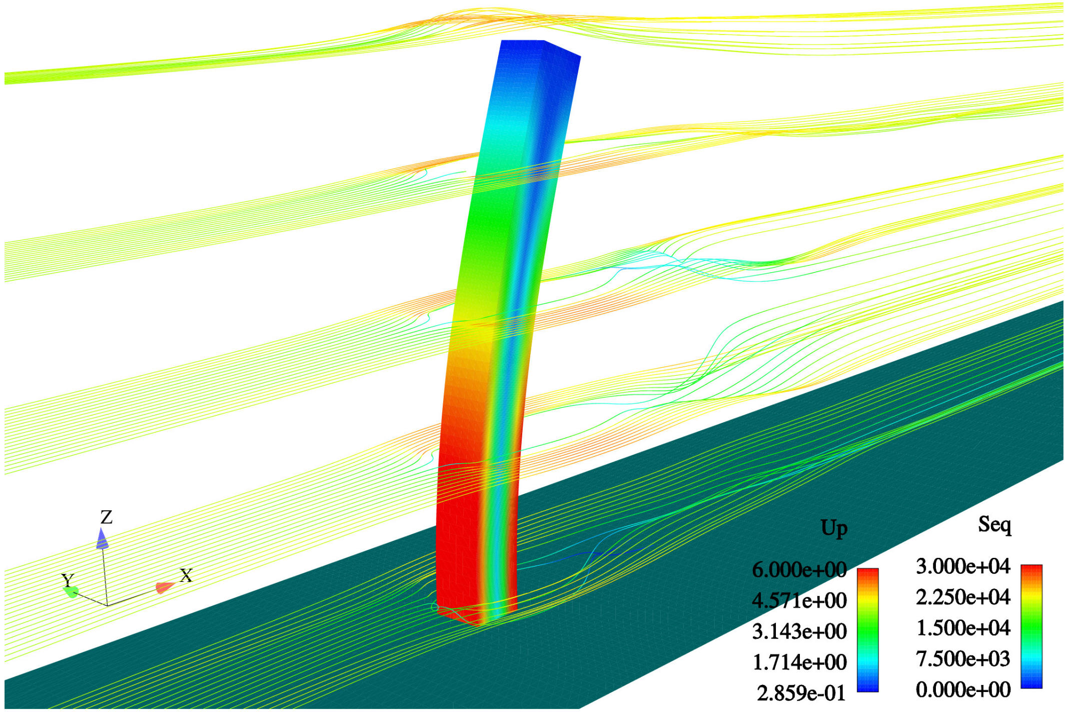
Flow-Induced Vibration

Flow-Induced Vibration: Solution for Limiting Density Ratio $\rho_s/\rho_f = 100$



Flow-Induced Vibration

Cantilevered Beam Vibration in an Oscillating Flow Field



Effect of Under-Relaxation

	Number of outer iterations per time-step	
ρ_s/ρ_f	Fixed under-relaxation	Adaptive under-relaxation
10	30	6
1	60	12

Summary and Future Work

Summary

- Self-contained FVM FSI solver implemented in OpenFOAM
  - Fluid flow solver with run-time selection of physics
  - Structural analysis: updated Lagrangian formulation, non-linear materials
  - OpenFOAM provides surface coupling utilities: patch-to-patch interpolation
- OpenFOAM is a good platform for a close-coupled FSI: extensive capabilities, easy physics model implementation, code sharing, mapping tools
- The solver shows reasonable efficiency for weak fluid-structure interaction
- Strong coupling algorithm with Aitken adaptive under-relaxation shows considerable improvement in comparison with fixed under-relaxation

Future Work

- Development of a coupled solver in OpenFOAM (work in progress) will considerably improve the efficiency of structural and FSI solver: **shared matrix classes** and block-matrix solution algorithm
- Further convergence acceleration for simulation of strong FSI can be achieved by using **Reduced Order Model (ROM)** for fluid and/or solid: a POD-based ROM will be used to improve convergence of the strong coupling algorithm
- Inclusion of **contact stresses and crack propagation** models for structures