A Coupled Pressure Based Solution Algorithm Based on the Volume-of-Fluid Approach for Two or More Immiscible Fluids

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Abstract

The current study deals with the problem of describing multiphase systems of N phases (N > 2), where neither of the single phases can be regarded as dominant. In the standard segregated approach using an iterativ scheme with sucessive solutions of the transport equations for each phase, the phase for which the corresponding transport equation is solved first is dominating the other phases. This is a result of the requirement to preserve boundedness of the volume fraction values. The investigation is based on the Volume-of-Fluid approach to model the behavior of immiscible fluids with interfaces. The surface tension is modeled with a Continuous Surface Force Approach [1].

In the present study a coupled solution approach of the equations to describe the transport of the phases and the pressure equation is presented. The pressure equation

$$\left[\nabla \cdot \left(\left(\frac{1}{\mathcal{A}_D}\right)_f \nabla\left[p^*\right]\right)\right] = \nabla \cdot \phi^*$$

is derived in the usual manner (see [3] for details) by inserting the flux corrector and flux predictor

$$\phi = \phi^* - \left(\frac{1}{\mathcal{A}_D}\right) |\mathbf{S}| \nabla_f^{\perp} p^* \qquad \phi^* = \left(\frac{\mathcal{A}_H}{\mathcal{A}_D}\right)_f \cdot \mathbf{S} - \left(\frac{1}{\mathcal{A}_D}\right)_f (\mathbf{f}\cdot\mathbf{x})_f |\mathbf{S}| \nabla_f^{\perp}\rho + \sum_{i=1}^{N}\sum_{k=i+1}^{N}\left(\left(\frac{1}{\mathcal{A}_D}\right)_f (\sigma_{ik}\kappa_{ik})_f |\mathbf{S}| \left(\nabla_f^{\perp}\alpha\right)_{ik}\right)$$

into the continuity equation. To derrive the equation to model the transport of the phases, flux corrector and predictor are inserted in the face based formulation of the transport equations of the phases, namely

$$\left[\frac{\partial\left[\alpha_i\right]}{\partial t}\right] + \left[\nabla \cdot \left(\phi\left[\alpha_i\right]_f\right)\right] + \left[\nabla \cdot \left(\left[\alpha_i\right]_f \sum_{k=1,k\neq i}^{N} \alpha_{k,f}\phi_{r,ik}\right)\right] = 0$$

resulting in the following equation

$$\left[\frac{\partial\left[\alpha_i\right]}{\partial t}\right] + \left[\nabla \cdot \left(\left(\left(\frac{\mathcal{A}_H}{\mathcal{A}_D}\right)_f \cdot \mathbf{S} + \sum_{i=1}^{N}\sum_{k=i+1}^{N}\left(\left(\frac{1}{\mathcal{A}_D}\right)_f (\sigma_{ik}\kappa_{ik})_f |\mathbf{S}| \left(\nabla_f^{\perp}\alpha\right)_{ik}\right) - \left(\frac{1}{\mathcal{A}_D}\right)_f (\mathbf{f}\cdot\mathbf{x})_f |\mathbf{S}| \nabla_f^{\perp}\rho\right)\left[\alpha_i\right]_f\right)\right]$$
$$- \left[\nabla \cdot \left(\left(\frac{1}{\mathcal{A}_D}\right)_f \nabla\left[p^*\right]\alpha_{i,f}\right)\right] + \left[\nabla \cdot \left(\left[\alpha_i\right]_f \sum_{k=1,k\neq i}^{N} \alpha_{k,f}\phi_{r,ik}\right)\right] = 0$$

The nonlinear terms are linearized and implicit couplings between pressure and volumetric phase fractions are introduced. The terms are distributed into the block matrix structure of Jasak [2] and solved with a suitable solver adapted to block matrices [2].

The solution algorithm is presented and simulation results will be discussed, which show the advantage of this new coupled solution approach compared to the segregated approach.

Key words: **Multiphase flow, Volume-of-Fluid, coupled solution approach**

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