OpenFOAM course - Lagrangian particle interaction

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A brief introduction to Lagrangian particle tracking

There are four levels of particle tracking or coupling between the continous and the particle phase in discrete element modelling. In one way coupling, the continous pahse is not affected by the particles. The flow field is solved before the particles is let into the flow and tracked as they go. The particles does not know of any other particles in the flow.
For two way coupling, the fluid flow is solved together with the movement for Lagrangian particles. The particle influence on the continous phase is taken into account. Both through the momentum transfer between the phases and the volume fraction of the particles.
As more knowledge is needed a third ond fourth way coupling is considered. In four way coupling, particle- particle interaction is also taken into account. In the less used three way coupling, the particles ar interacting without collisions. To move futher, by e.g. resolve the flow field around eachindividual particle, would be going into DNS.

Governing equation

The governing equation for the particles is Newtons second law.

$$m_p \frac{d^2 x_i}{dt^2} = \sum F_i \tag{1}$$

The force vector is a matter of choice. The level of detail in the vector can be large by adding many forces or simple by choosing the largest forces. The largest forces to account for are depending on the specific case but the drag-, gravity-, and bouyancy forces are important in many cases. Other forces may be the Basset force (accounting for particle history), the Saffmann force (Velocity gradients in the main flow) and the Magnus lift force (particle rotating). The particles also give rise to an extra source term in the Navier- Stokes equation that has to be included when solving the continous phase.

Particle collisions

Collisions between particles can be treated in different ways. One must choose between a hard and a soft spere approach. The hard sphere approach is more suited for collision dominated dilute flows and the soft spere approach is better for contact dominated dense flows.
The hard sphere approach is the more simple of the two, the collisions are instantaneous and is simple to calculate through the conservation of momentum before and after collision except

for losses. The losses are calculated with the aid of two constants, e the coefficient of normal restitution and ζ the coefficient of tangential restitution.

$$\mathbf{n} \cdot \mathbf{v'}_{12} = -e\mathbf{n} \cdot \mathbf{v}_{12} \tag{2}$$

$$\mathbf{n} \times \mathbf{v'}_{12} = -\zeta\mathbf{n} \times \mathbf{v}_{12} \tag{3}$$

$\mathbf{v'}_{12}$ is the relative speed between collision partners after collision, $\mathbf{v}_{12}$ is the relative speed before the collision and $\mathbf{n}$ is the normal vector from the contact surface. The particles may also stick to or slide against each other depending on the relation between the tangential and the normal components of the collision, $\frac{\mathbf{n}\times\mathbf{v}}{\mathbf{n}\cdot\mathbf{v}}$ and the Coulomb friction, μ.

The soft sphere approach to collision modelling is less straight forward. The collision between particles must be allowed to last a number of timesteps. In order to model the collision, the deformation of the particles and the contact between the particles must be taken into account. In this project, only the hard sphere model is implemented and the soft sphere model will not be discussed further.

Numerical modelling of hard sphere collisions

In this section, the governing equations of the hard sphere collisoin model are presented.
First, the possibility of collision within the next time step has to be calculated. This can be done by deterministic or by a stochastic method, here the deterministic method is presented. The possibility can be expressed by the following equation.

$$|\mathbf{n}_t + k(\mathbf{n}_{t+dt} - \mathbf{n}_t|^2 = (r_1 + r_2)^2 \tag{4}$$

Collision occurs if 0 < k < 1. $\mathbf{n}_t$ denotes the relative position at time t and $\mathbf{n}_{t+dt}$ denotes the relative position at time t + dt.
The particles are assumed to slide against each other, or in other words, $\mu < \frac{\mathbf{n}\times\mathbf{v}}{\mathbf{n}\cdot\mathbf{v}}$. The post collision velocities are expressed as:

$$\mathbf{v}_i\prime = \mathbf{v}_i - \{(1+e)(\mathbf{n} \cdot \mathbf{v}_{ij})\mathbf{n} + \frac{2}{7}|\mathbf{v}_{ct}|\mathbf{t}\}\frac{m_j}{m_i + m_j} \tag{5}$$

$$\omega_i\prime = \omega_i - \frac{5}{7r_i}|\mathbf{v}_{ct}|(\mathbf{n} \times \mathbf{t})\frac{m_j}{m_i + m_j} \tag{6}$$

$\mathbf{v}_{ct}$ is the relative tangential velocity at the point of contact. The equations are from Crowe, Sommerfeld and Tsuji (1998).

The solidParticleCloud class in OpenFOAM

The solidParticleCloud class in OpenFOAM is a class that calculates the movement of particles.

Particle properties

The particles are assumed to be rigid and spherical and are only described by their constants density, coefficient of restitution, coefficient of friction and diameter.

The solver only solves for the particle position and velocity. Above all, a particle rotation would improve the physics of the flow.

Particle forces

The forces acting on the particles are the drag, gravity and bouyancy force. The drag force is given by the expression

$$F_D = \frac{24\nu_c}{d}\frac{3\rho_c}{4d\rho_p}(1 + 0.15Re_p^{0.687}) \tag{7}$$

The drag coefficient is dependent on the Reynolds number and this correlation gives a good correspondance for Re $<$ 800.